2.4 Real Zeros of Polynomial Functions

Divide Using Long Division

1.) $\left(6x^3 - 19x^2 + 16x - 4\right) \div (x - 2)$

$$6x^2 - 7x + 2$$

$$x - 2 \enclose{longdiv}{\cancel{6x^3} - 19x^2 + 16x - 4}$$

$$-\ \ \cancel{6x^3} - 12x^2$$

$$-7x^2 + 16x$$

$$-\ \ \cancel{-7x^2} + 14x$$

$$2x - 4$$

$$-\ \ 2x - 4$$

$$0$$

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Divide Using Long Division

2.) $\left(x^3 - x + 1\right) \div (x - 1)$

$$x^2 + x + \frac{1}{x - 1}$$

$$x - 1 \enclose{longdiv}{x^3 + 0x^2 - x + 1}$$

$$-\ \ x^3 - x^2$$

$$x^2 - x$$

$$-\ \ x^2 - x$$

$$0 \quad + 1$$

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Divide Using Long Division

3.) $\left(2x^4+4x^3-5x^2+3x-2\right)\div\left(x^2+2x-3\right)$

$$\boxed{2x^2+1+\frac{x+1}{x^2+2x-3}}$$

$$x^2+2x-3 \,\big)\, 2x^4+4x^3-5x^2+3x-2$$

$$-\ \cancel{2x^4}+\cancel{4x^3}-6x^2$$

$$-\ \cancel{x^2}+3x-2$$

$$\cancel{x^2}+2x-3$$

$$x+1$$

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Divide Using Synthetic Division

4.) $\left(x^4-10x^2-2x+4\right)\div\left(x+3\right)$

$$\begin{array}{r|rrrrr} & 1 & 0 & -10 & -2 & 4 \\ -3 & \downarrow & -3 & 9 & 3 & -3 \\ \hline & 1 & -3 & -1 & 1 & 1 \end{array}$$

$$\boxed{x^3-3x^2-x+1+\frac{1}{x+3}}$$

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The Remainder Theorem

If a polynomial $f(x)$ *is divided by* $x-k$ *, the remainder is* $r = f(k)$

Use The Remainder Factor Theorem to evaluate the following function at $x=-2$

5.) $f(x)=3x^3+8x^2+5x-7$

$f(-2) = 3(-2)^3 + 8(-2)^2 + 5(-2) - 7$

$= -24 + 32 - 10 - 7$

$= -9$

$$\begin{array}{r|rrrr} -2 & 3 & 8 & +5 & -7 \\ & \downarrow & -6 & -4 & -2 \\ \hline & 3 & 2 & 1 & \boxed{-9} \end{array}$$

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Use The Remainder Theorem to evaluate the following function at $x=-3$

6.) $f(x)=2x^3-4x^2+1$

$$\begin{array}{r|rrrr} -3 & 2 & -4 & 0 & 1 \\ & \downarrow & -6 & 30 & -90 \\ \hline & 2 & -10 & 30 & -89 \end{array}$$

$f(-3) = 89$

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The Factor Theorem

A polynomial $f(x)$ ***has a factor*** $(x-k)$ ***if and only if*** $f(k)=0$

Show that $(x-2)$ ***and*** $(x+3)$ ***are factors and find the remaining factors.***

7.) $f(x)=2x^4+7x^3-4x^2-27x-18$

$$\begin{array}{r|rrrrr} 2 & 2 & 7 & -4 & -27 & -18 \\ & \downarrow & 4 & 22 & 36 & 18 \\ \hline -3 & 2 & 11 & 18 & 9 & \boxed{0} \\ & \downarrow & -6 & -15 & -9 & \\ \hline & 2 & 5 & 3 & \boxed{0} & \end{array}$$

$2x^2+5x+3=0$

$(2x+3)(x+1)=0$

$\boxed{x=-\frac{3}{2},-1,2,-3}$

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The Rational Zero Test

Possible Rational Zeros $=$ $\dfrac{\textit{Factors of the Constant Term}}{\textit{Factors of the Leading Coefficient}}$

Use The Rational Zero Test to determine the possible zeros. Then find the remaining zeros.

8.) $f(x)=2x^3+3x^2-8x+3$

$\dfrac{\pm 1,3}{\pm 1,2} \rightarrow \pm 1, \frac{1}{2}, 3, \frac{3}{2}$

$$\begin{array}{r|rrrr} 1 & 2 & 3 & -8 & 3 \\ & \downarrow & 2 & 5 & -3 \\ \hline & 2 & 5 & -3 & \boxed{0} \end{array}$$

$2x^2+5x-3=0$

$(2x-1)(x+3)=0$

$\boxed{x=\frac{1}{2},-3,1}$

9.) $f(x)=x^3-6x^2+7x+4$

$\pm 1,2,4$

$$\begin{array}{r|rrrr} 4 & 1 & -6 & 7 & 4 \\ & \downarrow & 4 & -8 & -4 \\ \hline & 1 & -2 & -1 & \boxed{0} \end{array}$$

$x^2-2x-1=0$

$\dfrac{2\pm\sqrt{4-4(1)(-1)}}{2}=\dfrac{2\pm\sqrt{8}}{2}=\dfrac{2\pm 2\sqrt{2}}{2}$

$\boxed{=1\pm\sqrt{2},4}$

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Upper and Lower Bound Tests for Real Zeros

Let f be a polynomial function of degree $n \geq 1$ with a positive leading coefficient. Suppose $f(x)$ is divided by $x - k$ using synthetic division.

- If $k \geq 0$ and every number in the last line is nonnegative (positive or zero), then k is an *upper bound* for the real zeros of f.
- If $k \leq 0$ and the numbers in the last line are alternately nonnegative and nonpositive, then k is a *lower bound* for the real zeros of f.

$$f(x) = 2x^4 - 7x^3 - 8x^2 + 14x + 8$$

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Use synthetic division to verify the upper and lower bounds of the real zeros of the function.

$$10.)\ f(x) = x^4 - 4x^3 + 15$$

Upper bound: $x = 4$ ***Lower bound:*** $x = -1$

$$\begin{array}{r|rrrrr} 4 & 1 & -4 & 0 & 0 & 15 \\ & \downarrow & 4 & 0 & 0 & 0 \\ \hline & 1 & 0 & 0 & 0 & 15 \end{array}$$

All +/0

Upper ✓

$$\begin{array}{r|rrrrr} -1 & 1 & -4 & 0 & 0 & 15 \\ & \downarrow & -1 & 5 & -5 & 5 \\ \hline & 1 & -5 & 5 & -5 & 20 \end{array}$$

Alt +/-

Lower

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11.) Find the polynomial function with leading coefficient 2 that has a degree of 3 with -2, 1, and 4 as zeros.

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