3.2-3.3 Logarithmic Functions

Every function of the form $f(x) = a^x$ passes the Horizontal Line Test and therefore must have an inverse function. This inverse function is called the logarithmic function with base a.

Definition of Logarithmic Function with Base *a*

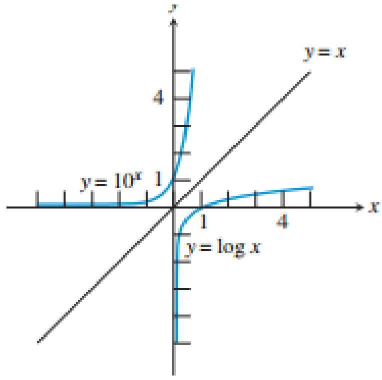
For $x > 0$, $a > 0$, and $a \neq 1$,

$$y = \log_a x \text{ if and only if } x = a^y.$$

The function given by

$$f(x) = \log_a x$$ Read as "log base *a* of *x*."

is called the **logarithmic function with base *a*.**



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Equivalent Equations

Logarithmic Form: $\log_a x = y$

Exponential Form: $a^y = x$

Rewrite the function in exponential form:

1.) $\log_5 \frac{1}{5} = -1$ $5^{-1} = \frac{1}{5}$

2.) $\log_{14} 196 = 2$ $14^2 = 196$

3.) $\log_{19} 1 = 0$ $19^0 = 1$

Rewrite the function in logarithmic form:

4.) $2^3 = 8$

$\log_2 8 = 3$

5.) $10^1 = 10$

$\log_{10} 10 = 1$

$\log 10 = 1$

6.) $\left(\frac{1}{5}\right)^{-3} = 125$

$\log_{\frac{1}{5}} 125 = -3$

Mar 20-7:56 AM

Evaluate the logarithmic expressions without a calculator.

7.) $\log_7 343$ 3

$7^{?} = 343$

8.) $\log_{12} 12$ 1

$12^{?} = 12$

9.) $\log_{16} 4$ $\frac{1}{2}$

$16^{?} = 4$

10.) $\log_{\frac{1}{5}} 25$ -2

$\frac{1}{5}^{?} = 25$

11.) $\log_3 \sqrt{3}$

$3^{?} = \sqrt{3}$

$3^{?} = 3^{1/2}$

$\frac{1}{2}$

12.) $\log_4 4^{.38}$

$4^{?} = 4^{.38}$

$.38$

Mar 20-7:57 AM

Basic Properties of Logarithms

For $0 < b \neq 1$, $x > 0$, and any real number y,

- $\log_b 1 = 0$ because $b^0 = 1$.
- $\log_b b = 1$ because $b^1 = b$.
- $\log_b b^y = y$ because $b^y = b^y$.
- $b^{\log_b x} = x$ because $\log_b x = \log_b x$.

Basic Properties of Common Logarithms

Let x and y be real numbers with $x > 0$.

- $\log 1 = 0$ because $10^0 = 1$.
- $\log 10 = 1$ because $10^1 = 10$.
- $\log 10^y = y$ because $10^y = 10^y$.
- $10^{\log x} = x$ because $\log x = \log x$.

Mar 20-7:57 AM

Using Properties of Logarithms

13.) Evaluate: $\log 100$ 2

$10^{?} = 100$

14.) Evaluate: $\log \sqrt[5]{10}$

$10^{?} = 10^{1/5}$ $\frac{1}{5}$

15.) Simplify: $\log_2 2^x$

$2^{?} = 2^x$

x

16.) Simplify: $\cancel{35}^{\cancel{\log_{35}} x}$ x

$\log_{35} \_\_ = \log_{35} x$

17.) Simplify: $\log_4 16^x$

$4^{?} = 16^x$

$4^{?} = 4^{2x}$

$2x$

18.) Simplify: $\log_{20} 8000^x$

$20^{?} = 8000^x$

$20^{?} = 20^{3x}$

$3x$

Mar 20-7:57 AM

The Natural Logarithmic Function

$f(x) = e^x$ is one-to-one so it has an inverse function.

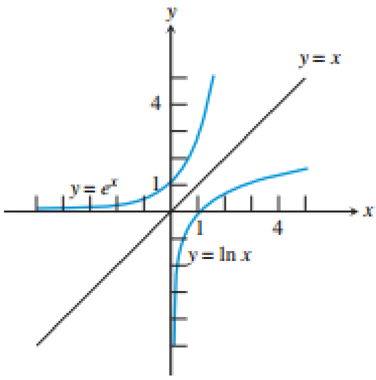
The inverse function is called the natural logarithmic function.
Denoted as: ln x
Note: The natural logarithm is written without a base. The base is understood to be e.

The Natural Logarithmic Function

The function defined by

$$f(x) = \log_e x = \ln x, \quad x > 0$$

is called the **natural logarithmic function.**



Basic Properties of Natural Logarithms

Let x and y be real numbers with $x > 0$.

- $\ln 1 = 0$ because $e^0 = 1$.
- $\ln e = 1$ because $e^1 = e$.
- $\ln e^y = y$ because $e^y = e^y$.
- $e^{\ln x} = x$ because $\ln x = \ln x$.

Mar 20-7:57 AM

Use properties of natural logarithms to rewrite each expression.

19.) $\ln \frac{1}{e} = \log_e \frac{1}{e}$

$e^{?} = \frac{1}{e}$ -1

20.) $e^{\ln 5}$

$\cancel{e}^{\log_{\cancel{e}} 5}$

5

21.) $\ln e^0$

$\log_e e^0$

$e^{?} = e^0$

0

22.) $2\ln e$

$2 \cdot \log_e e$

$e^{?} = e$

$2(1) = 2$

Mar 20-7:58 AM

3.3 Properties of Logarithmic Functions

Change of Base Formula:

Let u, b, and c be positive numbers with $c \neq 1$ *and* $b \neq 1$. *Then:*

$$\log_c u = \frac{\log_b u}{\log_b c}$$

In particular, $\log_c u = \frac{\log u}{\log c}$ *and* $\log_c u = \frac{\ln u}{\ln c}$.

valuate the logarithm using the change-of-base formula. Round our answers to 3 decimal places.

1.) $\log_4 25$

2.) $\log_2 12$

3.) $\log_3 \left(\frac{3}{5}\right)$

Mar 20-8:00 AM

Properties of Logarithms

Let b, R, and S be positive numbers such that $b \neq 1$ and c any real number.

Product Property: $\log_b(RS) = \log_b R + \log_b S$

Quotient Property: $\log_b\left(\frac{R}{S}\right) = \log_b R - \log_b S$

Power Property: $\log_b R^c = c\log_b R$

Mar 20-11:58 AM

EXPLORATION 2 Discovering Relationships and Nonrelationships

Of the eight relationships suggested here, four are *true* and four are *false* (using values of x within the domains of both sides of the equations). Thinking about the properties of logarithms, make a prediction about the truth of each statement. Then test each with some specific numerical values for x. Finally, compare the graphs of the two sides of the equation.

1. $\ln(x + 2) = \ln x + \ln 2$
2. $\log_3(7x) = 7\log_3 x$
3. $\log_2(5x) = \log_2 5 + \log_2 x$
4. $\ln \frac{x}{5} = \ln x - \ln 5$
5. $\log \frac{x}{4} = \frac{\log x}{\log 4}$
6. $\log_4 x^3 = 3\log_4 x$
7. $\log_5 x^2 = (\log_5 x)(\log_5 x)$
8. $\log|4x| = \log 4 + \log|x|$

Which four are true, and which four are false?

Mar 20-11:58 AM

Expanding a Logarithmic Expression

4.) *Expand* $\log_2 \frac{7x^3}{y}$ *. Assume x and y are positive.*

Use properties of logarithms to expand each expression.

5.) $\log_4 5x^3 y$

6.)

7.)

8.)

Mar 20-11:58 AM

Condensing a Logarithmic Expression

Condense the expression.

10.) $\log 6 + 2\log 2 - \log 3$

11.)

12.)

13.)

14

15.)

Mar 20-11:59 AM

Mar 20-11:59 AM