Honors Pre-Calc
Chapter 2 Test Review

Name KEY
Date 10/20/15 **Block** 0

2.1 Quadratic Functions

1. Write the quadratic in ~~standard form~~ vertex by completing the square: $f(x) = -3x^2 - 6x - 1$

Vertex: (-1, 2)

$f(x) = -3(x^2 + 2x) - 1$
$= -3(x^2 + 2x + 1 - 1) - 1$
$= -3(x+1)^2 + 3 - 1$
$= -3(x+1)^2 + 2$

2. Write the equation of the parabola that contains point P, $P(-1, 12)$, with vertex $V(1, 0)$.

$y = a(x-h)^2 + k$
$12 = a(-1-1)^2 + 0$
$12 = 4a$
$a = 3$
$y = 3(x-1)^2$

3. What is the average rate of change of $f(x) = -3x^2 - 6x - 1$ over the interval

A. [-3,0]

$\frac{f(0) - f(-3)}{0 - (-3)} = \frac{-1 + 10}{3} = 3$

B. [a,b]

$\frac{f(b) - f(a)}{b - a} = -3(b+a) - 6$

4. At a fourth of July celebration, fireworks are shot by remote control into the air from a pit that is 10 feet below the earth's surface. Round your answers to the nearest tenth.

Model: $f(t) = -\frac{1}{2}(32)t^2 + v_0 t + s_0$, *where t is time (sec), v_0 is the initial velocity and s_0 is the initial height (ft).*

A. Find an equation that models the height of an aerial firework t seconds after it is shot upward with an initial velocity of 80 ft/s.

$f(t) = -16t^2 + 80t - 10$

B. What is the maximum height above ground level that the firework will reach? How many seconds does it take to reach that maximum height?

Max Height: 90 ft at $t = 2.5$ seconds

C. Assuming a piece of the firework remains after it ignites, how many seconds will it take the firework to hit the ground after it is shot upward?

4.87 seconds

2.2 Power Functions

5. Write each statement as a power function equation. Use k for the constant of variation.

A. The volume, V, of a circular cone with fixed height is proportional to the square of its radius, r.

$V = kr^2$ or $V = kr^2h$ (constant since h is fixed.)

B. The centripetal force, F, acting on a satellite in orbit around the earth is inversely proportional to the square of the radius, r, of its orbit.

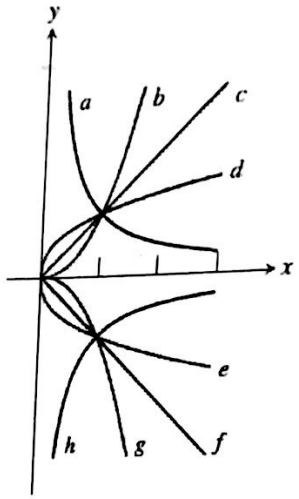
$F = \frac{k}{r^2}$

6. The current in a simple electrical circuit is inversely proportional to the resistance. If the current is 80 amps when the resistance is 50 ohms, find the current when the resistance is 22 ohms.

$C = \frac{k}{r}$ $80 = \frac{k}{50}$ $k = 4000$ $\frac{4000}{22} = C = 181.8$ amps

7. Complete the table using the graph to the right that represents all power functions: $f(x) = kx^a$, where k is the constant of variation and a is the power of x.

Graph	a	b	c	d	e	f	g	h
k +/-	+	+	+	+	−	−	−	−
a range	−	$a>1$	$a=1$	$0<a<1$	$0<a<1$	1	$a>1$	−



Match the equation to one of the curves labeled in the figure.
List whether the function is odd or even.

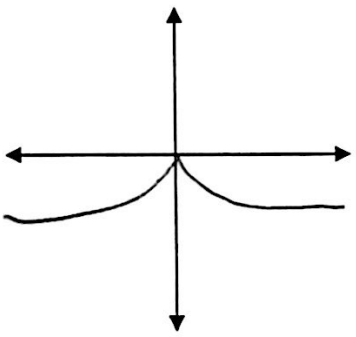
~~e~~ ~~O/E~~ A. $f(x) = -3x^{1/4}$

a O/E B. $f(x) = 2x^{-2/3}$

g O/E C. $f(x) = -2x^{4/3}$

a O/E D. $f(x) = x^{-3}$

8. For the function: $f(x) = -x^{2/3}$, list the following parameters and sketch the graph.



k	−1
a	2/3
Domain:	$(-\infty, \infty)$
Range:	$(-\infty, 0]$
Increasing:	$(-\infty, 0)$
Decreasing:	$(0, \infty)$
Symmetry:	Even – Reflect in y-axis
Boundedness:	Above
Extrema:	(0, 0)
Asympote(s):	NONE

2.3 Polynomial Functions of Higher Degree

9. For the function $f(x) = -4x^3 - 4x^2 + 24x$, find the zeros and describe the end behavior using limit notation.

$f(x) = -4x(x^2 + x - 6)$

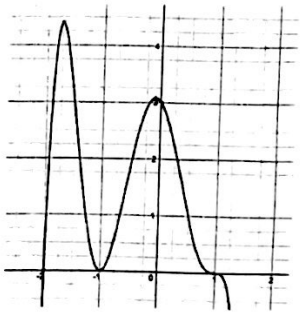
$= -4x(x+3)(x-2)$

$x = 0$ $x = -3$ $x = 2$

$\lim_{x \to -\infty} f(x) = \infty$ $\lim_{x \to \infty} f(x) = -\infty$

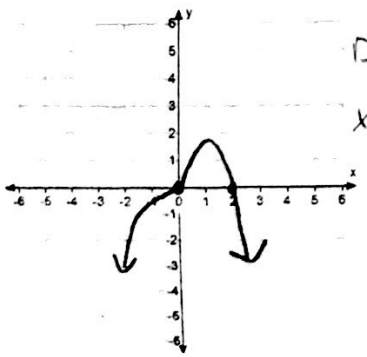
10. Give the equation for the polynomial function $p(x)$, in factored form, shown at the right, where $p(x)$ is of degree 6, and passes through the point (0, 3).

$f(x) = -\frac{3}{2}(x+2)(x+1)^2(x-1)^3$



11. Sketch a graph of the polynomial functions, state the degree and end behavior:

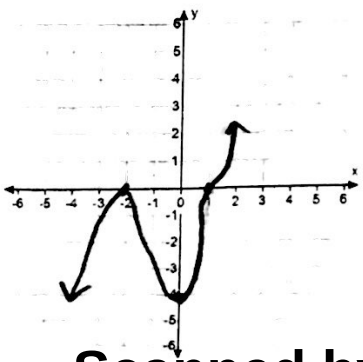
A. $f(x) = -x^3(x-2)$



Degree = 4

$x \to$

B. $f(x) = (x-1)^3(x+2)^2$



Degree = 5

4-2.5 Real & Complex Zeros of Polynomial Functions

12. Explain how you know that the function $p(x) = -x^3 + 5x - 2$ must have a zero in the interval [-3,-2].

$f(-3) = 10$

$f(-2) = -4$

Switch from + to − so must cross axis.

13. For $f(x) = 2x^5 + 9x^4 - 3x^2 - 10$, use synthetic division and the remainder theorem to evaluate *f(-1)*.

	2	9	0	−3	0	−10
−1	↓	−2	−7	7	−4	4
	2	7	−7	4	−4	−6

$f(-1) = -6$

14. For $f(x) = 4x^3 + 9x^2 - 3x - 10$, List all possible rational zeros. Show that $(x+2)$ is a factor. Identify all other zeros. Then write the polynomial in completely factored form.

$\pm \frac{1, 2, 5, 10}{\pm 1, 2, 4} = \pm 1, \frac{1}{2}, \frac{1}{4}, 2, 5, \frac{5}{2}, \frac{5}{4}, 10$

	4	9	−3	−10
−2	↓	−8	−2	10
	4	1	−5	0

$4x^2 + x - 5 = 0$

$(4x+5)(x-1) = 0$

$f(x) = (x+2)(4x+5)(x-1)$

$x = -\frac{5}{4}, 1, -2$

15. Find a polynomial function, in standard form, with real coefficients and a leading coefficient of 3 that has the given zeros: $x = 0, 0, \frac{2}{3}, 4-i$.

$f(x) = x^2(3x-2)(x-4-i)(x-4+i)$

$= x^2(3x-2)(x^2-8x+17)$

$= 3x^5 - 26x^4 + 67x^3 - 34x^2$

16. The function, $f(x) = x^4 - 6x^3 + 11x^2 + 12x - 26$ has a zero at $3-2i$. Find all of the zeros. Write the function as a product of irreducible quadratic factors. Then write a linear factorization.

$(x-3-2i)(x-3+2i)$

$(x^2 - 6x + 13)$

$x^2 - 2$

$x^2 - 6x + 13 \;\big|\; x^4 - 6x^3 + 11x^2 + 12x - 26$

$-x^4 - 6x^3 + 13x^2$

$-2x^2 + 12x - 26$

$-2x^2 + 12x - 26$

0

$f(x) = (x^2 - 6x + 13)(x^2 - 2)$

$f(x) = (x-3+2i)(x-3-2i)(x+\sqrt{2})(x-\sqrt{2})$

17. Show that all real zeros of $f(x) = 2x^3 - 5x^2 - 14x + 8$, must exist within [-4,8].

(Hint: Use synthetic division to verify the upper and lower bounds of the real zeros of f)

	2	−5	−14	8
−4	↓	−8	52	−152
	2	−13	38	−144

Alternating +/−

	2	−5	−14	8
8	↓	16	88	592
	2	11	74	600

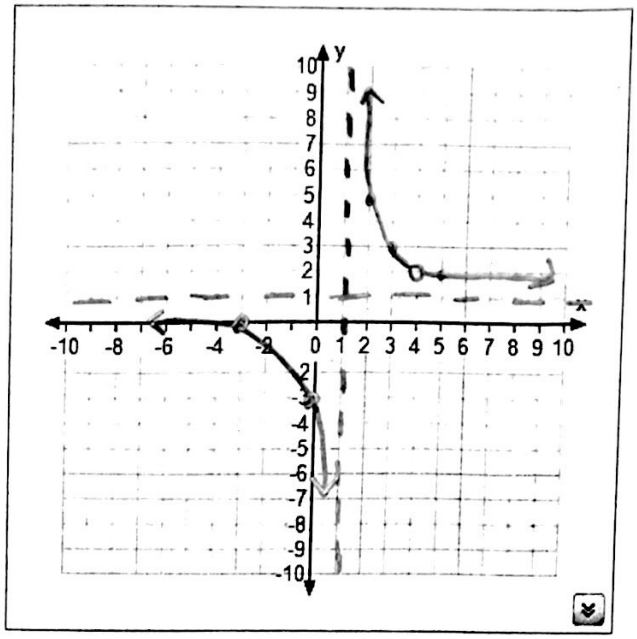
all +

No real zeros outside of given interval.

$\frac{(x-4)(x+3)}{(x-4)(x-1)}$

2.6 Graphs of Rational Functions

18. $f(x)=\dfrac{x^2-x-12}{x^2-5x+4}$ Graph number 18 to the right.



Vertical Asymptote(s):	$x=+1$ hole @ $x=4$
Horizontal Asymptote:	$y=1$
x-int (s):	$(-3,0)$
y-int:	$(0,-3)$
Asymptote Behavior:	$\lim_{x\to 1^-} f(x) = -\infty$ $\lim_{x\to 1^+} f(x) = \infty$
End Behavior:	$\lim_{x\to\infty} F(x) = 1 = \lim_{x\to-\infty} f(x)$

19. $f(x)=\dfrac{x^4-2x+1}{x+2}$

Vertical Asymptote(s):	$x=-2$
End Behavior Asymptote:	$y=x^3-2x^2+4x-10$
x-int (s):	$(1,0)$
y-int:	$(0,1/2)$
Asymptote Behavior:	$\lim_{x\to -2^+} f(x) = \infty$ $\lim_{x\to -2^-} = -\infty$
End Behavior: $\infty =$	$\lim_{x\to\infty} F(x) = \lim_{x\to-\infty} f(x) = -\infty$

20. $f(x)=\dfrac{2}{x^3-x}$

Vert Asymptote(s):	$x=0,\ x=1,\ x=-1$
End Behavior Asymptote:	$y=0$
x-int (s):	none
y-int:	none
Asymptote Behavior:	$\lim_{x\to -1^-} F(x) = -\infty$ $\lim_{x\to -1^+} f(x) = \infty$
End Behavior:	$\lim_{x\to 0^-} f(x) = \infty$ $\lim_{x\to 0^+} f(x) = -\infty$ $\lim_{x\to 1^-} f(x) = -\infty$ $\lim_{x\to 1^+} f(x) = \infty$

$\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 0$

2.7 Solving Rational Equations

Solve and check for extraneous solutions.

21. $\dfrac{4x^{(x-1)}}{x+4}+\dfrac{3^{(x+4)}}{x-1}=\dfrac{15}{x^2+3x-4}$

$4x^2-4x+3x+12=15$

$4x^2-x-3=0$

$(4x+3)(x-1)=0$

$x=-\frac{3}{4},\ 1$

$x=1$ is extraneous

22. $\dfrac{x+3}{x}-\dfrac{2}{x+3}=\dfrac{6}{x^2+3x}$ $(x^3)\ (x)$

$x^2+6x+9-2x=6$

$x^2+4x+3=0$

$(x+3)(x+1)=0$

~~$x=-3$~~

$x=-1$

23. Determine how many mls of a pure acid solution should be added to a 200 ml solution that is 45% concentration to produce a 75% acid solution.

$.75=\dfrac{90+x}{200+x}$

$150+.75x=90+x$

$60=.25x$

$x=240\text{mL}$

2.8 Solving Polynomial Inequalities

Solve the inequality using a sign chart. Support your answer graphically.

24. $f(x)=2x^4-3x^3-6x^2+5x+6<0$

	2	-3	-6	5	6
-1		-2	5	1	-6
-1	2	-5	-1	6	0
		-2	7	-6	
	2	-7	6	0	

$2x^2-7x+6=0$

$(x-2)(2x-3)$

$(\frac{3}{2},2)$

25. $f(x)=\dfrac{x^3-x}{x^2+1}\geq 0$

$\dfrac{x(x+1)(x-1)}{x^2+1}$

$[-1,0]\cup[1,\infty)$