Pre-Calc
5.4 – 5.5 Test Review

Name Key
Date ___ Period ___

Find the exact value.

1.) $\cos 25^\circ \cos 20^\circ - \sin 25^\circ \sin 20^\circ$

$\cos 45$ $\frac{\sqrt{2}}{2}$

2.) $\cos\frac{\pi}{16}\cos\frac{3\pi}{16} - \sin\frac{\pi}{16}\sin\frac{3\pi}{16}$

$\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

Find the exact value of the sine, cosine, and tangent of the angle by using the sum or difference formulas.

3.) 285° $= \cancel{\cos}(240+45) = \cancel{\cos}$

$\sin(240+45) = \sin 240 \cos 45 + \cos 240 \sin 45$

$= \frac{-\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{-\sqrt{6}}{4} + \frac{-\sqrt{2}}{4} = \frac{-\sqrt{6}-\sqrt{2}}{4}$

$\cos(240+45) = \cos 240 \cos 45 - \sin 240 \sin 45$

$\left(\frac{-1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{-\sqrt{2}+\sqrt{6}}{4}$

$\tan(240+45) = \frac{\tan 240 + \tan 45}{1 - \tan 240 \tan 45} = \frac{\sqrt{3}+1}{1-\sqrt{3}} \cdot \frac{1+\sqrt{3}}{1+\sqrt{3}} = \frac{\sqrt{3}+1+\sqrt{3}+3}{1-3}$

$= \frac{4+2\sqrt{3}}{-2} = -2-\sqrt{3}$

4.) $\frac{17\pi}{12}$ Hint: $\frac{9\pi}{4} - \frac{5\pi}{6}$

$\sin\frac{17\pi}{12} = \sin\frac{9\pi}{4}\cos\frac{5\pi}{6} - \cos\frac{9\pi}{4}\sin\frac{5\pi}{6}$

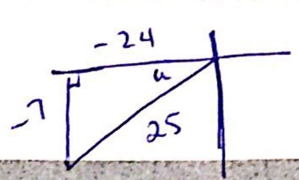
$\frac{\sqrt{2}}{2}\cdot\frac{-\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{-\sqrt{6}-\sqrt{2}}{4}$

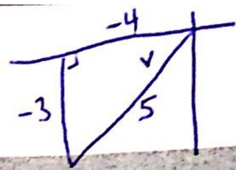
$\cos\frac{17\pi}{12} = \cos\frac{9\pi}{4}\cos\frac{5\pi}{6} + \sin\frac{9\pi}{4}\sin\frac{5\pi}{6}$

$\frac{\sqrt{2}}{2}\cdot\frac{-\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{-\sqrt{6}+\sqrt{2}}{2}$

$\tan\frac{17\pi}{12} = \frac{\tan\frac{9\pi}{4} - \tan\frac{5\pi}{6}}{1+\tan\frac{9\pi}{4}\tan\frac{5\pi}{6}} = \frac{1+\frac{\sqrt{3}}{3}}{1-\frac{\sqrt{3}}{3}} = \frac{3+\sqrt{3}}{3-\sqrt{3}} \cdot \frac{3+\sqrt{3}}{3+\sqrt{3}} = \frac{9+6\sqrt{3}+3}{9-3} = \frac{12+6\sqrt{3}}{6}$

$= 2+\sqrt{3}$





Find the exact value of trigonometric function given that $\sin u = -\frac{7}{25}$ ***and*** $\cos v = -\frac{4}{5}$***. (Both are in Quadrant III.)***

5.) $\cos(u+v)$

$\cos u \cos v - \sin v \sin u$

$\frac{-24}{25}\cdot\frac{-4}{5} - \frac{-3}{5}\cdot\frac{-7}{25}$

$\frac{96}{125} - \frac{21}{125} = \frac{75}{125}$

$= 3/5$

6.) $\cos(u-v)$

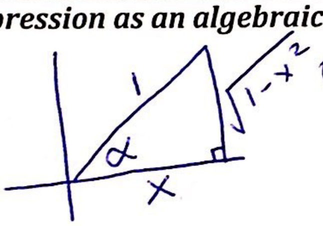
$\cos u \cos v + \sin u \sin v$

$\frac{-24}{25}\cdot\frac{-4}{5} + \frac{-7}{25}\cdot\frac{-3}{5}$

$\frac{96}{125} + \frac{21}{125} = \frac{117}{125}$

Write the trigonometric expression as an algebraic expression.

7.) $\cos(\overset{\alpha}{\arccos x} + \overset{B}{\arcsin x})$

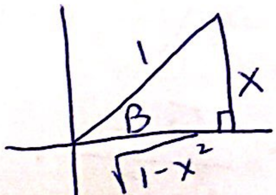


$\cos\alpha\cos B - \sin\alpha\sin B$

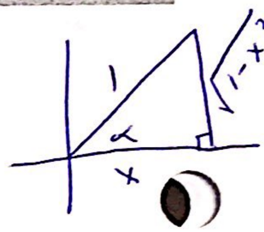
$x\cdot\sqrt{1-x^2} - \sqrt{1-x^2}\,x$

$x\sqrt{1-x^2} - x\sqrt{1-x^2}$

$= 0$



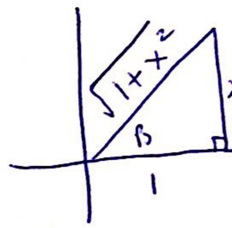
8.) $\cos(\overset{\alpha}{\arccos x} - \overset{B}{\arctan x})$



$\cos\alpha\cos B + \sin\alpha\sin B$

$x\cdot\frac{1}{\sqrt{1+x^2}} + \sqrt{1-x^2}\cdot\frac{x}{\sqrt{1+x^2}}$

$= \frac{x + x\sqrt{1-x^2}}{\sqrt{1+x^2}}$



Simplify the expression.

9.) $\sin\left(\frac{3\pi}{2}+\theta\right)$

$= \sin\frac{3\pi}{2}\cos\theta + \cancel{\cos\frac{3\pi}{2}\sin\theta}$

$= -\cos\theta$

10.) $\tan(\pi+\theta)$

$= \frac{\tan\pi + \tan\theta}{1 - \tan\pi\tan\theta} = \tan\theta$

Find all solutions of the equation in the interval $[0, 2\pi)$.

.) $\sin\left(x+\frac{\pi}{6}\right)-\sin\left(x-\frac{\pi}{6}\right)=\frac{1}{2}$

$\cancel{\sin x\cos\frac{\pi}{6}} + \cos x\sin\frac{\pi}{6} - \left(\cancel{\sin x\cos\frac{\pi}{6}} - \cos x\sin\frac{\pi}{6}\right) = \frac{1}{2}$

$\cos x\sin\frac{\pi}{6} + \cos x\sin\frac{\pi}{6} = \frac{1}{2}$

$2\cos x\sin\frac{\pi}{6} = \frac{1}{2}$

$\cancel{2}\cos x\ \frac{1}{\cancel{2}} = \frac{1}{2}$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

12.) $\tan(x+\pi)-\cos\left(x-\frac{\pi}{2}\right)=0$

$\frac{\tan x + \cancel{\tan\pi}}{1-\cancel{\tan x\tan\pi}} - \left(\cancel{\cos x\cos\frac{\pi}{2}} + \sin x\sin\frac{\pi}{2}\right) = 0$

$\tan x - \sin x = 0$

$\tan x = \sin x$

$\frac{\sin x}{\cos x} = \frac{\sin x}{1}$

$\cos x\sin x = \sin x$

$\cos x\sin x - \sin x = 0$

$\sin x(\cos x - 1) = 0$

$\sin x = 0$ $\qquad \cos x = 1$

$x = 0, \pi$ $\qquad x = 0$

13.) $\sin 2x - \sin x = 0$

$2\sin x\cos x - \sin x = 0$

$\sin x(2\cos x - 1) = 0$

$\sin x = 0$ $\qquad \cos x = 1/2$

$x = 0, \pi$ $\qquad x = \frac{\pi}{3}, \frac{5\pi}{3}$

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Use a double-angle formula to rewrite the expression.

14.) $6\sin x\cos x$

$3(2\sin x\cos x)$

$3\sin 2x$

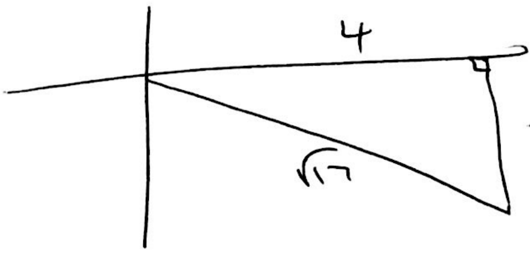
15.) $4-8\sin^2 x$

$4(1-2\sin^2 x)$

$4\cos 2x$

Using double-angle formulas, find the exact values of $\sin 2u$, $\cos 2u$, ***and*** $\tan 2u$.

16.) $\cot u=-4;\quad \frac{3\pi}{2}<u<2\pi$



$\sin 2u = 2\left(\frac{-1}{\sqrt{17}}\right)\left(\frac{4}{\sqrt{17}}\right) = \frac{-8}{17}$

$\cos 2u = \left(\frac{4}{\sqrt{17}}\right)^2 - \left(\frac{-1}{\sqrt{17}}\right)^2 = \frac{15}{17}$

$\tan 2u = \frac{\frac{-8}{17}}{15/17} = -8/15$

Using half-angle formulas, find the exact values of $\sin\frac{u}{2}$, $\cos\frac{u}{2}$, ***and*** $\tan\frac{u}{2}$.

$u = 135^\circ$

17.) $\theta = 67.5^\circ$

$\sin\frac{135}{2} = \sqrt{\frac{1-(-\sqrt{2}/2)}{2}} = \sqrt{\frac{1+\sqrt{2}/2}{2}} = \sqrt{\frac{2+\sqrt{2}}{2}\cdot\frac{1}{2}} = \frac{\sqrt{2+\sqrt{2}}}{2}$

$\cos\frac{135}{2} = \sqrt{\frac{1+-\sqrt{2}/2}{2}} = \sqrt{\frac{1-\sqrt{2}/2}{2}} = \sqrt{\frac{2-\sqrt{2}}{2}\cdot\frac{1}{2}} = \frac{\sqrt{2-\sqrt{2}}}{2}$

$\tan\frac{135}{2} = \frac{\frac{\sqrt{2}}{2}}{1+(-\sqrt{2}/2)} = \frac{\frac{\sqrt{2}}{2}}{1-\frac{\sqrt{2}}{2}} = \frac{\frac{\sqrt{2}}{2}}{\frac{2-\sqrt{2}}{2}} = \frac{\sqrt{2}}{\cancel{2}}\cdot\frac{\cancel{2}}{2-\sqrt{2}}$

$= \frac{\sqrt{2}}{2-\sqrt{2}}\cdot\frac{2+\sqrt{2}}{2+\sqrt{2}} = \frac{2\sqrt{2}+2}{2} = \sqrt{2}+1$

nd all solutions of the equation in the interval $[0, 2\pi)$.

18.) $\sin\frac{x}{2} + \cos x = 0$

$$\sqrt{\frac{1-\cos x}{2}}^{\,2} = (-\cos x)^2$$

$$\frac{1-\cos x}{2} = \cos^2 x$$

$$2\cos^2 x + \cos x - 1 = 0$$

$$(2\cos x - 1)(\cos x + 1) = 0$$

$\cos x = 1/2$ $\quad$ $\cos x = -1$

$x = \cancel{\frac{\pi}{3}}, \cancel{\frac{5\pi}{3}}, \pi$

Use the product-to-sum formulas to write each product as a sum or difference.

19.) $\cos 4\theta \sin 6\theta$

$$\frac{1}{2}\left[\sin(10\theta) - \sin(-2\theta)\right]$$

$$= \frac{1}{2}\left[\sin 10\theta + \sin 2\theta\right]$$

$$= \frac{1}{2}\left(\sin 10\theta + \sin 2\theta\right)$$

Use the sum-to-product formulas to write each sum or difference as a product.

20.) $\sin(x + y) - \sin(x - y)$

$$2\cos\left(\frac{x+y+x-y}{2}\right)\sin\left(\frac{x+y-(x-y)}{2}\right)$$

$$2\cos(x)\sin(y)$$

Find all solutions of the equation in the interval $[0, 2\pi)$.

21.) $\cos 4x + \cos 6x = 0$

$2\cos 5x \cos(-x) = 0$

$2\cos 5x \cos x = 0$

$\cos 5x = 0$

$\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\frac{5x}{5} = \frac{\frac{\pi}{2} + 2\pi n}{5}$

$x = \frac{\pi}{10} + \frac{2\pi}{5}n$

$x = \frac{\pi}{10}, \frac{5\pi}{10}, \frac{9\pi}{10}, \frac{13\pi}{10}, \frac{17\pi}{10}$

$\frac{3\pi}{10}, \frac{7\pi}{10}, \frac{11\pi}{10}, \frac{15\pi}{10}, \frac{19\pi}{10}$

$5x = \frac{\frac{3\pi}{2}}{5} + \frac{2\pi n}{5}$

$x = \frac{3\pi}{10} + \frac{2\pi}{5}n$

22.) $\sin x + \sin 3x = 0$

$= 2\sin(2x)\cos(x) = 0$

$\sin 2x = 0$

$\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\frac{2x}{2} = \frac{0 + 2\pi n}{2}$

$x = \pi n$

$\frac{2x}{2} = \frac{\pi + 2\pi n}{2}$

$x = \frac{\pi}{2} + \pi n$

$x = 0, \pi, \frac{\pi}{2}, \frac{3\pi}{2}$