$21x^2 + 25x - 4$

$-84x^2$

$28x \quad -3x$

$|21x^2 + 28x| - 3x - 4|$

$7x(3x+4) - 1(3x+4)$

$(7x-1)(3x+4)$

Sep 13-7:42 AM

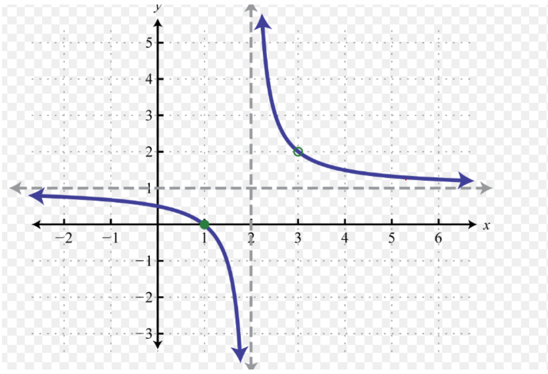
Sep 13-7:46 AM

2.6-2.7 Rational Functions

Simple Rational Function : a function whose rule can be written as a ratio of two polynomials. Its graph is a hyperbola, which has two separate branches.

$$f(x) = \frac{p(x)}{q(x)}$$

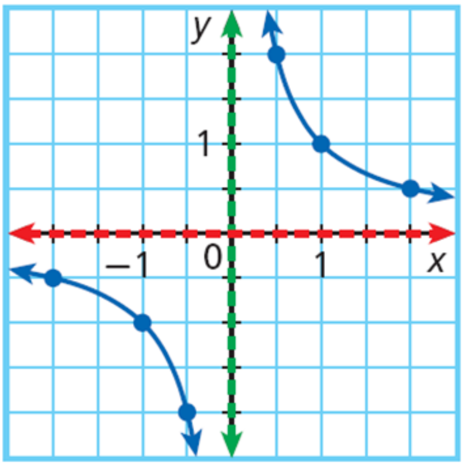
where p(x) and q(x) are polynomial functions, and $q(x) \neq 0$



Sep 10-12:04 PM

Parent Rational Function

$$f(x) = \frac{1}{x}$$



Domain: $x \neq 0$

Range: $y \neq 0$

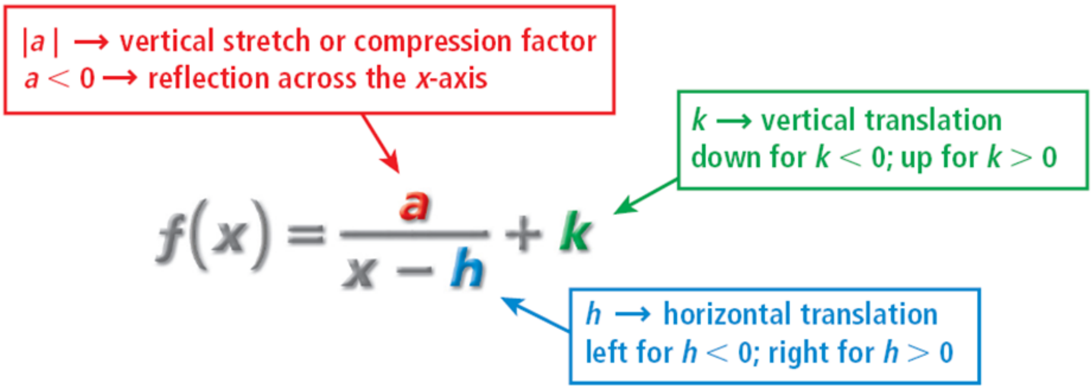
Vertical Asymptote(s): $x = 0$

Horizontal Asymptote(s): $y = 0$

Nov 20-12:02 PM

Transformations of Simple Rational Functions

Simple Rational Functions: $y = \frac{a}{x-h} + k$

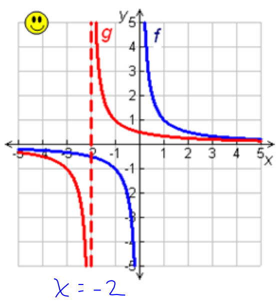


Sep 10-11:43 AM

Using the graph of $f(x) = \frac{1}{x}$ as a guide, describe the transformation and graph each function.

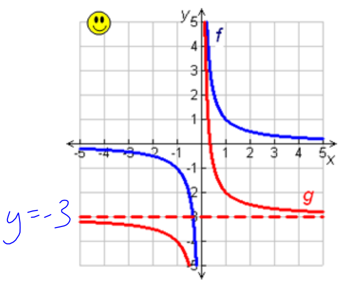
A. $g(x) = \frac{1}{x+2}$

* Left 2



B. $g(x) = \frac{1}{x} - 3$

* Down 3



What do you notice about the location of the vertical and horizontal asymptotes?

Vertical Asymptote: $x = h$

opposite of what you see

Horizontal Asymptote: $y = k$

Sep 10-11:46 AM

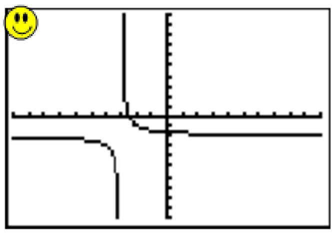
Rational Functions

For a rational function of the form $f(x) = \dfrac{a}{x - h} + k$,

- the graph is a hyperbola.
- there is a vertical asymptote at the line $x = h$, and the domain is $\{x \mid x \neq h\}$.
- there is a horizontal asymptote at the line $y = k$, and the range is $\{y \mid y \neq k\}$.

Identify the domain, range and all asymptotes of the given functions.

1.) $y = \dfrac{1}{x+3} - 2$



Vertical Asymptote(s): $x = -3$

Domain: $x \neq -3$

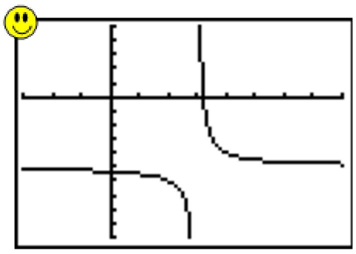
Horizontal Asymptote(s): $y = -2$

Range: $y \neq -2$

Sep 10-11:50 AM

Identify the domain, range and all asymptotes of the given functions.

2.) $y = \dfrac{1}{x-3} - 5$



Vertical Asymptote(s): $x = 3$

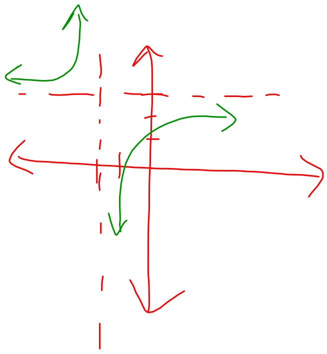
Domain: $x \neq 3$

Horizontal Asymptote(s): $y = -5$

Range: $y \neq -5$

Sep 10-11:58 AM

Describe the transformations of the simple rational function. Identify the domain, range and all asymptotes of the given functions. *Draw a sketch of the graph to help.



3.) $y = \frac{-2}{x+2} + 3$

Transformations: Rx, Vertical Stretch x2, Left 2, Up 3

Vertical Asymptote(s): x = -2

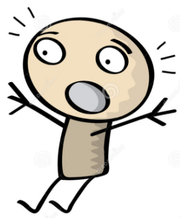
Domain: x ≠ -2

Horizontal Asymptote(s): y = 3

Range: y ≠ 3

Sep 10-12:01 PM

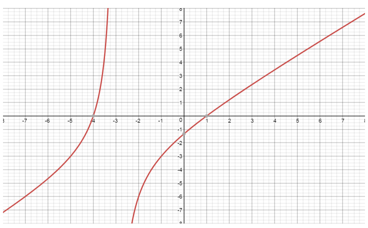
Not all Rational Functions are Simple!



General Form: $f(x) = \frac{p(x)}{q(x)}; q(x) \neq 0$

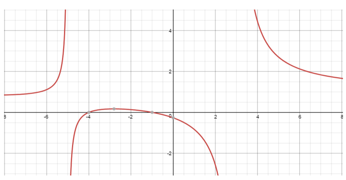
Examples:

$a.)\ f(x) = \frac{x^2 + 3x - 4}{x+3}$



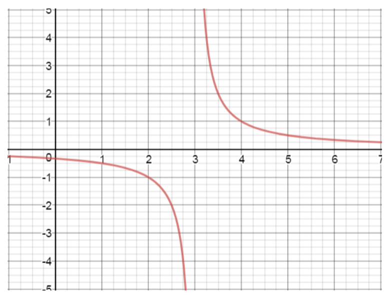
*Not a Hyperbola

$b.)\ f(x) = \frac{x^2 + 5x + 4}{x^2 + 2x - 15}$



*Not a Hyperbola

$c.)\ f(x) = \frac{x + 4}{x^2 + x - 12}$



Sep 10-12:12 PM

Calculating Vertical Asymptotes of ALL Rational Functions:

Find the value of x for which the denominator = 0.

a.) $f(x)=\dfrac{x^2+3x-4}{x+3}$ V.A. $x=-3$

b.) $f(x)=\dfrac{x^2+5x+4}{x^2+2x-15} = \dfrac{(x+4)(x+1)}{(x+5)(x-3)}$ V.A. $x=-5$, $x=3$

c.) $f(x)=\dfrac{x+4}{x^2+x-12}$ $\dfrac{x+4}{(x+4)(x-3)} = \dfrac{1}{x-3}$ V.A. $x=3$

hole @ $x=-4$ ↳ $\left(-4, \frac{1}{-7}\right)$

Hmmm....Something's up with example c....

A **hole** is an omitted point in a graph.

Holes in Graphs **Rational Functions**

If a rational function has the same factor $x - b$ in both the numerator and the denominator, then there is a hole in the graph at the point where $x = b$, unless the line $x = b$ is a vertical asymptote.

Sep 10-12:27 PM

You try...
Find any holes and vertical asymptotes in each rational function.

4.) $f(x)=\dfrac{x^2-25}{x^2+12x+35}$

$= \dfrac{(x+5)(x-5)}{(x+5)(x+7)} = \dfrac{x-5}{x+7}$

hole @ $x=-5$ $(-5,-5)$ V.A. $x=-7$

5.) $f(x)=\dfrac{x+4}{2x^2+5x-3}$

$= \dfrac{x+4}{(2x-1)(x+3)}$

V.A. $x=-3$, $x=1/2$

6.) $f(x)=\dfrac{x^2-9}{3x^2+8x-3}$

$= \dfrac{(x+3)(x-3)}{(3x-1)(x+3)} = \dfrac{x-3}{3x-1}$

hole @ -3 $(-3, 3/5)$

V.A. $x=1/3$

Sep 10-12:36 PM

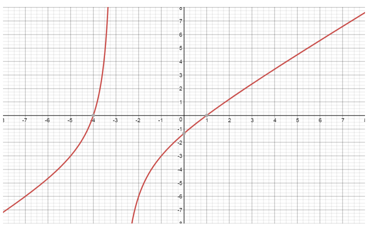
Sep 13-8:24 AM

Calculating Horizontal Asymptotes of ALL Rational Functions:

The existence and location of a horizontal asymptote depends on the degree of the polynomials that make up the rational function.

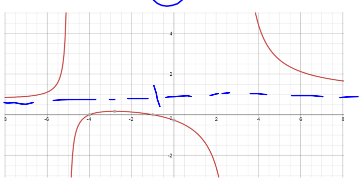
Examples:

a.) $f(x)=\dfrac{x^2+3x-4}{x+3}$



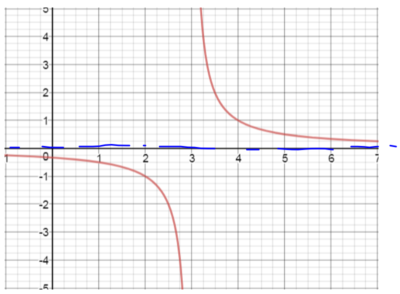
Horizontal Asymptote?

b.) $f(x)=\dfrac{x^2+5x+4}{x^2+2x-15}$



Horizontal Asymptote?

c.) $f(x)=\dfrac{x+4}{x^2+x-12}$



Horizontal Asymptote?

Sep 10-12:41 PM

Calculating Horizontal Asymptotes of ALL Rational Functions:

$$f(x)=\frac{N(x)}{D(x)};\ D(x)\neq 0$$

If *N* degree > *D* degree: H.A.: Does not exist → slant asymptote exists

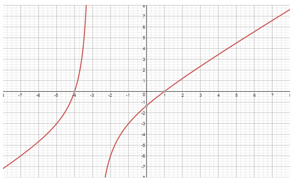
If *N* degree = *D* degree: H.A.: *y* = ratio of leading coefficients

$$f(x)=\frac{\text{leading coefficient of } N}{\text{leading coefficient of } D}$$

If *N* degree < *D* degree: H.A.: *y* = 0

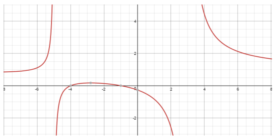
Examples:

$a.)\ f(x)=\dfrac{x^2+3x-4}{x+3}$



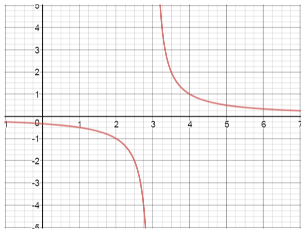
Horizontal Asymptote?

$b.)\ f(x)=\dfrac{x^2+5x+4}{x^2+2x-15}$



Horizontal Asymptote?

$c.)\ f(x)=\dfrac{x+4}{x^2+x-12}$



Horizontal Asymptote?

Sep 10-12:50 PM

You try...
Find any horizontal asymptotes if they exist.

$7.)\ f(x)=\dfrac{x^2-25}{x^2+12x+35}$

$y=1$

$8.)\ f(x)=\dfrac{x+4}{2x^2+5x-3}$

$y=0$

$9.)\ f(x)=\dfrac{x^2-9}{3x^2+8x-3}$

$y=1/3$

Sep 10-12:57 PM

Putting it all together..
Find all vertical/horizontal asymptotes and identify any holes if they exist.

10.) $f(x)=\frac{x^2-16}{x-4}$

$\frac{(x+4)(x-4)}{x-4}$ → $x+4$

hole @ 4
(4, 8) NO H.A./V.A.

11.) $f(x)=\frac{x^2}{x^2-9}$

$\frac{x^2}{(x+3)(x-3)}$

V.A. $x=\pm 3$
H.A. $y=1$

12.) $f(x)=\frac{5x-25}{x^2-x-20}$

$\frac{5(x-5)}{(x-5)(x+4)}$

hole @ 5
(5, 5/9)
V.A. $x=-4$ $y=0$

May 5-7:47 AM

Sep 10-1:07 PM