

Do Now: Rewrite in radical notation.

root of _____ to the _____

Recall: $x^{\frac{m}{n}} = \sqrt[n]{x^m}$

Rewrite each expression using radical notation.

1.) $13^{\frac{3}{4}}$ $\sqrt[4]{13^3}$

2.) $7^{\frac{5}{3}}$ $\sqrt[3]{7^5}$

3.) $8^{\frac{1}{6}}$ $\sqrt[6]{8}$

4.) $4^{\frac{2}{3}}$ $\sqrt[3]{4^2}$

5.) $2^{\frac{5}{4}}$ $\sqrt[4]{2^5}$

Rewrite each expression using rational exponent notation.

6.) $\sqrt[3]{5}$ $5^{\frac{1}{3}}$

7.) $(\sqrt[3]{8})^7$ $8^{\frac{7}{3}}$

8.) $(\sqrt[4]{13})^3$ $13^{\frac{3}{4}}$

9.) $(\sqrt[5]{11})^5$ $11^{\frac{5}{5}}$

10.) $(\sqrt[3]{4})^9$ $4^{\frac{9}{3}}$

11.) $64^{\frac{1}{3}}$ $\sqrt[3]{64} = 4$

12.) $(-125)^{\frac{2}{3}}$ $\sqrt[3]{-125}^2 = 25$

13.) $-\left(243^{\frac{3}{5}}\right)$

$-\left(\sqrt[5]{243^3}\right)$

-27

14.) $(-216)^{-\frac{1}{3}}$

$\sqrt[3]{-216}^{-1}$

$-6^{-1} = -\frac{1}{6}$

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3.1 Exponential Functions

Exponential Function: $f(x) = a \cdot b^x = ab^x$

Determine if the function is an exponential function.

7.2^{-1 \cdot x}

1.) $f(x) = 3^x$ Yes

4.) $k(x) = 7 \cdot 2^{-x}$ Yes
 $7 \left(\frac{1}{2}\right)^x$

2.) $g(x) = 6x^{-4}$ No

5.) $q(x) = 5 \cdot 6^x$

No

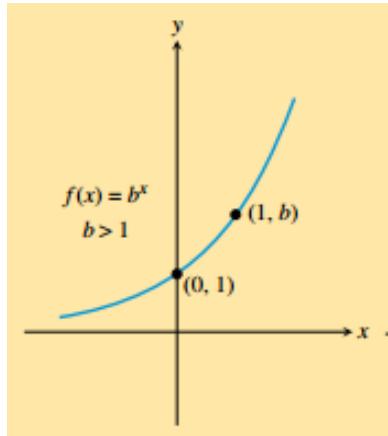
3.) $h(x) = -2 \cdot 1.5^x$ Yes

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Exponential Growth vs Exponential Decay

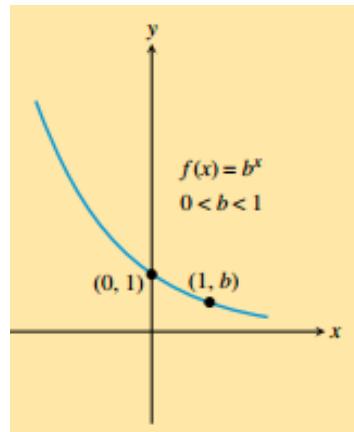
Exponential Growth:

$$f(x) = a \cdot b^x; a > 0 \text{ and } b > 1$$



Exponential Decay:

$$f(x) = a \cdot b^x; a > 0 \text{ and } 0 < b < 1$$



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Finding an Exponential Function from Its Table of Values

Determine a formula for the exponential function whose values are given in the table.

$$ab^x$$

6.) $f(x)$

$$f(x) = \frac{3}{2} \cdot \frac{1}{2}^x$$

7.) $g(x)$

$$g(x) = 12 \cdot \frac{1}{3}^x$$

Table 3.6 Values for Two Exponential Functions

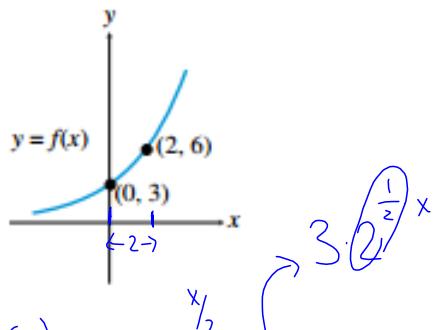
x	$f(x)$	$g(x)$
-2	6	108
-1	3	36
0	3/2	12
1	3/4	4
2	3/8	4/3

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Finding an Exponential Function from Its Table of Values

Determine a formula for the exponential function whose values are given.

8.)

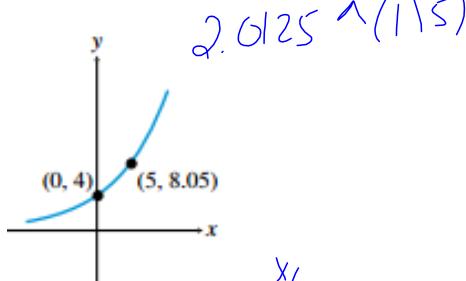


$$f(x) = 3 \cdot 2^{\frac{x}{2}}$$

$$f(x) = 3\sqrt{2}^x$$

$$f(x) = 3(1.4142)^x$$

9.)



$$f(x) = 4(2.0125)^{\frac{x}{5}}$$

$$f(x) = 4\sqrt[5]{2.0125}^x$$

$$f(x) = 4(1.1501)^x$$

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Table 3.11 Values for Two Exponential Functions

x	$f(x)$	$g(x)$
-2	1.472	-9.0625
-1	1.84	-7.25
0	2.3	-5.8
1	2.875	-4.64
2	3.59375	-3.7123

10.) $f(x)$

$$f(x) = 2.3(1.25)^x$$

Exponential growth

11.) $g(x)$

$$g(x) = -5.8(-4/5)^x$$

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Exponential Growth and Decay Model

$$f(x) = P(1 \pm r)^t$$

initial amount time
 ↓ decay factor growth factor

Tell whether the function is exponential growth or decay and find the constant percentage rate of growth or decay.

$$1 - r = .968$$

12.) $P(t) = 3.5 \cdot (0.968)^t$
 Decay $1 - .968 = .032$
 3.2%

13.) $P(t) = 4.3 \cdot (1.018)^t$
 growth 1.8%

Determine the exponential function that satisfies the given condition.

- 14.) Initial Value: 5
 Decreasing at a rate of .59%/week

$$y = 5(1 - .0059)^x \quad ; x = \text{weeks}$$

$$y = 5(.9941)^x$$

- 15.) Initial Value: 0.6g
 Doubling every 3 days

$$y = .6(2)^{\frac{x}{3}}$$

$$y = .6(1.2599)^x$$

$$y = ab^x$$

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**16.) Radioactive Decay**

The half-life of a certain radioactive substance is 14 days.

There are 6.6 grams present initially.

- a.) Express the amount of substance remaining as a function of time t .

$$f(t) = 6.6 \left(\frac{1}{2}\right)^{t/14} \quad \begin{matrix} \text{Winkaw} \\ t = x = \text{time} \end{matrix}$$

- b.) When will there be less than 1 gram remaining?

$\approx 38.11 \text{ days}$

$y = \text{chemical}$

$$\hookrightarrow 0.7$$

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17.) Population Models

Using the midyear data in Table 3.7 and assuming the growth is exponential, when did the population of Austin surpass 800,000 persons?

$$f(t) = 465622 \left(\frac{656562}{465622} \right)^t$$

Table 3.7 Populations of Two Major U.S. Cities

City	1990 Population	2000 Population
Austin, Texas	465,622	656,562
Columbus, Ohio	632,910	711,265

Source: World Almanac and Book of Facts 2005.

Window:

$$t = 15.75 \text{ years}$$

$$x: 0 - 25 \text{ years}$$

1990

2006

$$y: 300,000 - 900,000$$

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Investigating the Natural Base e

Use a calculator to complete the table:

n	10^1	10^2	10^3	10^4	10^5	10^6
1	2.594	2.705	2.717	2.718	2.718	2.718

Do the values approach a number? If so, what is the number rounded to three decimal places?

The Natural Base e:

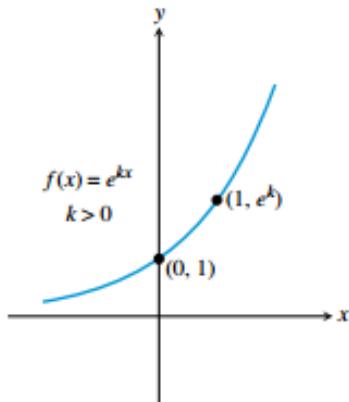
As n approaches $+\infty$, $\left(1 + \frac{1}{n}\right)^n$ approaches $e \approx 2.718$

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Exponential Functions and the Base e

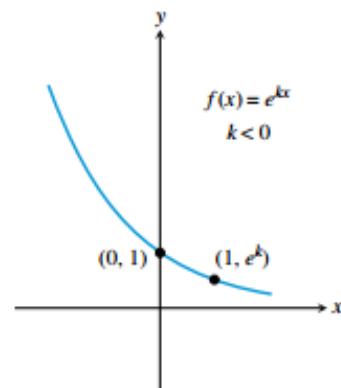
Exponential Growth

$$f(x) = a \cdot e^{kx}; a > 0 \text{ and } k > 0$$



Exponential Decay

$$f(x) = a \cdot e^{kx}; a > 0 \text{ and } k < 0$$



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3.2-3.3 Logarithmic Functions

Every function of the form $f(x)$ passes the Horizontal Line Test and therefore must have an inverse function. This inverse function is called the logarithmic function with base a .

Definition of Logarithmic Function with Base a

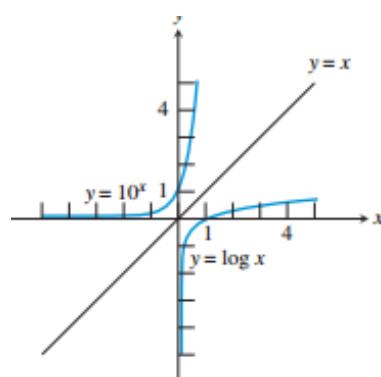
For $x > 0$, $a > 0$, and $a \neq 1$,

$$y = \log_a x \text{ if and only if } x = a^y.$$

The function given by

$$f(x) = \log_a x \quad \text{Read as "log base } a \text{ of } x\text{"}$$

is called the **logarithmic function with base a** .



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Equivalent Equations

Logarithmic Form: $\log_a x = y$

Exponential Form: $a^y = x$

Rewrite the function in exponential form:

$$1.) \log_5 \frac{1}{5} = -1$$

$$2.) \log_{14} 196 = 2$$

$$3.) \log_{19} 1 = 0$$

Rewrite the function in logarithmic form:

$$4.) 2^3 = 8$$

$$5.) 10^1 = 10$$

$$6.) \left(\frac{1}{5}\right)^{-3} = 125$$

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Evaluate the logarithmic expressions without a calculator.

$$7.) \log_7 343$$

$$8.) \log_{12} 12$$

$$9.) \log_{16} 4$$

$$10.) \log_{\frac{1}{5}} 25$$

$$11.) \log_3 \sqrt{3}$$

$$12.) \log_4 4^{.38}$$

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Basic Properties of Logarithms

For $0 < b \neq 1$, $x > 0$, and any real number y ,

- $\log_b 1 = 0$ because $b^0 = 1$.
- $\log_b b = 1$ because $b^1 = b$.
- $\log_b b^y = y$ because $b^y = b^y$.
- $b^{\log_b x} = x$ because $\log_b x = \log_b x$.

Basic Properties of Common Logarithms

Let x and y be real numbers with $x > 0$.

- $\log 1 = 0$ because $10^0 = 1$.
- $\log 10 = 1$ because $10^1 = 10$.
- $\log 10^y = y$ because $10^y = 10^y$.
- $10^{\log x} = x$ because $\log x = \log x$.

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Using Properties of Logarithms

13.) Evaluate: $\log 100$

14.) Evaluate: $\log \sqrt[5]{10}$

15.) Simplify: $\log_2 2^x$

16.) Simplify: $35^{\log_{35} x}$

17.) Simplify: $\log_4 16^x$

18.) Simplify: $\log_{20} 8000^x$

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The Natural Logarithmic Function

$f(x) = e^x$ is one-to-one so it has an inverse function.

The inverse function is called the natural logarithmic function.

Denoted as: $\ln x$

Note: The natural logarithm is written without a base. The base is understood to be e .

The Natural Logarithmic Function

The function defined by

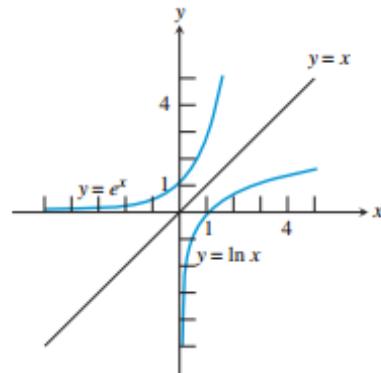
$$f(x) = \log_e x = \ln x, \quad x > 0$$

is called the **natural logarithmic function**.

Basic Properties of Natural Logarithms

Let x and y be real numbers with $x > 0$.

- $\ln 1 = 0$ because $e^0 = 1$.
- $\ln e = 1$ because $e^1 = e$.
- $\ln e^y = y$ because $e^y = e^y$.
- $e^{\ln x} = x$ because $\ln x = \ln x$.



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Use properties of natural logarithms to rewrite each expression.

19.) $\ln \frac{1}{e}$

20.) $e^{\ln 5}$

21.) $\ln e^0$

22.) $2 \ln e$

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3.3 Properties of Logarithmic Functions**Change of Base Formula:**

Let u , b , and c be positive numbers with $b \neq 1$. Then: $c \neq 1$

$$\log_c u = \frac{\log_b u}{\log_b c}$$

$$\text{In particular, } \log_c u = \frac{\log u}{\log c} \text{ and } \log_c u = \frac{\ln u}{\ln c}.$$

Evaluate the logarithm using the change-of-base formula. Round our answers to 3 decimal places.

1.) $\log_4 25$

2.) $\log_2 12$

3.) $\log_3 \left(\frac{3}{5} \right)$

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Properties of Logarithms

Let b , R , and S be positive numbers such that $b \neq 1$ and c any real number.

Product Property: $\log_b(RS) = \log_b R + \log_b S$

Quotient Property: $\log_b \left(\frac{R}{S} \right) = \log_b R - \log_b S$

Power Property: $\log_b R^c = c \log_b R$

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EXPLORATION 2 Discovering Relationships and Nonrelationships

Of the eight relationships suggested here, four are *true* and four are *false* (using values of x within the domains of both sides of the equations). Thinking about the properties of logarithms, make a prediction about the truth of each statement. Then test each with some specific numerical values for x . Finally, compare the graphs of the two sides of the equation.

1. $\ln(x + 2) = \ln x + \ln 2$

2. $\log_3(7x) = 7 \log_3 x$

3. $\log_2(5x) = \log_2 5 + \log_2 x$

4. $\ln \frac{x}{5} = \ln x - \ln 5$

5. $\log \frac{x}{4} = \frac{\log x}{\log 4}$

6. $\log_4 x^3 = 3 \log_4 x$

7. $\log_5 x^2 = (\log_5 x)(\log_5 x)$

8. $\log |4x| = \log 4 + \log |x|$

Which four are true, and which four are false?

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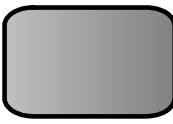
Expanding a Logarithmic Expression

4.) Expand $\log_2 \frac{7x^3}{y}$. Assume x and y are positive.

Use properties of logarithms to expand each expression.

5.) $\log_4 5x^3y$

6.) 

7.) 

8.) 

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Condensing a Logarithmic Expression*Condense the expression.*

10.) $\log 6 + 2\log 2 - \log 3$

11.)

12.)

13.)

14)

15.)

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