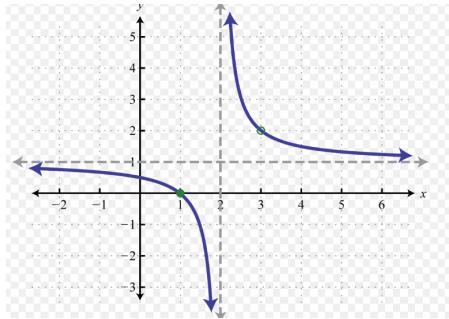


2.6-2.7 Rational Functions

Rational Function: a function whose rule can be written as a ratio of two polynomials. Its graph is a hyperbola, which has two separate branches.

$$f(x) = \frac{p(x)}{q(x)}$$

where $p(x)$ and $q(x)$ are polynomial functions, and $q(x) \neq 0$



Sep 10-12:04 PM

Parent Rational Function

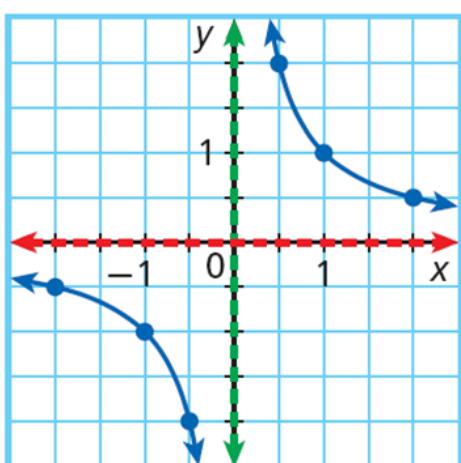
$$f(x) = \frac{1}{x}$$

Domain: $x \neq 0$

Range: $y \neq 0$

Vertical Asymptote(s): $x = 0$

Horizontal Asymptote(s): $y = 0$



Nov 20-12:02 PM

Transformations of Simple Rational Functions

Simple Rational Function:

$$f(x) = \frac{a}{x-h} + k$$

$|a| \rightarrow$ vertical stretch or compression factor
 $a < 0 \rightarrow$ reflection across the x-axis

$k \rightarrow$ vertical translation
 down for $k < 0$; up for $k > 0$

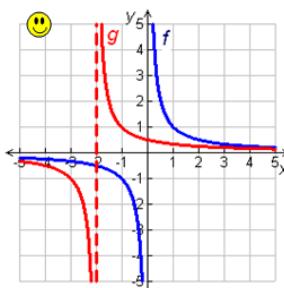
$$f(x) = \frac{a}{x-h} + k$$

$h \rightarrow$ horizontal translation
 left for $h < 0$; right for $h > 0$

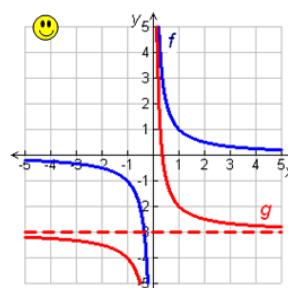
Sep 10-11:43 AM

Using the graph of $f(x) = \frac{1}{x}$ as a guide, describe the transformation and ~~graph~~ each function.

A. $g(x) = \frac{1}{x+2}$
 Left 2



B. $g(x) = \frac{1}{x} - 3$
 Sketch
 Down 3



What do you notice about the location of the vertical and horizontal asymptotes?

H.A. $y = k$

V.A. $x = h$



Sep 10-11:46 AM

Rational Functions

For a rational function of the form $f(x) = \frac{a}{x - h} + k$,

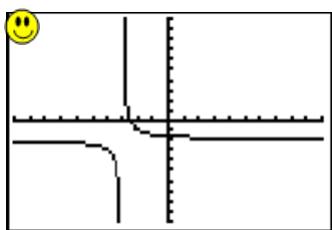
- the graph is a hyperbola.
- there is a vertical asymptote at the line $x = h$, and the domain is $\{x | x \neq h\}$.
- there is a horizontal asymptote at the line $y = k$, and the range is $\{y | y \neq k\}$.

Identify the domain, range and all asymptotes of the given functions.

$$1.) f(x) = \frac{1}{x+3} - 2$$

Vertical Asymptote(s): $x = -3$

Domain: $x \neq -3$



Horizontal Asymptote(s): $y = -2$

Range: $y \neq -2$

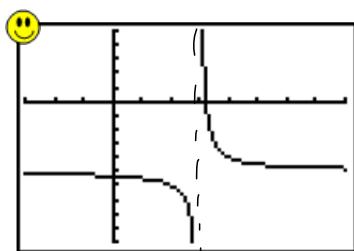
Sep 10-11:50 AM

Identify the domain, range and all asymptotes of the given functions.

$$2.) f(x) = \frac{1}{x-3} - 5$$

Vertical Asymptote(s): $x = 3$

Domain: $x \neq 3$
 $(-\infty, 3) \cup (3, \infty)$



Horizontal Asymptote(s): $y = -5$

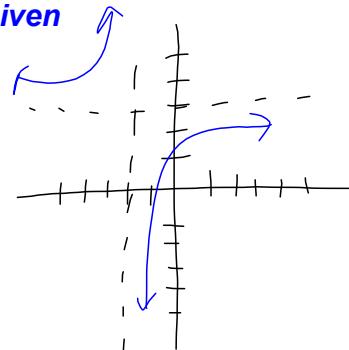
Range: $y \neq -5$

Sep 10-11:58 AM

Describe the transformations of each rational function.

Identify the domain, range and all asymptotes of the given function. *Draw a sketch of the graph to help.

$$3.) f(x) = \frac{-2}{x+2} + 3$$



Vertical Stretch $\times 2$

Transformations: $R_x ; \leftarrow 2, \uparrow 3$

Vertical Asymptote(s): $x = -2$

Domain: $x \neq -2$

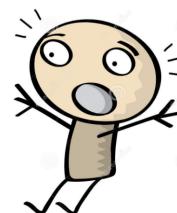
Horizontal Asymptote(s): $y = 3$

Range: $y \neq 3$

Sep 10-12:01 PM

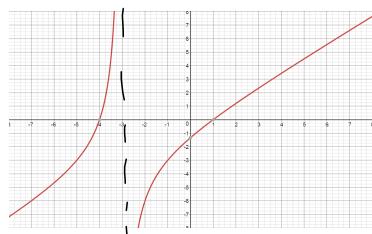
Not all Rational Functions are Simple!

General Form: $f(x) = \frac{p(x)}{q(x)}$; $q(x) \neq 0$



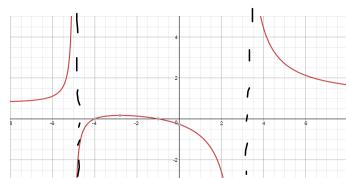
Examples:

$$a.) f(x) = \frac{x^2 + 3x - 4}{x + 3}$$



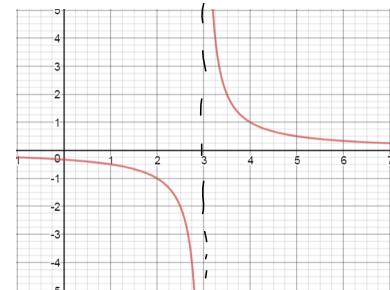
*Not a Hyperbola

$$b.) f(x) = \frac{x^2 + 5x + 4}{x^2 + 2x - 15}$$



*Not a Hyperbola

$$c.) f(x) = \frac{x + 4}{x^2 + x - 12}$$



Hyperbola

Sep 10-12:12 PM

Identifying Vertical Asymptotes of any Rational Function:

Find the value of x for which the denominator = 0.

$$a.) f(x) = \frac{x^2 + 3x - 4}{x + 3} \rightarrow \frac{(x+4)(x-1)}{x+3} = V.A. \quad x = -3$$

$$b.) f(x) = \frac{x^2 + 5x + 4}{x^2 + 2x - 15} \rightarrow \frac{(x+4)(x+1)}{(x+5)(x-3)} = V.A. \quad x = -5 \\ x = 3$$

$$c.) f(x) = \frac{x+4}{x^2 + x - 12} \rightarrow \frac{x+4}{(x+4)(x-3)} = \frac{1}{x-3} \quad V.A. \quad x = 3 \\ \text{hole } @ x = -4 \\ (-4, -\frac{1}{7})$$



Hmmm....Something's up with example c....

A **hole** is an omitted point in a graph.

Holes in Graphs Rational Functions

If a rational function has the same factor $x - b$ in both the numerator and the denominator, then there is a hole in the graph at the point where $x = b$, unless the line $x = b$ is a vertical asymptote.

Sep 10-12:27 PM

You try...

Identify any holes and vertical asymptotes for each rational function.

$$4.) f(x) = \frac{(x+5)(x-5)}{x^2 + 12x + 35} = \frac{(x+5)(x-5)}{(x+7)(x+5)} = \frac{x-5}{x+7}$$

Hole @ $x = -5$ $(-5, -5)$

V.A.: $x = -7$

$$5.) f(x) = \frac{x+4}{2x^2 + 5x - 3} = \frac{x+4}{(2x-1)(x+3)}$$

V.A. $x = -3$

$x = \frac{1}{2}$

$$6.) f(x) = \frac{x^2 - 9}{3x^2 + 8x - 3}$$

V.A. $x = \frac{1}{3}$

Hole @ $x = -3$ $(-3, \frac{3}{5})$

Sep 10-12:36 PM

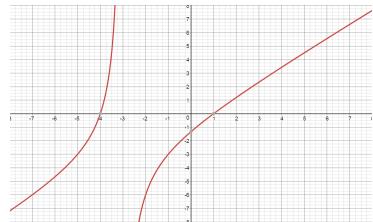
Identifying Horizontal Asymptotes of any Rational Function:

The existence and location of a horizontal asymptote depends on the degree of the polynomials that make up the rational function.

Examples:

$$a.) f(x) = \frac{x^2 + 3x - 4}{x + 3}$$

$N > D$



Horizontal Asymptote?

No.

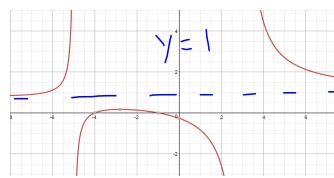
$N = D$

$$b.) f(x) = \frac{x^2 + 5x + 4}{x^2 + 2x - 15}$$

$$c.) f(x) = \frac{x + 4}{x^2 + x - 12}$$

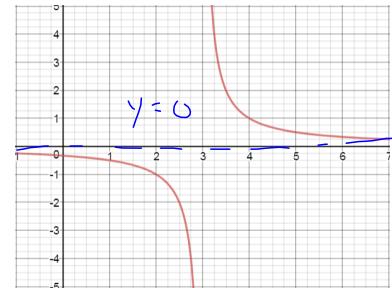
$N < D$

$$c.) f(x) = \frac{x + 4}{x^2 + x - 12}$$



Horizontal Asymptote?

Yes



Horizontal Asymptote?

Sep 10-12:41 PM

Identifying Horizontal Asymptotes of any Rational Function:

$$f(x) = \frac{N(x)}{D(x)}; D(x) \neq 0$$

If N degree $>$ D degree: H.A.: Does not exist

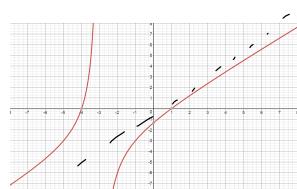
If N degree $=$ D degree: H.A.: $y =$ ratio of leading coefficients

$$f(x) = \frac{\text{leading coefficient of } N}{\text{leading coefficient of } D}$$

If N degree $<$ D degree: H.A.: $y = 0$

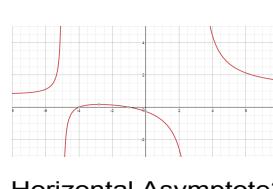
Examples:

$$a.) f(x) = \frac{x^2 + 3x - 4}{x + 3}$$



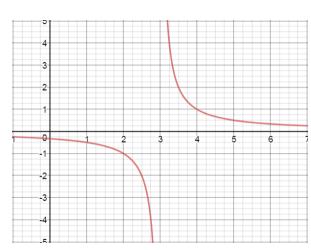
Horizontal Asymptote?

$$b.) f(x) = \frac{x^2 + 5x + 4}{x^2 + 2x - 15}$$



Horizontal Asymptote?

$$c.) f(x) = \frac{x + 4}{x^2 + x - 12}$$



Horizontal Asymptote?

Sep 10-12:50 PM

You try...

Identify the horizontal asymptote for each rational function. If there is not a horizontal asymptote, explain why.

$$7.) f(x) = \frac{x^2 - 25}{x^2 + 12x + 35}$$

$$y = 1$$

$$8.) f(x) = \frac{x+4}{2x^2 + 5x - 3}$$

$$y = 0$$

$$9.) f(x) = \frac{x^2 - 9}{3x^2 + 8x - 3}$$

$$y = 1/3$$

Sep 10-12:57 PM

Putting it all together..

Find all vertical/horizontal asymptotes and identify any holes if they exist.

$$(x+4)(x-4)$$

$$10.) f(x) = \frac{x^2 - 16}{x \neq 4} = x+4$$

$x \neq 4$ hole @ $x = 4$

$$11.) f(x) = \frac{x^2}{x^2 - 9}$$

$$y = 1$$

$N > D$

~~DNE~~

$$12.) f(x) = \frac{5x - 25}{x^2 - x - 20} = \frac{5(x-5)}{(x-5)(x+4)}$$

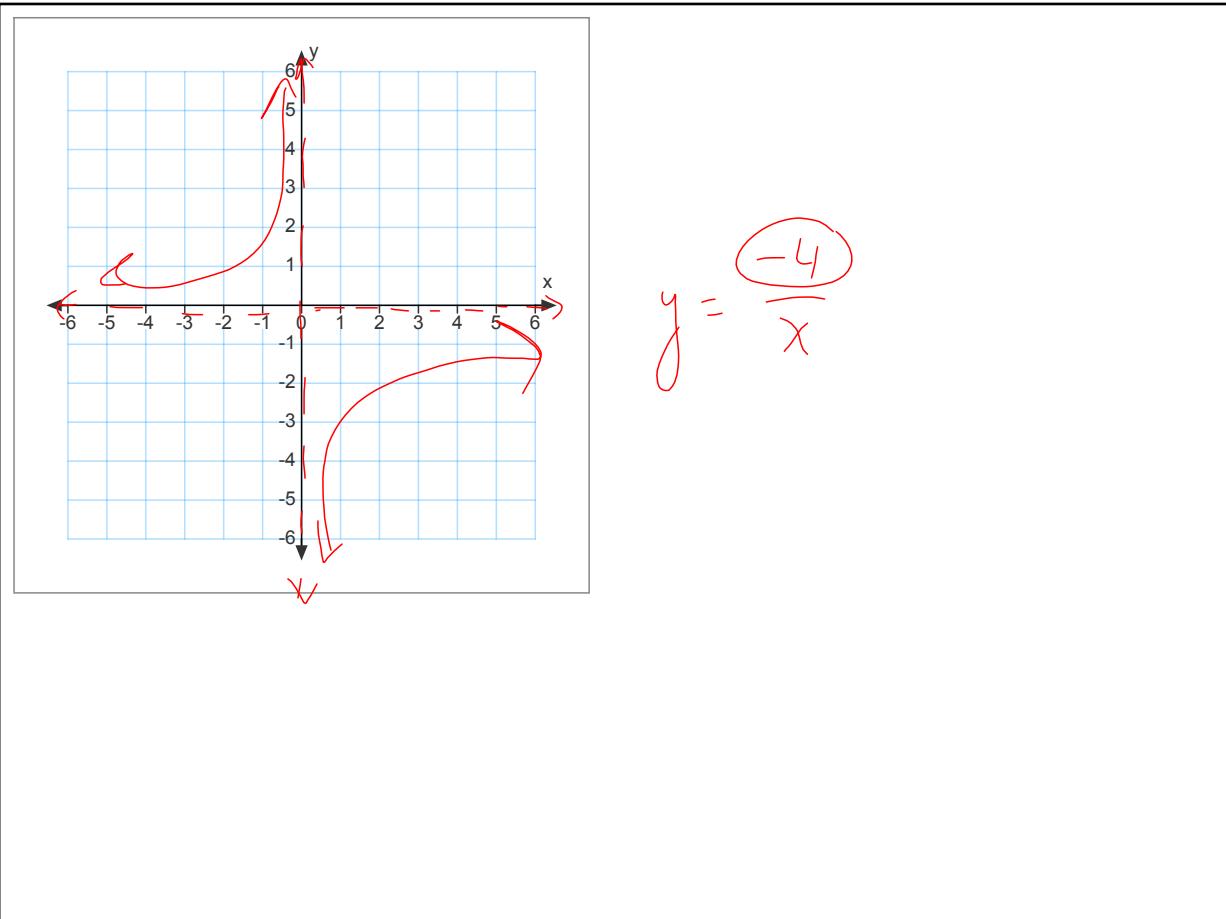
$x = \pm 3$

$$y = 0$$

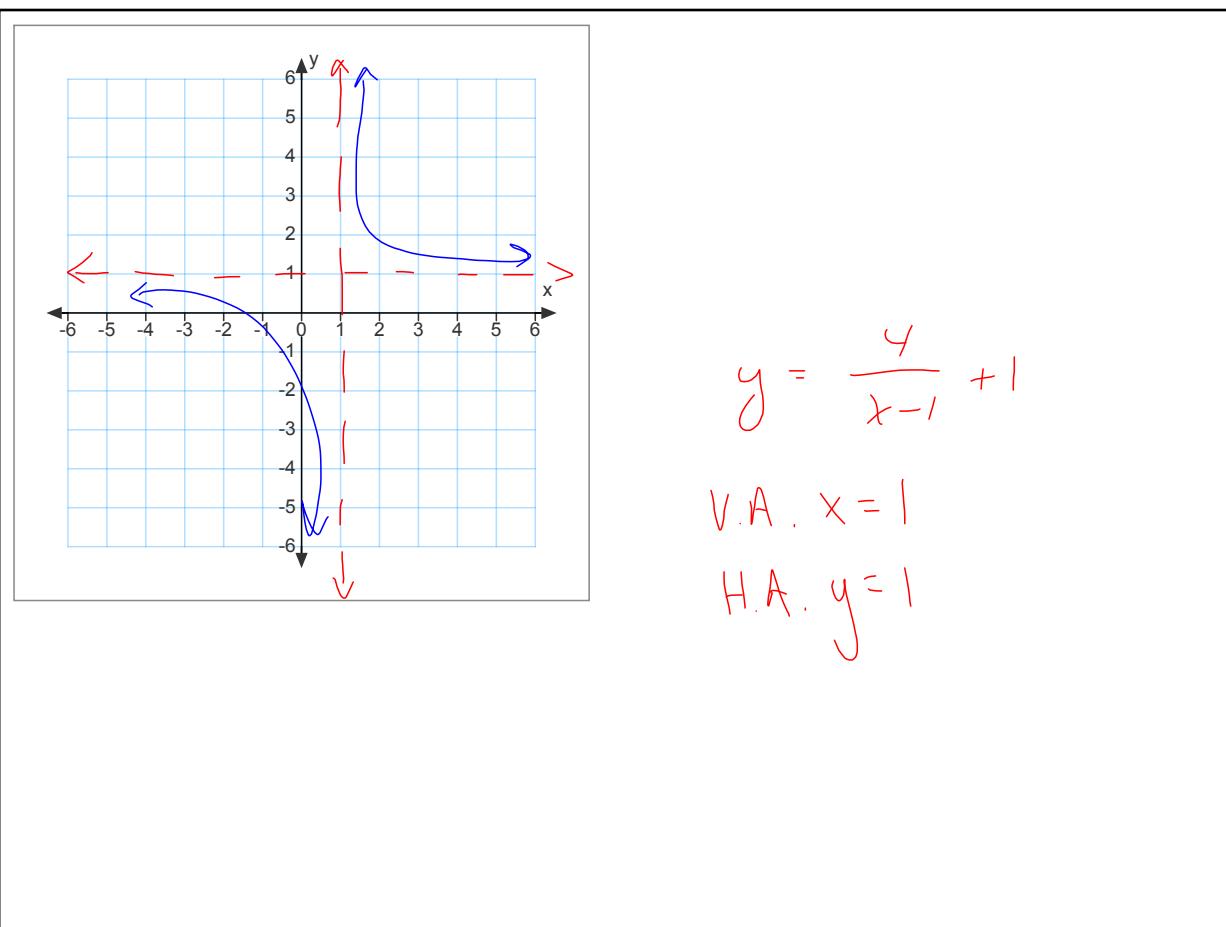
hole @ $x = 5$ $(5, 5)$

$$x = -4$$

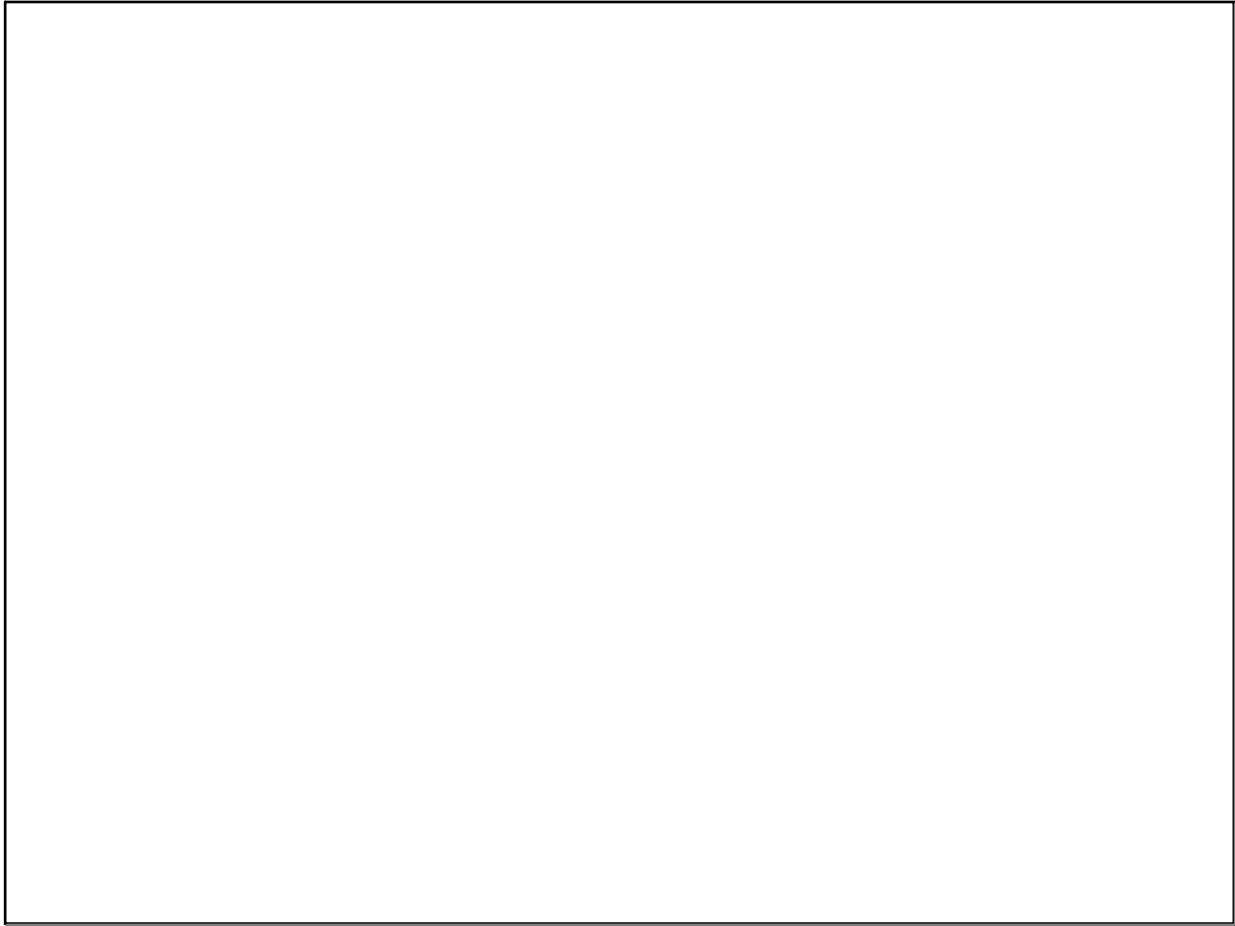
May 5-7:47 AM



Sep 13-10:06 AM



Sep 13-10:08 AM



Sep 10-1:07 PM