$(x+\sqrt{3}y)(x+\sqrt{3}y)$

⑤ $7x^2 - 6\sqrt{3}xy + 13y^2 - 16 = 0$

$\cot 2\theta = \frac{A-C}{B} = \frac{7-13}{-6\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\tan 2\theta = \sqrt{3}$

$2\theta = \pi/3$

$\theta = \pi/6$

$x = x'\cos\pi/6 - y'\sin\pi/6$
$= \sqrt{3}/2\, x' - 1/2\, y'$
$= \frac{1}{2}(\sqrt{3}x' - y')$

$y = x'\sin\pi/6 + y'\cos\pi/6$
$= \frac{1}{2}x' + \sqrt{3}/2\, y'$
$= \frac{1}{2}(x' + \sqrt{3}y')$

$7\left(\frac{1}{4}\right)(3x^2 - 2\sqrt{3}xy + y^2) \quad -6\sqrt{3}\left(\frac{1}{4}\right)(\sqrt{3}x^2 + 2xy - \sqrt{3}y^2)$

$+ 13\left(\frac{1}{4}\right)(x^2 + 2\sqrt{3}xy + 3y^2) - 16 = 0$

$21x^2 - 14\sqrt{3}xy + 7y^2 \quad -18x^2 - 12\sqrt{3}xy + 18y^2$

$+ 13x^2 + 26\sqrt{3}xy + 39y^2 = 64$

$\frac{16x^2}{64} + \frac{64y^2}{64} = \frac{64}{64} = \boxed{\frac{(x')^2}{4} + \frac{(y')^2}{1} = 1}$

(6) $23x^2 + 26\sqrt{3}xy - 3y^2 - 144 = 0$

$A = 23$
$B = 26\sqrt{3}$
$C = -3$

$\cot 2\theta = \frac{A-C}{B} = \frac{26}{26\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\theta = \pi/6$

$x = \frac{1}{2}(\sqrt{3}x' - y')$ $\quad y = \frac{1}{2}(x' + \sqrt{3}y')$

(xy) / primes

$23\left(\frac{1}{4}\right)(3x^2 - 2\sqrt{3}xy + y^2) + 26\sqrt{3}\left(\frac{1}{4}\right)(\sqrt{3}x^2 + 2xy - \sqrt{3}y^2)$

$-3\left(\frac{1}{4}\right)(x^2 + 2\sqrt{3}xy + 3y^2) - 144 = 0$

$69x^2 - 46\sqrt{3}xy + 23y^2 + 78x^2 + 52\sqrt{3}xy - 78y^2$

$-3x^2 - 6\sqrt{3}xy - 9y^2 = 576$

$\frac{144x^2}{576} - \frac{64y^2}{576} = \frac{576}{576}$

$\frac{(x')^2}{4} - \frac{(y')^2}{9} = 1$

(7) $3x^2 - 2\sqrt{3}xy + y^2 + 2x + 2\sqrt{3}y = 0$

$A = 3$
$B = -2\sqrt{3}$
$C = 1$

$\cot 2\theta = \frac{A-C}{B} = \frac{2}{-2\sqrt{3}} = -\frac{1}{\sqrt{3}}$

$$\frac{\cot^2\theta - 1}{2\cot\theta} = \frac{-1}{\sqrt{3}}$$

$$\sqrt{3}\cot^2\theta - \sqrt{3} = -2\cot\theta$$

$$(\sqrt{3}\cot\theta - 1)(\cot\theta + \sqrt{3}) = 0$$

$\cot\theta = \frac{1}{\sqrt{3}} \longrightarrow \tan\theta = \sqrt{3}$

$\theta = \pi/3$

$x = \frac{1}{2}x' - \sqrt{3}/2\, y'$

$x = \frac{1}{2}(x' - \sqrt{3}y')$

$y = x'\sqrt{3}/2 + \frac{1}{2}y'$

$y = \frac{1}{2}(\sqrt{3}x + y')$

(+4) primes.

$$3\left(\tfrac{1}{4}\right)(x^2 - 2\sqrt{3}xy + 3y^2) - 2\sqrt{3}\left(\tfrac{1}{4}\right)(\sqrt{3}x^2 - 2xy - \sqrt{3}y^2) + \left(\tfrac{1}{4}\right)(3x^2 + 2\sqrt{3}xy + y^2)$$
$$+ 2\left(\tfrac{1}{2}\right)(x - \sqrt{3}y) + 2\sqrt{3}\left(\tfrac{1}{2}\right)(\sqrt{3}x + y) = 0$$

$$3x^2 - 6\sqrt{3}xy + 9y^2 - 6x^2 + 4\sqrt{3}xy + 6y^2 + 3x^2 + 2\sqrt{3}xy + y^2$$
$$+ 4x - 4\sqrt{3}y + 12x + 4\sqrt{3}y = 0$$

$16y^2 + 4x + 12x = 0$

$16y^2 + 16x = 0$

$y^2 = -x$

$(y')^2 = -x$

(8) $xy - 2y - 4x = 0$

$A = 0$
$B = 1$
$C = 0$

$\cot 2\theta = \frac{A-C}{B} = 0$

$\tan 2\theta = \pi/2$

$\theta = \pi/4$

$x = x'\cos\pi/4 - y'\sin\pi/4$
$= \frac{\sqrt{2}}{2}(x' - y')$

$y = x'\sin\pi/4 + y'\cos\pi/4$
$= \frac{\sqrt{2}}{2}(x' + y')$

(x 2) $\frac{1}{2}(x^2 - y^2) - \sqrt{2}(x+y) - 5\sqrt{2}(x-y) = 0$

$x^2 - y^2 - 2\sqrt{2}x - 2\sqrt{2}y - 4\sqrt{2}x + 4\sqrt{2}y = 0$

$x^2 - 6\sqrt{2}x - y^2 + 2\sqrt{2}y = 0$

$x^2 - 6\sqrt{2}x + 18 - (y^2 - 2\sqrt{2}y + 2) = 18 - 2$

$\frac{(x - 3\sqrt{2})^2}{16} - \frac{(y - \sqrt{2})^2}{16} = \frac{16}{16}$

$= \frac{(x' - 3\sqrt{2})^2}{16} - \frac{(y' - \sqrt{2})^2}{16} = 1$

(10) $2x^2 + \sqrt{3}xy + y^2 - 10 = 0$

$A = 2$
$B = \sqrt{3}$
$C = 1$

$\cot 2\theta = \frac{A-C}{B} = \frac{1}{\sqrt{3}}$ $\quad \tan 2\theta = \pi/3$
$\theta = \pi/6$

$x = \frac{1}{2}(\sqrt{3}x' - y')$ $\quad y = \frac{1}{2}(x' + \sqrt{3}y')$

(x y)

$2\left(\frac{1}{4}\right)(3x^2 - 2\sqrt{3}xy + y^2) + \sqrt{3}\left(\frac{1}{4}\right)(\sqrt{3}x^2 + 2xy - \sqrt{3}y^2)$

$+ \left(\frac{1}{4}\right)(x^2 + 2\sqrt{3}xy + 3y^2) - 10 = 0$

$= 6x^2 - 4\sqrt{3}xy + 2y^2 + 3x^2 + 2\sqrt{3}xy - 3y^2$

$+ x^2 + 2\sqrt{3}xy + 3y^2 = 40$

$\frac{10}{40}x^2 + \frac{2}{40}y^2 = \frac{40}{40}$

$$\frac{(x')^2}{4} + \frac{(y')^2}{20} = 1$$

(12) $x^2 + 4xy - 2y^2 - 1 = 0$

$A = 1$
$B = 4$
$C = -2$

$\cot 2\theta = \frac{A-C}{B} = \frac{1+2}{4} = 3/4$

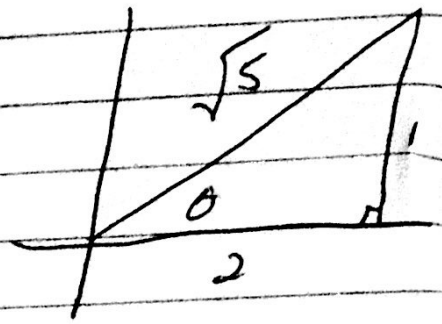
$\frac{\cot^2\theta - 1}{2\cot\theta} = \frac{3}{4}$

$4\cot^2\theta - 4 = 6\cot\theta$

$4\cot^2\theta - 6\cot\theta - 4 = 0$

$(4\cot\theta + 2)(\cot\theta - 2) = 0$

$\cot\theta = 2$



$x = x'(2/\sqrt{5}) - y'(1/\sqrt{5})$
$= \frac{1}{\sqrt{5}}(2x' - y')$

$y = x'(1/\sqrt{5}) + y'(2/\sqrt{5})$
$= \frac{1}{\sqrt{5}}(x' + 2y')$

$\times 5 \quad \left(\frac{1}{5}\right)(4x^2 - 4xy + y^2) + 4\left(\frac{1}{5}\right)(2x^2 + 3xy - 2y^2)$
$-2\left(\frac{1}{5}\right)(x^2 + 4xy + 4y^2) = 1$

$4x^2 - 4xy + y^2 + 8x^2 + 12xy - 8y^2 - 2x^2 - 8xy - 8y^2 = 5$

$\frac{10x^2 - 15y^2}{5} = \frac{5}{5}$

$\frac{(x')^2}{1/2} - \frac{(y')^2}{1/3} = 1$