Pre-Calc
5.1 – 5.3 Test Review

Name KEY
Date 5/2/14 **Period** ___

Verify the identity.

1.) $(\sec^2 x - 1)(\cos^2 x) = \sin^2 x$

$= (\tan^2 x)(\cos^2 x)$

$= \frac{\sin^2 x}{\cos^2 x} \cdot \frac{\cos^2 x}{1}$

$= \sin^2 x$ ✓

2.) $\cot^2 x \csc^2 x - \cot^2 x = \cot^4 x$

$= \cot^2 x (\csc^2 x - 1)$

$= \cot^2 x (\cot^2 x)$

$= \cot^4 x$ ✓

3.) $\sec^2 x \csc^2 x = \sec^2 x + \csc^2 x$

$= \sec^2 x (1 + \cot^2 x)$

$\frac{1}{\cos^2 x} + \frac{1}{\cos^2 x} \cdot \frac{\cos^2 x}{\sin^2 x}$

$= \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}$

$= \sec^2 x + \csc^2 x$ ✓

4.) $\frac{\sec x}{\sin x} - \frac{\sin x}{\cos x} = \cot x$

$= \frac{\sec x \cos x - \sin^2 x}{\sin x \cos x}$

$= \frac{\frac{1}{\cos x} \cdot \frac{\cos x}{1} - \sin^2 x}{\sin x \cos x} = \frac{1 - \sin^2 x}{\sin x \cos x}$

$= \frac{\cos^2 x}{\sin x \cos x} = \frac{\cos x}{\sin x} = \cot x$ ✓

5.) $\frac{\cos x}{1 - \sin x} + \tan x = \sec x$

$= \frac{\cos x}{1 + \sin x} + \frac{\sin x}{\cos x}$

$= \frac{\cos^2 x + \sin x (1 + \sin x)}{\cos x (1 + \sin x)}$

$\frac{\cos^2 x + \sin x + \sin^2 x}{LCD}$

$\frac{\sin^2 x + \cos^2 x + \sin x}{LCD} = \frac{1 + \sin x}{\cos x (1 + \sin x)}$

$= \frac{1}{\cos x} = \sec x$ ✓

6.) $\frac{\csc(-x)}{\sec(-x)} = -\cot x$

$\frac{-\csc x}{\sec x} = \frac{-\frac{1}{\sin x}}{\frac{1}{\cos x}}$

$= -\frac{1}{\sin x} \cdot \frac{\cos x}{1} = \frac{-\cos x}{\sin x}$

$= -\cot x$ ✓

y the identity.

7.) $\dfrac{\sin x}{1-\cos x} = \dfrac{1+\cos x}{\sin x}$

$\frac{\sin x}{1-\cos x} \cdot \frac{1+\cos x}{1+\cos x} = \frac{\sin x + \sin x\cos x}{1-\cos^2 x}$

$= \frac{\sin x(1+\cos x)}{\sin^2 x} = \frac{1+\cos x}{\sin x}$ ✓

8.) $\csc^4 x - \cot^4 x = 2\csc^2 x - 1$

$= (\csc^2 x - \cot^2 x)(\csc^2 x + \cot^2 x)$

$= (1)(\csc^2 x + \cot^2 x)$

$= \csc^2 x + \csc^2 x - 1$

$= 2\csc^2 x - 1$ ✓

Find all solutions of the equation in the interval $[0, 2\pi)$ **.**

(37)
9.) $2\sin x - 1 = 0$

$2\sin x = 1$

$\sin x = 1/2$

$[0,2\pi)$: $\frac{\pi}{6}, \frac{5\pi}{6}$

$x = \frac{\pi}{6} + 2\pi n$

$= \frac{5\pi}{6} + 2\pi n$

(39)
10.) $\sin x = \sqrt{3} - \sin x$

$\frac{2\sin x}{2} = \frac{\sqrt{3}}{2}$

$\sin x = \sqrt{3}/2$

$x = \frac{\pi}{3} + 2\pi n$

$= \frac{2\pi}{3} + 2\pi n$

$[0, 2\pi)$: $\frac{\pi}{3}, \frac{2\pi}{3}$

(38)
11.) $\tan x + 1 = 0$

$\tan x = -1$

$x = \frac{3\pi}{4} + \pi n$

$= \frac{7\pi}{4} + \pi n$

$[0, 2\pi)$: $\frac{3\pi}{4}, \frac{7\pi}{4}$

(40)
12.) $\frac{1}{2}\sec x - 1 = 0$

$\frac{1}{2}\sec x = 1$

$\sec x = 2$

$\cos x = 1/2$

$x = \frac{\pi}{3} + 2\pi n$

$\frac{5\pi}{3} + 2\pi n$

$[0, 2\pi)$: $\frac{\pi}{3}, \frac{5\pi}{3}$

5 HW
13.) $\sec^4 x - 3\sec^2 x - 4 = 0$

$(\sec^2 x - 4)(\sec^2 x + 1) = 0$

$\sec^2 x = 4$ ~~$\sec^2 x = -1$~~

$\sec x = \pm 2$ ~~$\sec x =$~~

$\cos x = \pm 1/2$

$\frac{\pi}{3} + 2\pi n$

$\frac{2\pi}{3} + 2\pi n$

$\frac{4\pi}{3} + 2\pi n$

$\frac{5\pi}{3} + 2\pi n$

$[0, 2\pi)$: $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

(51)
14.) $\cos^2 x + \sin x = 1$

$(1 - \sin^2 x) + \sin x = 1$

$-\sin^2 x + \sin x + 1 = 1$

$\sin^2 x - \sin x = 0$

$\sin x(\sin x - 1) = 0$

$\sin x = 0$ $\sin x = 1$

$x = 0 + 2\pi n$ $x = \frac{\pi}{2} + 2\pi n$

$\pi + 2\pi n$

$[0, 2\pi)$: $0, \frac{\pi}{2}, \pi$

all solutions of the equation in the interval $[0, 2\pi)$.

(53)

5.) $2\sin 2x - \sqrt{2} = 0$

$\sin 2x = \frac{\sqrt{2}}{2}$

$\frac{2x}{2} = \frac{\frac{\pi}{4} + 2\pi n}{2}$

$\frac{3\pi}{4} + 2\pi n$ / 2

$[0, 2\pi)$:

$\frac{\pi}{8}, \frac{9\pi}{8}, \frac{3\pi}{8}, \frac{11\pi}{8}$

$x = \frac{\pi}{8} + \pi n$

$\frac{3\pi}{8} + \pi n$

(54)

16.) $\sqrt{3}\tan 3x = 0$

$\tan 3x = 0$

$\frac{3x}{3} = \frac{0 + \pi n}{3}$

$x = \frac{\pi}{3} n$

$[0, 2\pi)$: $0, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{3\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

#42 (5.3)

17.) $\tan^2 3x = 3$

$\tan 3x = \pm\sqrt{3}$

$\frac{3x}{3} = \frac{\frac{\pi}{3} + \pi n}{3}$

$\frac{2\pi}{3} + \pi n$

$\frac{4\pi}{3} + \pi n$

$\frac{5\pi}{3} + \pi n$

$[0, 2\pi)$:

$\frac{\pi}{9}, \frac{4\pi}{9}, \frac{7\pi}{9}, \frac{10\pi}{9}, \frac{13\pi}{9}, \frac{16\pi}{9}, \frac{2\pi}{9}, \frac{5\pi}{9}, \frac{8\pi}{9}, \frac{11\pi}{9}, \frac{14\pi}{9}, \frac{17\pi}{9}$

$x = \frac{\pi}{9} + \frac{\pi}{3} n, \frac{2\pi}{9} + \frac{\pi n}{3}, \frac{4\pi}{9} + \frac{\pi}{3} n, \frac{5\pi}{9} + \frac{\pi}{3} n$

(61)

18.) $\tan^2 x + \tan x - 12 = 0$

$(\tan x + 4)(\tan x - 3) = 0$

$\tan x = -4$ $\quad \tan x = 3$

$x = -1.3258$ $\quad x = 1.2490$

$[0, 2\pi)$: $4.9574, 1.8158, 1.2490, 4.3906$

#40 5.3

19.) $\cos^2 x - 2\cos x - 1 = 0$

$x = \frac{2 \pm \sqrt{4 - 4(1)(-1)}}{2}$

$= \frac{2 \pm \sqrt{8}}{2} = 1 \pm \sqrt{2}$

$\cos x = 1 + \sqrt{2}$ $\quad \cos x = 1 - \sqrt{2}$

-1.414

$x = 1.9979$

$\cos(x) = -1.414$

$x =$ ____

$x = 1.9979, 4.2853$

(62)

20.) $\sec^2 x + 6\tan x + 4 = 0$

$1 + \tan^2 x + 6\tan x + 4 = 0$

$\tan^2 x + 6\tan x + 5 = 0$

$(\tan x + 5)(\tan x + 1) = 0$

$\tan x = -5$ $\quad \tan x = -1$

$x = -1.3734$ $\quad x = \frac{3\pi}{4} + \pi n$

$x = 4.9098, 1.7682$

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$