3.3 Properties of Logarithmic Functions

Change of Base Formula:

Let u, b, and c be positive numbers with $b \neq 1$ and $c \neq 1$. Then:

$$\log_c u = \frac{\log_b u}{\log_b c}$$

In particular, $\log_c u = \dfrac{\log u}{\log c}$ *and* $\log_c u = \dfrac{\ln u}{\ln c}$.

Evaluate the logarithm using the change-of-base formula. Round your answers to 3 decimal places.

1.) $\log_4 25$

$\dfrac{\log 25}{\log 4} = 2.322$

2.) $\log_2 12$

$\dfrac{\log 12}{\log 2} = 3.585$

3.) $\log_3\left(\dfrac{3}{5}\right) = .465$

$\dfrac{\log 3/5}{\log 3}$

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$\ln 4 = 1.386$

$\log_3 4^3 = x$

$3^x = 4^3$

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Properties of Logarithms

Let b, R, and S be positive numbers such that $b \neq 1$ and c any real number.

Product Property: $\log_b(RS) = \log_b R + \log_b S$

Quotient Property: $\log_b\left(\frac{R}{S}\right) = \log_b R - \log_b S$

Power Property: $\log_b R^c = c\log_b R$

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EXPLORATION 2 Discovering Relationships and Nonrelationships

Of the eight relationships suggested here, four are *true* and four are *false* (using values of x within the domains of both sides of the equations). Thinking about the properties of logarithms, make a prediction about the truth of each statement. Then test each with some specific numerical values for x. Finally, compare the graphs of the two sides of the equation.

1. $\ln(x+2) = \ln x + \ln 2$ $\ln(2x)$ False
2. $\log_3(7x) = 7\log_3 x$ $\log_3 7 + \log_3 x$ False $\log_3 x^7$
3. $\log_2(5x) = \log_2 5 + \log_2 x$ True
4. $\ln\frac{x}{5} = \ln x - \ln 5$ True
5. $\log\frac{x}{4} = \frac{\cancel{\log x}}{\log 4}$ False $\log x - \log 4$
6. $\log_4 x^3 = 3\log_4 x$ True
7. $\log_5 x^2 = \cancel{(\log_5 x)(\log_5 x)}$ False $2\log_5 x$
8. $\log|4x| = \log 4 + \log|x|$ True

Which four are true, and which four are false?

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Expanding a Logarithmic Expression

4.) *Expand* $\log_2 \frac{7x^3}{y}$ *. Assume x and y are positive.*

$\log_2 7 + 3\log_2 x - \log_2 y$

Use properties of logarithms to expand each expression.

5.) $\log_4 5x^3y$

$\log_4 5 + 3\log_4 x + \log_4 y$

6.) [redacted] $= \ln \frac{(3x-5)^{1/2}}{7}$

$\boxed{\frac{1}{2}\ln(3x-5) - \ln 7}$

7.) [redacted]

$\ln 3 + \ln x + 3\ln y$

8.) [redacted]

$\log_8 64 + 2\log_8 x$

$2 + 2\log_8 x$

9.) $\log_2 \sqrt{4x}$

$\log_2 (4x)^{1/2}$

$\frac{1}{2}\log_2 4 + \frac{1}{2}\log_2 x$ (2)

$1 + \frac{1}{2}\log_2 x$

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Condensing a Logarithmic Expression

Condense the expression.

10.) $\log 6 + 2\log 2 - \log 3$

$\log\left(\frac{6 \cdot 2^2}{3}\right) = \log 8$

11.) [redacted]

$\ln \frac{16}{4}$

$\ln 4$

12.) [redacted]

$\log_{16}\left(\frac{12^4}{2^4}\right) = \log_{16} 1296$

13.) [redacted]

$\log_4 2^7 x^5 y^3 = \log_4 128 x^5 y^3$

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$$14.)\ 3(\ln 3 - \ln x) + (\ln x - \ln 9)$$

$$3\left(\ln \frac{3}{x}\right) + \ln \frac{x}{9}$$

$$\ln \left(\frac{3}{x}\right)^3 \cdot \frac{x}{9}$$

$$\ln \frac{27x}{x^3 \cdot 9}$$

$$\ln \frac{3}{x^2}$$

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$$15.)\ \frac{1}{4}\log_5 81 - \left(2\log_5 6 - \frac{1}{2}\log_5 4\right)$$

$$\log_5 \frac{\sqrt[4]{81}}{\left(\frac{6^2}{\sqrt{4}}\right)} = \log_5 \frac{3}{18}$$

$$\log_5 \frac{1}{6}$$

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