

PreCalculus  
Midterm Exam Review

Name KEY  
Date \_\_\_\_\_ Block \_\_\_\_\_

1. Convert the angle measure from degrees to radians.

a.)  $-270^\circ \quad \frac{\pi}{2}, -\frac{3\pi}{2}$

b.)  $144^\circ \quad \frac{4\pi}{5}$

2. Convert the angle measure from radians to degrees.

a.)  $\frac{7\pi}{3} \quad 420^\circ$

b.)  $\frac{-13\pi}{60} \quad -39^\circ$

3. a.) If the Earth rotates once every 24 hours, find the angular speed in radians/hour.

$$\frac{1 \text{ rev}}{24 \text{ hrs}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = \frac{\pi}{12} = .2618 \text{ rad/hr}$$

b.) If a fan rotates 30 times in a minute, find the angular speed in radians/hour.

$$\frac{30 \text{ rev}}{\text{min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} = 3600\pi \text{ rad/hr} = 11309.73 \text{ rad/hr}$$

c.) If a ferris wheel rotates 4 times per minute, find the angular speed in radians/second.

$$\frac{4 \text{ rev}}{\text{min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = \frac{8\pi}{60} = \frac{2\pi}{15} \text{ rad/sec} = .4189 \text{ rad/sec}$$

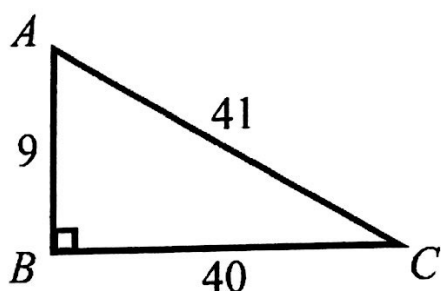
4. The minute hand of a clock is 10.2 cm. How far does the tip travel in 15 minutes? 33 minutes? 54 minutes?

$S = r\theta$   
 $S = 10.2 \left(\frac{\pi}{2}\right)$   
 ~~$S = 10.2 \left(\frac{\pi}{2}\right)$~~   
 $S = 16 \text{ cm}$

$\frac{33}{60} = \frac{x}{2\pi}$   
 $x = 3.456 \text{ rad}$   
 $S = 10.2(3.456)$   
 $S = 35.2 \text{ cm}$

$\frac{54}{60} = \frac{x}{2\pi}$   
 $S = 10.2\left(\frac{54}{60}\right)(2\pi)$   
 $= 57.7 \text{ cm}$

5. Find the six trigonometric ratios of  $\angle A$ .



$\sin A = \frac{40}{41}$

$\csc A = \frac{41}{40}$

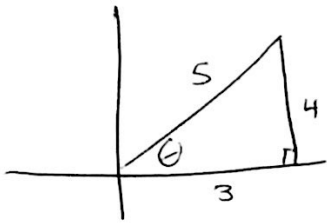
$\cos A = \frac{9}{41}$

$\sec A = \frac{41}{9}$

$\tan A = \frac{40}{9}$

$\cot A = \frac{9}{40}$

6. a.) Given  $\sin \theta = \frac{4}{5}$  in Quadrant I, find the remaining 5 trig ratios.



$$\cos \theta = \frac{3}{5}$$

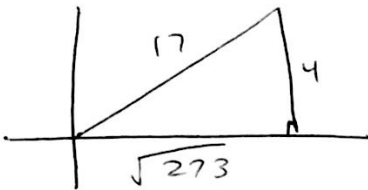
$$\sec \theta = \frac{5}{3}$$

$$\tan \theta = \frac{4}{3}$$

$$\cot \theta = \frac{3}{4}$$

$$\csc \theta = \frac{5}{4}$$

b.) Given  $\csc \theta = \frac{17}{4}$  in Quadrant I, find the remaining 5 trig ratios.



$$\sin \theta = \frac{4}{17}$$

$$\sec \theta = \frac{17\sqrt{273}}{273}$$

$$\cos \theta = \frac{\sqrt{273}}{17}$$

$$\cot \theta = \frac{\sqrt{273}}{4}$$

$$\tan \theta = \frac{4\sqrt{273}}{273}$$

7. Use a calculator to evaluate each function.

a.)  $\sin 41^\circ$

.6561

b.)  $\cot 71.5^\circ$

.3346

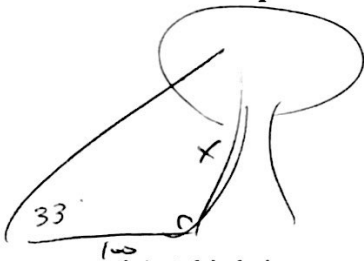
c.)  $\cot \frac{\pi}{16}$

5.0273

d.)  $\tan \frac{\pi}{8}$

.4142

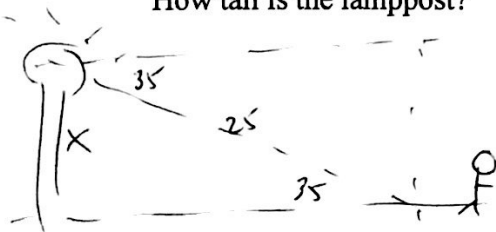
8. a.) John wants to measure the height of a tree. He walks exactly 100 feet from the base of the tree and looks up. The angle from the ground to the top of the tree is  $33^\circ$ . How tall is the tree?



$$\tan 33 = \frac{x}{100}$$

$$x = 64.9 \text{ Feet}$$

b.) A bird sits on top of a lamppost. The angle of depression from the bird to the feet of an observer standing away from the lamppost is  $35^\circ$ . The distance from the bird to the observer is 25 meters. How tall is the lamppost?



$$\sin 35 = \frac{x}{25}$$

$$x = 14.34 \text{ m}$$

9. Determine two co-terminal angles (one positive and one negative) for each angle.

a.)  $\theta = 52^\circ$   $\theta = 412^\circ$

$\theta = -308^\circ$

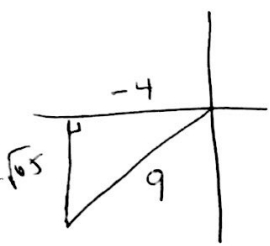
b.)  $\theta = \frac{7\pi}{8}$

$\theta = \frac{23\pi}{8}$

$\theta = -\frac{9\pi}{8}$

10. Find the indicated trigonometric value in the specified quadrant.

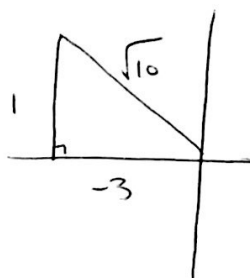
a.)  $\sec \theta = -\frac{9}{4}$ ; QIII;  $\tan \theta$



$$\tan \theta = \frac{-\sqrt{65}}{-4}$$

$$= \frac{\sqrt{65}}{4}$$

b.)  $\cot \theta = -3$ ; QII;  $\sin \theta$



$$\sin \theta = \frac{\sqrt{10}}{10}$$

11. Find the period and amplitude.

a.)  $y = 3 \sin 2x$

$$A = 3$$

$$P = \frac{2\pi}{2} = \pi$$

b.)  $y = \frac{2}{3} \sin \pi x$

$$A = \frac{2}{3}$$

$$P = \frac{2\pi}{\pi} = 2$$

c.)  $y = \frac{3}{4} \cos \frac{\pi}{12} x$

$$A = \frac{3}{4}$$

$$P = \frac{2\pi}{\frac{\pi}{12}} = 24$$

12. Identify the transformation from  $f$  to  $g$ .

a.)  $f(x) = \sin x$   
 $g(x) = -4 \sin x$

-  $R_x$

- Vertical stretch  
 Amplitude 1 to 4

b.)  $f(x) = \cos x$   
 $g(x) = -\cos(x - \pi)$

-  $R_x$

- Phase Shift  
 Right  $\pi$

c.)  $f(x) = 4 \sin x$   
 $g(x) = 4 \sin 3x - 2$

- Vertical Shift  
 down 2

13. Find the max and min.

a.)  $y = 3 \sin x - 2$

$$\text{Max} = 3$$

$$\text{Min} = -3$$

b.)  $y = \frac{1}{2} \sin(x - \pi)$

$$\text{Max} = +\frac{1}{2}$$

$$\text{Min} = -\frac{1}{2}$$

14. Graph the following:

a.)  $y = \sin x$

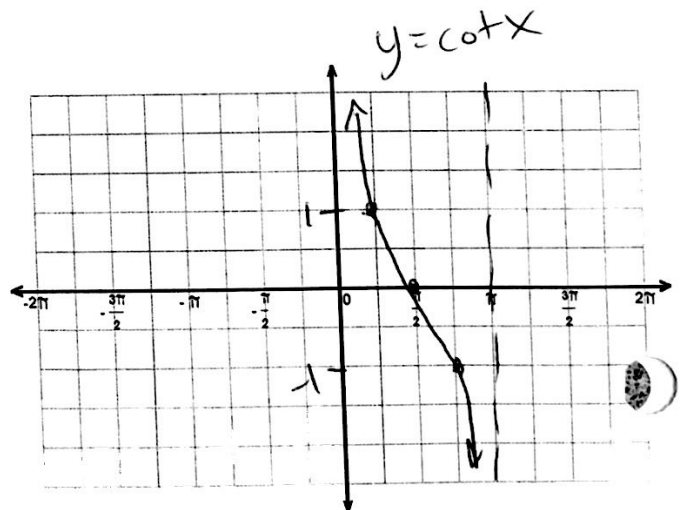
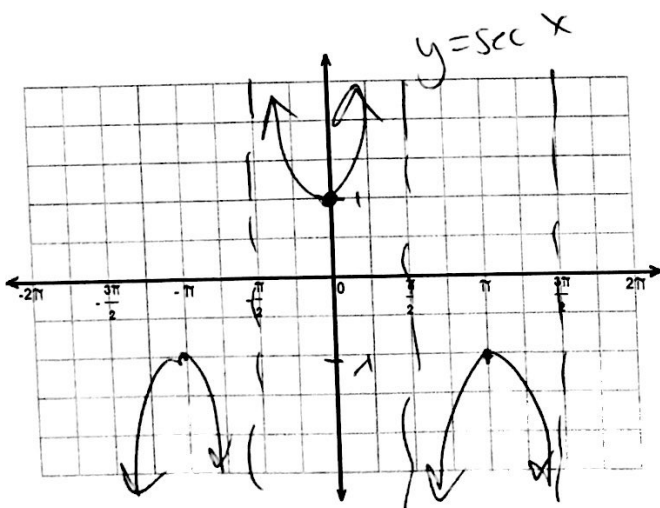
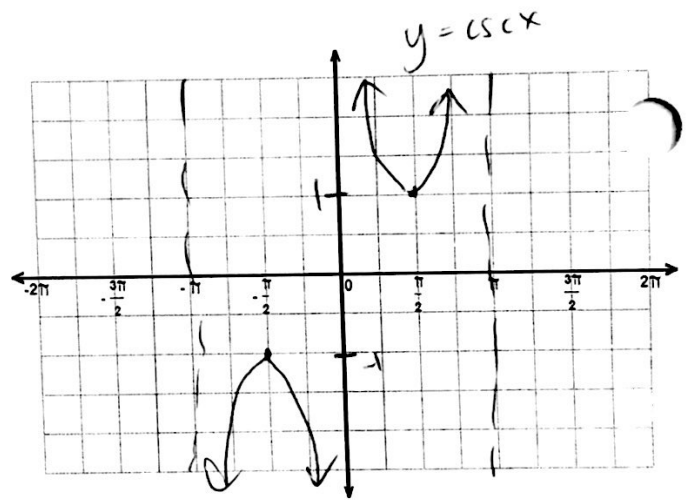
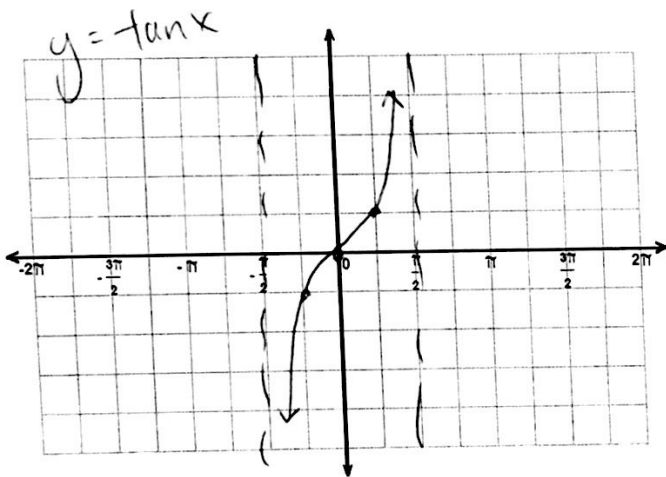
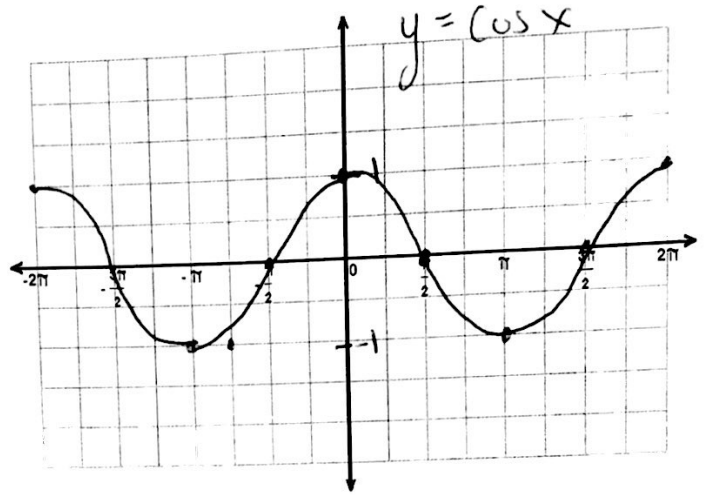
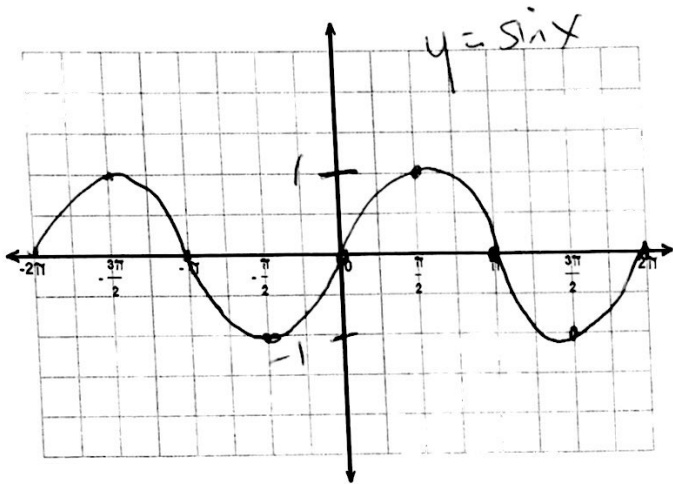
b.)  $y = \cos x$

c.)  $y = \tan x$

d.)  $y = \csc x$

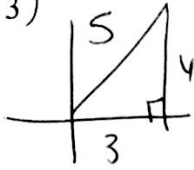
e.)  $y = \sec x$

f.)  $y = \cot x$



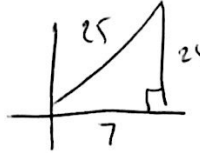
15. Find the exact value of the expression.

a.)  $\sin\left(\arctan\frac{4}{3}\right)$



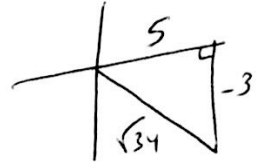
$$\sin \theta = \frac{4}{5}$$

b.)  $\cos\left(\arcsin\frac{24}{25}\right)$



$$\cos \theta = \frac{7}{25}$$

c.)  $\sec\left(\arctan\left(-\frac{3}{5}\right)\right)$



$$\sec \theta = \frac{\sqrt{34}}{5}$$

16. Use a calculator to approximate the value of the expression.

a.)  $\sin^{-1}(0.19)$

$$10.95^\circ$$

b.)  $\arccos(-0.51)$

$$120.66^\circ$$

c.)  $\tan^{-1}(1.32)$

$$52.85^\circ$$

17. Find all solutions for the variable in the interval  $[0, 2\pi)$ .

a.)  $2\sin^2 x + 3\cos x - 3 = 0$

b.)  $2\cos^2 x = \cos x$

c.)  $3\tan^3 x - \tan x = 0$

d.)  $\sin 2x - \cos x = 0$

(b)  $2\cos^2 x - \cos x = 0$

$$\cos x (2\cos x - 1) = 0$$

$$\cos x = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

(a)  $2\sin^2 x + 3\cos x - 3 = 0$

$$2(1 - \cos^2 x) + 3\cos x - 3 = 0$$

$$2 - 2\cos^2 x + 3\cos x - 3 = 0$$

$$2\cos^2 x - 3\cos x + 1 = 0$$

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\cos x = 1$$

$$x = 0$$

(c)  $3\tan^3 x - \tan x = 0$

$$\tan x (3\tan^2 x - 1) = 0$$

$$\tan x = 0$$

$$x = 0, \pi$$

$$\tan^2 x = \frac{1}{3}$$

$$\tan x = \pm \frac{\sqrt{3}}{3}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

(d)  $\sin 2x - \cos x = 0$

$$2\sin x \cos x - \cos x = 0$$

$$\cos x (2\sin x - 1) = 0$$

$$\cos x = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

18. Use half-angle formulas to find the exact value.

$$a.) \cos\left(\frac{\pi}{8}\right) = \sqrt{\frac{1 + \cos\frac{\pi}{4}}{2}} = \sqrt{\frac{1 + \sqrt{2}/2}{2}} = \frac{\sqrt{2 + \sqrt{2}}}{2}$$

$$u = \frac{\pi}{4}$$

$$b.) \tan\left(\frac{3\pi}{8}\right) = \frac{1 - \cos\frac{3\pi}{4}}{\sin\frac{3\pi}{4}} = \frac{1 - (-\sqrt{2}/2)}{\sqrt{2}/2} = \frac{2 + \sqrt{2}}{\sqrt{2}}$$

$$u = \frac{3\pi}{4}$$

$$\frac{2 + \sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}} + \frac{2}{\sqrt{2}} = \frac{2\sqrt{2} + 2}{2} = \sqrt{2} + 1$$

$$c.) \sin\left(\frac{\pi}{12}\right) = \sqrt{\frac{1 - \sqrt{3}/2}{2}} = \frac{\sqrt{2 - \sqrt{3}}}{2}$$

$$u = \frac{\pi}{6}$$

19. Write the expression as the sine, cosine, or tangent of an angle.

$$a.) \cos 60^\circ \cos 10^\circ - \sin 60^\circ \sin 10^\circ \quad \cos 70^\circ$$

$$b.) \frac{\tan 152^\circ - \tan 47^\circ}{1 + \tan 152^\circ \tan 47^\circ} \quad \tan 105^\circ$$

$$c.) \sin \frac{4\pi}{9} \cos \frac{\pi}{8} + \cos \frac{4\pi}{9} \sin \frac{\pi}{8} \quad \sin \frac{41\pi}{72}$$