2017

PreCalculus
Final Exam Review

Name Key
Date _____ Block 2B, 1A

Chapter 6

In 1 – 3, solve the triangle for all angles and sides. If two solutions exist, find both.

1. $c = 13, b = 8, B = 31°$

$\frac{8}{\sin 31°} = \frac{13}{\sin C}$ $\quad \frac{a}{\sin 92.18} = \frac{8}{\sin 31°}$ $\quad \frac{a}{\sin 25.82°}$

$\sin C = \frac{13 \sin 31°}{8}$

$C = 56.82°$	OR	$123.18° = C$
$A = 92.18°$		$25.82° = A$
$a = 15.52$		$6.77 = a$

2. $A = 55°, b = 12, c = 7$

$a^2 = 12^2 + 7^2 - 2(12)(7)\cos 55°$

$a = 9.83$

$\frac{9.83}{\sin 55°} = \frac{12}{\sin B}$

$B = 89.66°$

$C = 35.34°$

3. $A = 33°, B = 70°, b = 7$

$C = 77°$

$\frac{7}{\sin 70°} = \frac{a}{\sin 33}$

$a = 4.06$

$\frac{7}{\sin 70°} = \frac{c}{\sin 77°}$

$c = 7.26$

In 4 - 5, find the area of the triangle to the nearest tenth.

4. $A = 52°, b = 14$ m, $c = 21$ m

$\text{Area} = \frac{1}{2}bc\sin A = \frac{1}{2}(14)(21)\sin 52° = 115.8 m^2$

5. $a = 7$ cm, $b = 8$ cm, $c = 9$ cm $\quad s = \frac{7+8+9}{2} = 12$

$\text{Area} = \sqrt{12(12-7)(12-8)(12-9)}$

$\text{Area} = 26.8 \text{ cm}^2$

Chapter 9

In 6 – 8, write the equation in standard form and then classify the graph as a parabola, circle, ellipse, or hyperbola.

6. $x^2 + y^2 - 6x + 4y + 9 = 0$ — Circle

$(x^2 - 6x + 9) + (y^2 + 4y + 4) = -9 + 9 + 4$

$(x-3)^2 + (y+2)^2 = 4$

7. $x^2 - 6x + 16y + 21 = -4y^2$ — Ellipse

$(x^2 - 6x + 9) + 4(y^2 + 4y + 4) = -21 + 9 + 16$

$(x-3)^2 + 4(y+2)^2 = 4$

$\frac{(x-3)^2}{4} + (y+2)^2 = 1$

8. $y^2 - 6y - 4x + 21 = 0$ — Parabola

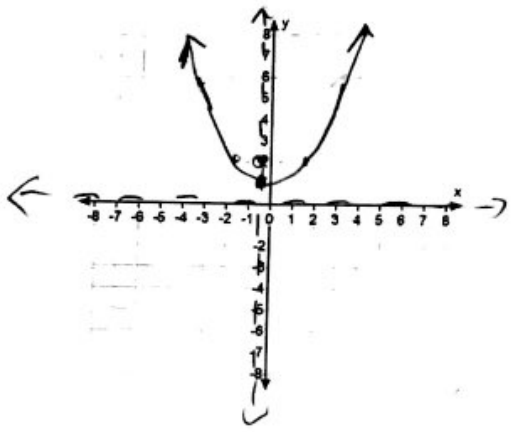
$y^2 - 6y + 9 = 4x - 21 + 9$

$(y-3)^2 = 4x - 12$

$(y-3)^2 = 4(x-3)$

9. Find the vertex, axis of symmetry, focus, and directrix of the parabola and sketch its graph.

$$\left(x+\frac{1}{2}\right)^2 = 4(y-1)$$



Vertex: $(-1/2, 1)$ opens up

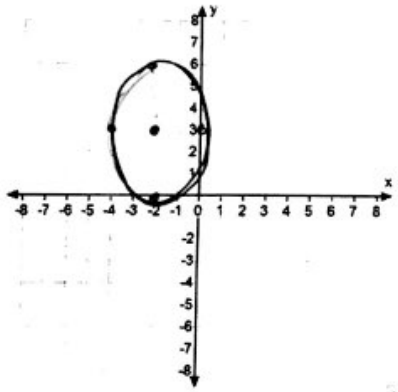
A.O.S. $x = -1/2$ $p = 1$

Focus: $(-1/2, 2)$

directrix: $y = 0$

10. Identify the conic as a circle or ellipse. Then find the center and radius (if it's a circle); find the center, vertices, co-vertices, and foci (if it's an ellipse). Sketch its graph.

Ellipse



$$9x^2 + 4y^2 + 36x - 24y + 36 = 0$$

$9(x^2+4x+4) + 4(y^2-6y+9) = -36+36+36$

$9(x+2)^2 + 4(y-3)^2 = 36$

$\frac{(x+2)^2}{4} + \frac{(y-3)^2}{9} = 1$

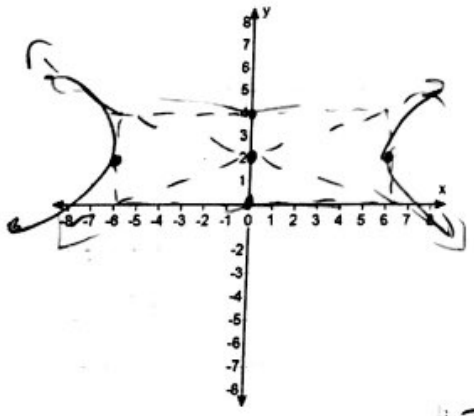
Foci: $(-2, 3\pm\sqrt{5})$

Center: $(-2, 3)$ $a = 3$, $b = 2$

Vertices: $(-2, 6), (-2, 0)$ $c^2 = a^2 - b^2$ $c = \sqrt{5}$

Co-vertices $(0, 3), (-4, 3)$

11. Find the center, vertices, foci, lines containing the axes, and the equations of the asymptotes of the hyperbola and then sketch its graph.



$$x^2 - 9y^2 + 36y - 72 = 0$$

Center: $(0, 2)$

$a = 6, b = 2$

$c^2 = a^2 + b^2$

$c = 2\sqrt{10}$

$x^2 - 9(y^2-4y+4) = 72 - 36$

$x^2 - 9(y-2)^2 = 36$

$\frac{x^2}{36} - \frac{(y-2)^2}{4} = 1$

Vertices: $(-6, 2), (6, 2)$

Foci: $(\pm 2\sqrt{10}, 2)$

Lines containing axes:

transverse: $y = 2$

conjugate $x = 0$

Asymptotes: $y = 2 \pm \frac{1}{3}x$

12. Write the equation of a circle that has a center at (-1, 3) and passes through the point (-5, 6).

radius $= \sqrt{(-1+5)^2 + (3-6)^2} = \sqrt{16+9} = \sqrt{25} = 5$

$$(x+1)^2 + (y-3)^2 = 25$$

Chapter 3

In 13 - 15, evaluate each expression WITHOUT A CALCULATOR.

13. $\dfrac{\log_{12} 12^{36}}{\log_4 4^{18}} = \dfrac{36}{18} = 2$

14. $\ln e^{5a} = 5a$

15. $\log_4 320 - \log_4 5$

$\log_4 \frac{320}{5} = \log_4 64 = 3$

16. Use the change of base formula to evaluate: $\log_5 7$

$\frac{\log 7}{\log 5}$ OR $\frac{\ln 7}{\ln 5} = 1.209$

17. Use $\log_a 2 \approx 0.4307$ and $\log_a 3 \approx 0.6826$ to rewrite and evaluate $\log_a 24$ (1.9747)

$\log_a(2^3 \cdot 3) = \log_a 2^3 + \log_a 3 = 3\log_a 2 + \log_a 3 = 3(0.4307) + 0.6826$

18. Use the properties of logarithms to expand: $\ln \dfrac{\sqrt{x^3 y^2}}{z}$.

$\frac{\ln (x^3 y^2)^{1/2}}{z} = \frac{\ln x^{3/2} y}{z} =$

$\frac{3}{2}\ln x + \ln y - \ln z$

19. Use the properties of logarithms to express the following expression as a single logarithm:

$3\ln(x-2) + 2\ln(x+2)$ = $\ln(x-2)^3 + \ln(x+2)^2 = \ln(x-2)^3(x+2)^2$

In 20 – 22, solve each equation algebraically. When necessary, round your result to the nearest thousandth.

20. $3^{2x} - 5 = 9$

$3^{2x} = 14$

$\log_3 3^{2x} = \log_3 14$

$2x = \log_3 14$

$x = \left(\frac{\log 14}{\log 3}\right) \div 2$

$x = 1.201$

21. $3 + \log_2 3x = 5$

$\log_2 3x = 2$

$2^2 = 3x$

$x = 1.333$ or $\frac{4}{3}$

22. $\log(x) + \log(x - 21) = 2$

$\log x(x-21) = 2$

$10^2 = x^2 - 21x$

$x^2 - 21x - 100 = 0$

$(x-25)(x+4) = 0$

$x = 25$

23. The number of bacteria present in culture $N(t)$ at time t hours is given by $N(t) = 3000(2)^t$.

a. What is the initial population? 3000

b. How much bacteria are present after 24 hours? $N(24) = 3000(2^{24}) = 5.0333 \times 10^{10}$

c. How long will it take the population to triple in size?

$9000 = 3000(2^t)$

$2^t = 3$

$\log_2 2^t = \log_2 3$

$t = \frac{\log 3}{\log 2} = 1.58$ hours

24. The number of students infected with flu after t days at Washington High School is modeled by the following function: $P(t) = \dfrac{1600}{1+99e^{-0.4t}}$

a. What was the initial number of infected students? $P(0) = \dfrac{1600}{1+99e^{0}} = \dfrac{1600}{100}$

16 students

b. After 5 days, how many students will be infected? $P(5) = \dfrac{1600}{1+99e^{-0.4(5)}} =$

111.13 students

c. What is the maximum number of students that will be infected?

1600 students

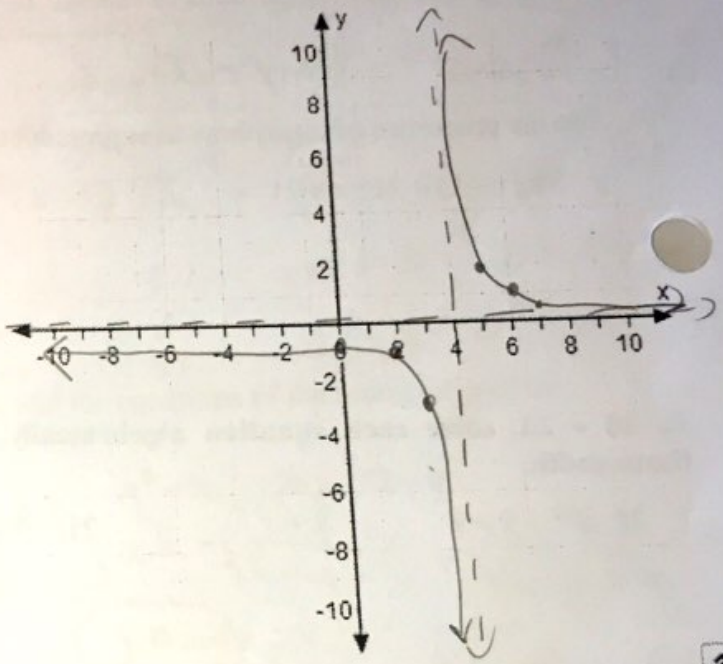
Chapters 2/7

In 25 – 27, find the indicated information for each function. Note in each example if $N > D$, $N = D$ or $N < D$. Then graph each function.

25. $f(x) = \dfrac{2}{x-4}$

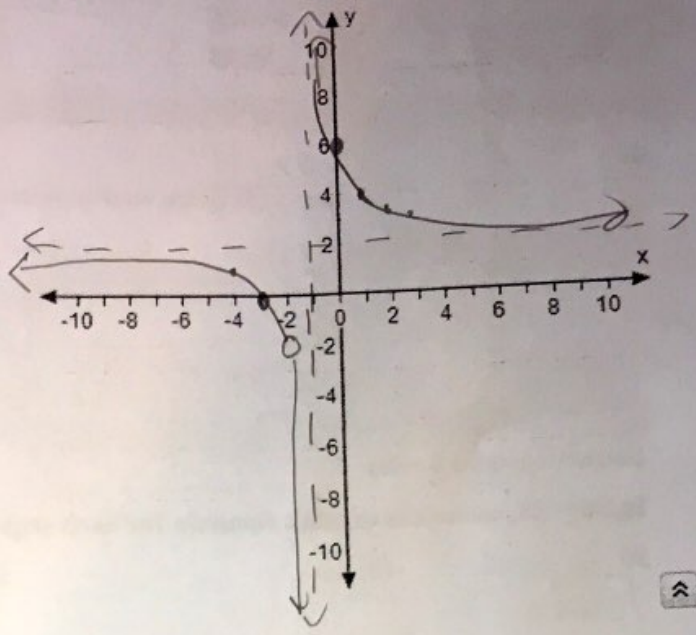
Higher Degree N/D?	$N<D$
Asymptotes:	V.A. $x=4$ HA $y=0$
x-int (s):	none
y-int:	$(0, -1/2)$
hole(s):	none
Test Points:	x | $f(x)$ 2 | -1 3 | -3 5 | 2 6 | 1 7 | $2/3$

≤



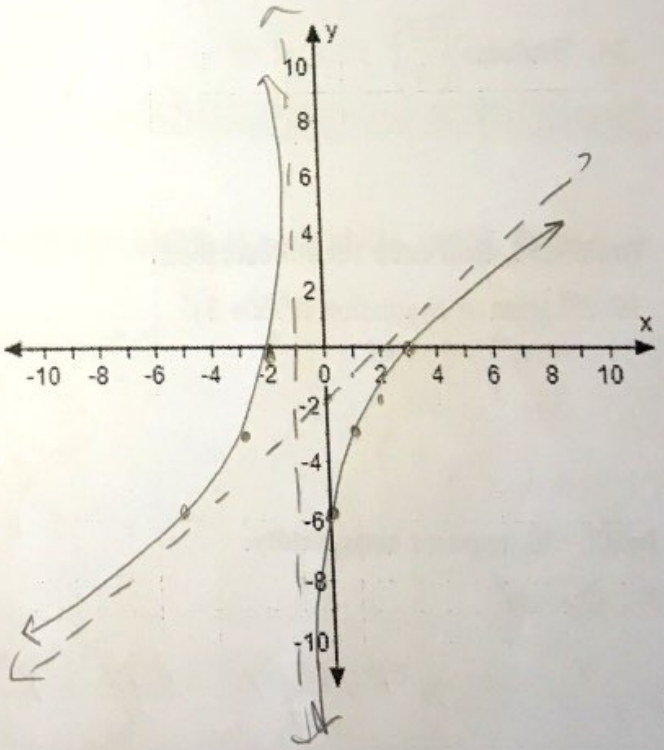
26. $f(x)=\dfrac{2x^2+10x+12}{x^2+3x+2}$ $= \dfrac{2(x+3)(x+2)}{(x+2)(x+1)}$

Higher Degree N/D?	$N=D$
Asymptotes:	H.A. $y=2$ V.A. $x=-1$
x-int (s):	$(-3, 0)$
y-int:	$(0, 6)$
hole(s):	$(-2, -2)$
Test Points:	X | f(x) 1 | 4 2 | 3 1/3 3 | 3 -5 | 1



27. $f(x)=\dfrac{x^2-x-6}{x+1}$ $= \dfrac{(x-3)(x+2)}{x+1}$

Higher Degree N/D?	$N>D$
Asymptotes:	V.A. $x=-1$ S.A. $y=x-2$
x-int (s):	$(3,0)$, $(-2,0)$
y-int:	$(0,-6)$
hole(s):	none
Test Points:	X | f(x) 1 | -3 2 | -1 1/3 -3 | -3 -5 | -6



In 28 – 29, find the partial fraction decomposition of each.

28. $\frac{-5x+4}{x^2-x} = \frac{-5x+4}{x(x-1)} = \frac{-4}{x} - \frac{1}{x-1}$

$\frac{A}{x} + \frac{B}{x-1} = \frac{-5x+4}{x(x-1)}$

$A(x-1) + Bx = -5x+4$

$Ax - A + Bx = -5x+4$

$A+B=-5 \quad -A=4$

$B=-1 \quad A=-4$

29. $\frac{-7x-15}{x^2+6x+9} = \frac{-7x-15}{(x+3)^2}$

$\frac{A}{x+3} + \frac{B}{(x+3)^2}$

$\frac{-7}{x+3} + \frac{6}{(x+3)^2}$

$A(x+3) + B = -7x-15$

$Ax + 3A + B = -7x-15$

$A=-7 \quad 3A+B=-15 \quad B=6$

Sequences and Series

In 30 – 31, write the explicit formula for each sequence.

30. -3, -6, -12, -24, -48, ...

$a_n = -3 \cdot 2^{n-1}$

31. -4, -14, -24, -34, -44, ...

$a_n = -4 - 10(n-1)$

32. Find "n" if you know that $S_n = 59{,}046$ in the series $6 + 18 + 54 + 162\ldots$

$59046 = 6\left(\frac{1-3^n}{1-3}\right)$

$9841 = \frac{1-3^n}{-2}$

$19683 = 3^n$

$\log_3 19683 = n$

$n=9$

33. Evaluate $\sum_{n=0}^{5}(20-n^2)$ $= 20+19+16+11+4+(-5) = 65$

34. Evaluate $\binom{12}{3} = 220$ $\quad \frac{12!}{9!3!}$

In 35 - 36, find each term described.

35. 2nd term in expansion of $(x+3)^3$

$k=1$
$n=3$
$a=x$
$b=3$

$\binom{3}{1}(x)^2(3)^1$

$9x^2$

36. 4th term in expansion of $(3u-1)^4$

$k=3$
$n=4$
$a=3u$
$b=(-1)$

$\binom{4}{3}(3u)^1(-1)^3$

$4(3u)(-1)$

$-12u$

In 37 - 38, expand completely.

37. $(2y-x)^4$

$16y^4 - 32y^3x + 24y^2x^2 - 8yx^3 + x^4$

38. $(2y+3x)^3$

$8y^3 + 36y^2x + 54yx^2 + 27x^3$