1.4 Building Functions from Functions

Sum, Difference, Product, and Quotient of Functions

Let *f* and *g* be two functions with overlapping domains. Then, for all x common to both domains, the sum, difference, product, and quotient of *f* and *g* are defined as follows:

1.) $Sum: (f+g)(x) = f(x) + g(x)$

2.) $Difference: (f-g)(x) = f(x) - g(x)$

3.) $Product: \quad (fg)(x) = f(x) \cdot g(x)$

4.) $Quotient: \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$

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Given: $f(x) = x^2$ **and** $g(x) = \sqrt{x+1}$

State the domain of f(x) and g(x):

$f(x): (-\infty, \infty)$ $\quad g(x): x+1 \geq 0$, $x \geq -1$ $\quad [-1, \infty)$

Find the following and state the domain:

1.) $(f+g)(x)$

$= x^2 + \sqrt{x+1}$

D: $[-1, \infty)$

2.) $(f-g)(x)$

$= x^2 - \sqrt{x+1}$

D: $[-1, \infty)$

3.) $(fg)(x)$

$= x^2\sqrt{x+1}$

D: $[-1, \infty)$

4.) $\left(\frac{f}{g}\right)(x)$

$= \frac{x^2}{\sqrt{x+1}}$

D: $(-1, \infty)$

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Composition of Functions:

The composition of function *f* with the function *g* is:

$$(f \circ g)(x) = f(g(x)).$$

The domain of $f \circ g$ is the set of all x in the domain of g such that $g(x)$ is in the domain of *f.*

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Finding the Domain of a Composite Function

1.) Find the domain of the inside function. If there are restrictions, keep them!

2.) Construct the new function. If there are additional restrictions, add them to those in Step 1.

$$f(x) = x^2 - 9 \qquad g(x) = \sqrt{9 - x^2}$$

1.) *Find* : $(f \circ g)(x)$ *and state the domain.*

D of $g(x)$: $9 - x^2 \geq 0$

$(3-x)(3+x) \geq 0$

$[-3, 3]$

$-3 \quad 3$

$(f \circ g)(x) = \sqrt{9-x^2}^{\,2} - 9$

$= 9 - x^2 - 9$

$(f \circ g)(x) = -x^2$

D: $[-3, 3]$

$$f(x) = x^2 \qquad g(x) = \sqrt{2 - x}$$

2.) *Find* : $(f \circ g)(x)$ *and state the domain.*

$(f \circ g)(x) = 2 - x$

D: $(-\infty, 2]$

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Decomposing a Function

For each function h, find functions f and g such that $h(x) = f(g(x))$.

1.) Write the function $h(x) = (3x-5)^3$ as a composition of two functions.

$g(x) = 3x-5$

$f(x) = x^3$

2.) Write the function $h(x) = (x+1)^2 - 3(x+1) + 4$ as a composition of two functions.

$g(x) = x+1$

$f(x) = x^2 - 3x + 4$

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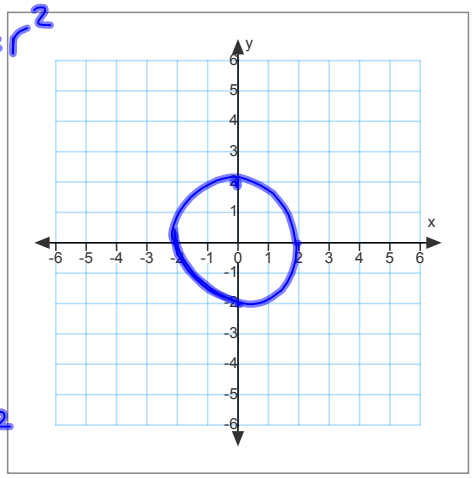
Relations and Implicitly Defined Functions

$Graph: x^2 + y^2 = 4$ $(x-h)^2 + (y-k)^2 = r^2$

$\sqrt{y^2} = \sqrt{-x^2+4}$

$y = \pm\sqrt{4-x^2}$

$y = \sqrt{4-x^2}$ $y = -\sqrt{4-x^2}$



Is $x^2 + y^2 = 4$ a function? Prove your answer algebraically.

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Practice: Find two functions defined implicity by the given relation.

1.) $x^2 + 2xy + y^2 = 1$

$\sqrt{(x+y)^2} = \sqrt{1}$

$x+y = \pm 1$

$y = -x \pm 1$

$y = -x+1$

$y = -x-1$

2.) $x + |y| = 1$

$|y| = -x+1$

$y = -x+1$

$y = -(-x+1)$

$y = x-1$

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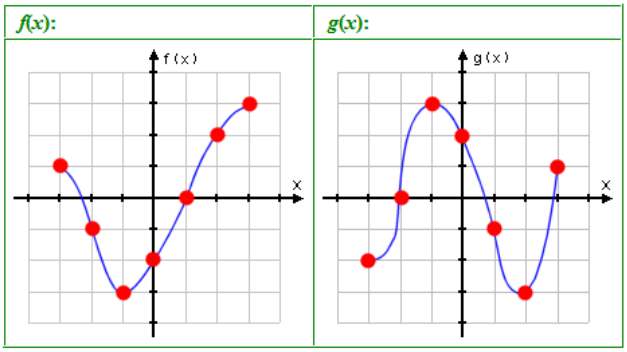
Composition of Functions Graphically



Find: 1.) $(g \circ f)(3) = 1$

$g(3)$

2.) $(f \circ g)(-1) = 3$

$f(3) = 3$

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