6. $\log_2 x^{-2} = -2 \log_2 x$

7. $\log x^3y^2 = \log x^3 + \log y^2 = 3 \log x + 2 \log y$

8. $\log xy^3 = \log x + \log y^3 = \log x + 3 \log y$

9. $\ln \frac{x^2}{y^3} = \ln x^2 - \ln y^3 = 2 \ln x - 3 \ln y$

10. $\log 1000x^4 = \log 1000 + \log x^4 = 3 + 4 \log x$

11. $\log \sqrt[4]{\frac{x}{y}} = \frac{1}{4}(\log x - \log y) = \frac{1}{4}\log x - \frac{1}{4}\log y$

12. $\ln \frac{\sqrt[3]{x}}{\sqrt[3]{y}} = \frac{1}{3}(\ln x - \ln y) = \frac{1}{3}\ln x - \frac{1}{3}\ln y$

13. $\log x + \log y = \log xy$

14. $\log x + \log 5 = \log 5x$

15. $\ln y - \ln 3 = \ln(y/3)$

16. $\ln x - \ln y = \ln(x/y)$

17. $\frac{1}{3}\log x = \log x^{1/3} = \log \sqrt[3]{x}$

18. $\frac{1}{5}\log z = \log z^{1/5} = \log \sqrt[5]{z}$

19. $2 \ln x + 3 \ln y = \ln x^2 + \ln y^3 = \ln (x^2y^3)$

20. $4 \log y - \log z = \log y^4 - \log z = \log \left(\frac{y^4}{z}\right)$

21. $4 \log (xy) - 3 \log (yz) = \log (x^4y^4) - \log (y^3z^3)$
$= \log \left(\frac{x^4y^4}{y^3z^3}\right) = \log \left(\frac{x^4y}{z^3}\right)$

22. $3 \ln (x^3y) + 2 \ln (yz^2) = \ln (x^9y^3) + \ln (y^2z^4)$
$= \ln (x^9y^5z^4)$

In #23–28, natural logarithms are shown, but common (base-10) logarithms would produce the same results.

23. $\frac{\ln 7}{\ln 2} \approx 2.8074$

24. $\frac{\ln 19}{\ln 5} \approx 1.8295$

25. $\frac{\ln 175}{\ln 8} \approx 2.4837$

26. $\frac{\ln 259}{\ln 12} \approx 2.2362$

27. $\frac{\ln 12}{\ln 0.5} = -\frac{\ln 12}{\ln 2} \approx -3.5850$

28. $\frac{\ln 29}{\ln 0.2} = -\frac{\ln 29}{\ln 5} \approx -2.0922$

29. $\log_3 x = \frac{\ln x}{\ln 3}$

30. $\log_7 x = \frac{\ln x}{\ln 7}$

31. $\log_2(a + b) = \frac{\ln(a + b)}{\ln 2}$

32. $\log_5(c - d) = \frac{\ln(c - d)}{\ln 5}$

33. $\log_2 x = \frac{\log x}{\log 2}$

34. $\log_4 x = \frac{\log x}{\log 4}$

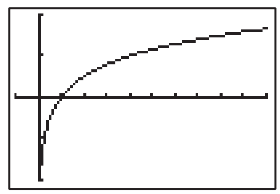
35. $\log_{1/2}(x + y) = \frac{\log(x + y)}{\log (1/2)} = -\frac{\log(x + y)}{\log 2}$

36. $\log_{1/3}(x - y) = \frac{\log(x - y)}{\log(1/3)} = -\frac{\log(x - y)}{\log 3}$

37. Let $x = \log_b R$ and $y = \log_b S$.
Then $b^x = R$ and $b^y = S$, so that
$\frac{R}{S} = \frac{b^x}{b^y} = b^{x-y}$
$\log_b\left(\frac{R}{S}\right) = \log_b b^{x-y} = x - y = \log_b R - \log_b S$

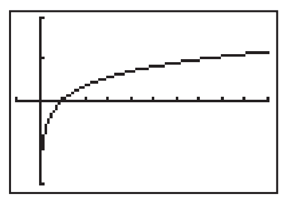
38. Let $x = \log_b R$. Then $b^x = R$, so that
$R^c = (b^x)^c = b^{c \cdot x}$
$\log_b R^c = \log_b b^{c \cdot x} = c \cdot x = c \log_b R$

39. Starting from $g(x) = \ln x$: vertically shrink by a factor $1/\ln 4 \approx 0.72$.



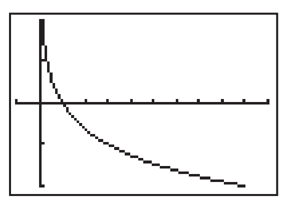
$[-1, 10]$ by $[-2, 2]$

40. Starting from $g(x) = \ln x$: vertically shrink by a factor $1/\ln 7 \approx 0.51$.



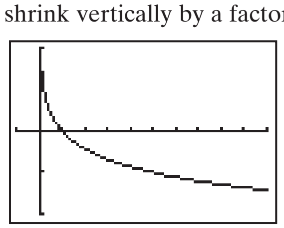
$[-1, 10]$ by $[-2, 2]$

41. Starting from $g(x) = \ln x$: reflect across the x-axis, then vertically shrink by a factor $1/\ln 3 \approx 0.91$.



$[-1, 10]$ by $[-2, 2]$

42. Starting from $g(x) = \ln x$: reflect across the x-axis, then shrink vertically by a factor of $1/\ln 5 \approx 0.62$.



$[-1, 10]$ by $[-2, 2]$

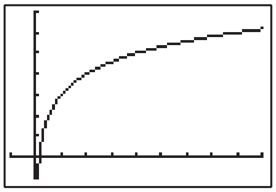
43. (b): $[-5, 5]$ by $[-3, 3]$, with Xscl = 1 and Yscl = 1 (graph $y = \ln(2 - x)/\ln 4$).

44. (c): $[-2, 8]$ by $[-3, 3]$, with Xscl = 1 and Yscl = 1 (graph $y = \ln(x - 3)/\ln 6$).

45. (d): $[-2, 8]$ by $[-3, 3]$, with Xscl $= 1$ and Yscl $= 1$ (graph $y = \ln(x - 2)/\ln 0.5$).

46. (a): $[-8, 4]$ by $[-8, 8]$, with Xscl $= 1$ and Yscl $= 1$ (graph $y = \ln(3 - x)/\ln 0.7$).

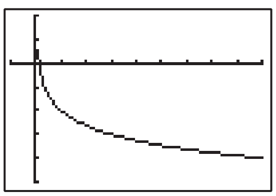
47.



$[-1, 9]$ by $[-1, 7]$

Domain: $(0, \infty)$
Range: $(-\infty, \infty)$
Continuous
Always increasing
Asymptote: $x = 0$
$\lim_{x\to\infty} f(x) = \infty$
$f(x) = \log_2 (8x) = \dfrac{\ln (8x)}{\ln (2)}$

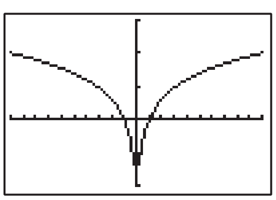
48.



$[-1, 9]$ by $[-5, 2]$

Domain: $(0, \infty)$
Range: $(-\infty, \infty)$
Continuous
Always decreasing
Asymptote: $x = 0$
$\lim_{x\to\infty} f(x) = -\infty$
$f(x) = \log_{1/3} (9x) = \dfrac{\ln (9x)}{\ln \left(\frac{1}{3}\right)}$

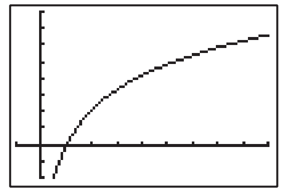
49.



$[-10, 10]$ by $[-2, 3]$

Domain: $(-\infty, 0) \cup (0, \infty)$
Range: $(-\infty, \infty)$
Discontinuous at $x = 0$
Decreasing on interval $(-\infty, 0)$; increasing on interval $(0, \infty)$
Asymptote: $x = 0$
$\lim_{x\to\infty} f(x) = \infty$, $\lim_{x\to-\infty} f(x) = \infty$,

50.



$[-1, 9]$ by $[-2, 8]$

Domain: $(0, \infty)$
Range: $(-\infty, \infty)$
Continuous
Always increasing
Asymptote: $x = 0$
$\lim_{x\to\infty} f(x) = \infty$

51. In each case, take the exponent of 10, add 12, and multiply the result by 10.

(a) 0
(b) 10
(c) 60
(d) 80
(e) 100
(f) 120 $(1 = 10^0)$

52. **(a)** $R = \log \dfrac{250}{2} + 4.25 = \log 125 + 4.25 \approx 6.3469.$

(b) $R = \log \dfrac{300}{4} + 3.5 = \log 75 + 3.5 \approx 5.3751$

53. $\log \dfrac{I}{12} = -0.00235(40) = -0.094$, so
$I = 12 \cdot 10^{-0.094} \approx 9.6645$ lumens.

54. $\log \dfrac{I}{12} = -0.0125(10) = -0.125$, so
$I = 12 \cdot 10^{-0.125} \approx 8.9987$ lumens.

55. From the change-of-base formula, we know that
$f(x) = \log_3 x = \dfrac{\ln x}{\ln 3} = \dfrac{1}{\ln 3} \cdot \ln x \approx 0.9102 \ln x.$
$f(x)$ can be obtained from $g(x) = \ln x$ by vertically stretching by a factor of approximately 0.9102.

56. From the change-of-base formula, we know that
$f(x) = \log_{0.8} x = \dfrac{\log x}{\log 0.8} = \dfrac{1}{\log 0.8} \cdot \log x \approx -10.32 \log x.$
$f(x)$ can be obtained from $g(x) = \log x$ by reflecting across the x-axis and vertically stretching by a factor of approximately 10.32.

57. True. This is the product rule for logarithms.

58. False. The logarithm of a positive number less than 1 is negative. For example, $\log 0.01 = -2$.

59. $\log 12 = \log (3 \cdot 4) = \log 3 + \log 4$ by the product rule. The answer is B.

60. $\log_9 64 = (\ln 64)/(\ln 9)$ by the change-of-base formula. The answer is C.

61. $\ln x^5 = 5 \ln x$ by the power rule. The answer is A.