preCalculus 16/17
Review: Chapter 3

Name Key
Date 4/28/17 Block ______

$y = ab^x$

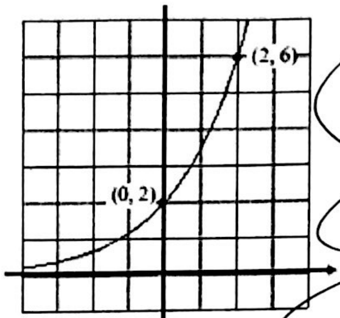
In 1 – 3, write the exponential equation modeled by each table, graph, or description.

1.

x	$f(x)$
-2	20
-1	10
0	5
1	$\frac{5}{2}$
2	$\frac{5}{4}$

$y = 5\left(\frac{1}{2}\right)^x$

2.



$y = 2(3)^{x/2}$

$y = 2(\sqrt{3})^x$

$y = 2(1.732)^x$

3. You buy a new computer for \$2100. The computer decreases by 50% annually.

$y = 2100(.5)^x$

In 4 – 9, evaluate each expression WITHOUT A CALCULATOR.

4. $\log_2 32^7$ $\quad$ 35

5. $\log_{64} 8$ $\quad$ $1/2$

6. $\dfrac{\log_{12} 12^{36}}{\log_4 4^{18}} = \dfrac{36}{18}$ $\quad$ 2

7. $\ln e$ $\quad$ 1

8. $\log_4 320 - \log_4 5$

$\log_4\left(\frac{320}{5}\right)$

$= \log_4 64$ $\quad$ 3

9. $e^{\ln 3xy^2}$ $\quad$ $3xy^2$

10. Use the change of base formula to evaluate:

a. $\log_5 7$ $\quad$ 1.209

b. $\log_{\frac{1}{3}} \frac{1}{5}$ $\quad$ 1.465

c. $\log_3 17$ $\quad$ 2.579

11. Use $\log_a 2 \approx 0.4307$ and $\log_a 3 \approx 0.6826$ to rewrite and evaluate the following expressions.

a. $\log_a\left(\dfrac{2}{3}\right)$

$\log_a 2 - \log_a 3$

$.4307 - .6826$

$-.2519$

b. $\log_a 6$

$\log_a 3 + \log_a 2$

$.6826 + .4307$

1.1133

c. $\log_a 24$

$\log_a (3 \cdot 2 \cdot 2 \cdot 2)$

$.6826$
$+ 3(.4307)$

1.9747

12. Expand each expression.

a. $\log_5 7x^3y$

$\boxed{\log_5 7 + 3\log_5 x + \log_5 y}$

b. $\ln\left(\frac{x^2y^3}{x-y}\right)$

$\boxed{2\ln x + 3\ln y - \ln(x-y)}$

c. $\ln\sqrt{x^3y^2}$

$\frac{1}{2}\ln x^3 + \frac{1}{2}\ln y^2$

$\boxed{\frac{3}{2}\ln x + \ln y}$

13. Condense each expression.

a. $\frac{1}{3}\log_4(x+y)$

$\boxed{\log_4 \sqrt[3]{x+y}}$

b. $3\ln(x-2)-2\ln(x+2)$

$\boxed{\ln \frac{(x-2)^3}{(x+2)^2}}$

c. $\log 8 + 3\log x - \log 7$

$\boxed{\log \frac{8x^3}{7}}$

14. Solve each equation algebraically. When necessary, round your result to the nearest thousandth.

a. $3^{2x} - 5 = 9$

$3^{2x} = 14$

$\frac{\log_3 14}{2} = \frac{2x}{2}$

$x = 1.201$

b. $3^{-2x} - 6 = 2$

$3^{-2} = 8$

$\frac{\log_3 8}{-2} = \frac{-2x}{-2}$

$x = -.944$

c. $e^{4x} - 7 = 10$

$e^4 x = 17$

$\frac{\ln 17}{4} = x$

$x = .708$

d. $3 + e^{-2x} = 11$

$e^{-2x} = 8$

$\frac{\ln 8}{-2} = x$

$x = -1.040$

$\cancel{x = 1.039}$

e. $\ln 5x - 9 = 11$

$\ln 5x = 20$

$\frac{e^{20}}{5} = \frac{5x}{5}$

$x = 97033039.08$

f. $3 + \log_2 3x = 5$

$\log_2 3x = 2$

$\frac{2^2}{3} = \frac{3x}{3}$

$\frac{4}{3} = x$

g. $\log_2 3 + \log_2 x = \log_2 5 + \log_2(x-2)$

$\log_2 3x = \log_2 5(x-2)$

$3x = 5x - 10$

$-2x = -10$

$x = 5$

h. $\log_3(x-5) = 4$

$3^4 = x - 5$

$81 + 5 = x$

$86 = x$

i. $\log(3x+2) = \log(2x-1)$

$3x + 2 = 2x - 1$

$\cancel{x = -3}$

j. $\log_2 x + \log_2(x-1) = \log_2(4x)$

$\log_2 x(x-1) = \log_2(4x)$

$x^2 - x = 4x$

$x^2 - 5x = 0$

$x(x-5) = 0$

$\cancel{x = 0}$

$x = 5$

k. $3e^{2x} + 8e^x - 3 = 0$

$3x^2 + 9x - x - 3$

$3x(x+3) - 1(x+3)$

$-9x^2$

$9x^n - x$

$(3e^x - 1)(e^x + 3) = 0$

$e^x = 1/3$

$\ln 1/3 = x$

$x = -1.099$

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15. **Determine the balance if you invest $3500 at 6.5% for 10 years compounded:**

a. annually

$3500(1+.065)^{10}$

6569.98

b. weekly

$3500\left(1+\frac{.065}{52}\right)^{520}$

6701.67

c. continuously

$3500e^{.065(10)}$

6764.39

16. **A deposit of $10,000 is made in a savings account for which the interest is compounded continuously. The balance will double in 5 years.**

a. What is the annual interest rate for this account?

$2=e^{5r}$

$\frac{\ln 2}{5}=r=.1386$

$r=13.86\%$

b. Find the balance after 3 years.

$10000e^{(.1386\cdot 3)}=A$ ~~= 14997.06~~ $=15155.83$

17. **The population of Glenbrook in the year 1910 was 4200. Assume the population increased at the rate of 2.25% per year.**

a. Write an exponential model for the population of Glenbrook. Define your variables.

1910 $t=0$

$y=4200(1.0225)^x$

b. Determine the population in 1930 and 1900.

6554.14

3362.14

c. Determine when the population is double the original amount.

$x=31.15$ years

$1910+31.15=1941.15=1941$

18. **The half-life for ^{14}C is 5715 years.** $y=I\left(\frac{1}{2}\right)^{x/5175}$

a. If there are 2 grams after 1000 years, what was the initial quantity?

$2=I\left(\frac{1}{2}\right)^{1000/5715}$

$I=2.258$

b. If the initial quantity is 3 grams, how much is present after 1000 years?

$A=3\left(\frac{1}{2}\right)^{1000/5715}$

$A=2.659$

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19. **The number of bacteria present in culture $N(t)$ at time t hours is given by $N(t) = 3000(2)^t$.**

a. What is the initial population?

3000

b. How much bacteria are present after 24 hours?

$3000(2)^{24} = N = 5.033 \times 10^{10}$

c. How long will it take the population to triple in size? $3 = 2^t$ $\quad \log_2 3 = t$

1.585 hours

20. **A species of bat is in danger of becoming extinct. Five years ago, the total population of the species was 2000. Two years ago, the total population of species was 1400. Using an exponential model ($y = ae^{kx}$), determine the total population of the species one year ago.**

$1400 = 2000e^{3k}$

$.7 = e^{3k}$

$\frac{\ln .7}{3} = k = -.1189$

$y = 2000e$ $\quad (-.1189 \cdot 4)$

1243 bats

(3, 1400)

(4, ?)

21. **The number of students infected with flu after t days at Washington High School is modeled by the following function:**

$$P(t) = \frac{1600}{1+99e^{-0.4t}}$$

a. What was the initial number of infected students? 16

b. After 5 days, how many students will be infected?

$111.13 \approx 111$

c. What is the maximum number of students that will be infected?

1600

d. According to this model, when will the number of students infected be 800?

$800 = \frac{1600}{1+99e^{-.4t}}$ $\quad 800(1+99e^{-.4t}) = 1600$ $\quad e^{-.4t} = \frac{1}{99}$

$1+99e^{-.4t} = 2$ $\quad -.4t = \ln(1/99)$

$99e^{-.4t} = 1$

$t = 11.49$ days

22. **Find a logistic equation of the form $y = \frac{c}{1+ab^x}$ that fits the graph below, if the y-intercept is 5 and the point (24, 135) is on the curve.**

$c = 500$

$5 = \frac{500}{1+a}$

$5 + 5a = 500$

$5a = 495$

$a = 99$

$135 = \frac{500}{1+99b^{24}}$

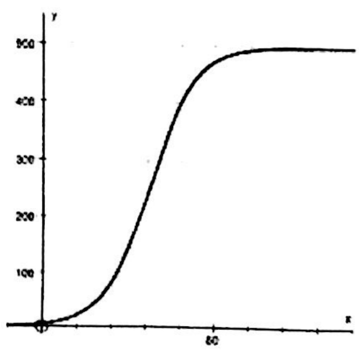
$135(1+99b^{24}) = 500$

$1+99b^{24} = (500/135)$

$\frac{99b^{24}}{99} = \frac{(500/135)-1}{99}$

$b^{24} =$

$b = .8607$



$y = \frac{500}{1+99(.8607)^x}$