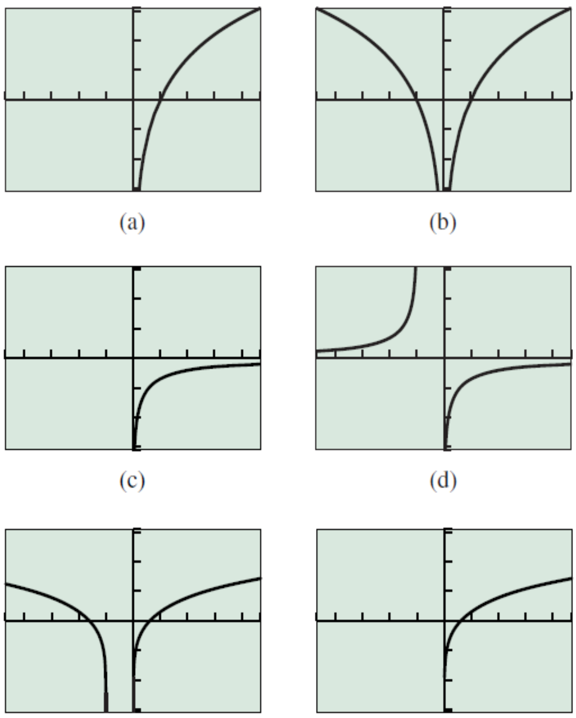
21. $f(x) = \ln \dfrac{x}{x+1}$

22. $g(x) = \ln x - \ln(x+1)$

23. $f(x) = 2 \ln x$

24. $g(x) = \ln x^2$



(a) (b) (c) (d) (e) (f)

Solving Logarithmic Equations

8.) $\ln(x-2)+\ln(2x-3)=2\ln x$

$\cancel{\ln}(x-2)(2x-3)=\cancel{\ln}x^2$

$2x^2-7x+6=x^2$

$x^2-7x+6=0$

$(x-6)(x-1)=0$

$x=6$

$\cancel{x=1}$

9.) $\log_6(3x+14)-\log_6 5=\log_6 2x$

$\cancel{\log_6}\left(\frac{3x+14}{5}\right)=\cancel{\log_6}2x$

$3x+14=10x$

$14=7x$

$2=x$

10.) $\log 5x+\log(x-1)=2$

$\log 5x(x-1)=2$

$10^2=5x(x-1)$

$100=5x^2-5x$

$5x^2-5x-100=0$

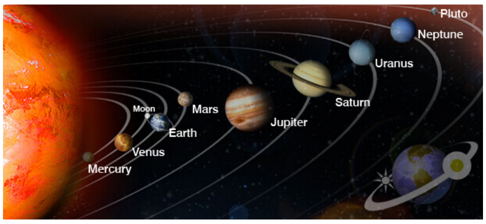
$5(x^2-x-20)=0$

$5(x-5)(x+4)=0$

$x=5, \cancel{-4}$

Oct 25-8:22 AM

Orders of Magnitude



Mercury: 57.9 billion meters from the Sun

Pluto: 5900 billion meters from the Sun

Mercury: 5.79×10^{10} Pluto: 5.9×10^{12}

Check the log of each quantity.

Pluto's distance is 2 orders of magnitude greater than Mercury.

Order of Magnitude: the common log of a positive quantity

Determine by how many orders of magnitude the quantities differ.

100 1

1.) A dollar to a penny 2

1000 1

2.) A kilometer to a meter 3

3.) New York City: Population: 8,000,000
to Earmuff Junction: Population: 8 6

The Richter Scale magnitude R of an earthquake is:

$$R = \log\frac{a}{T} + B \qquad R = \log\left(\frac{I}{I_0}\right);\ I_0 = 1$$

where a is the amplitude in micrometers of the vertical ground motion at the receiving station, T is the period of the associated seismic wave in seconds, and B accounts for the weakening of the seismic wave with increasing distance from the epicenter of the earthquake.

B, T are constant.

Comparing Earthquakes

How many times more severe was the 2001 earthquake in Gujarat, India (R_1, $R = 7.9$) than the 1999 earthquake in Athens, Greece (R_2, $R = 5.9$)?

$$R_1 - R_2 = \log\frac{a_1}{T} + B - \left(\log\frac{a_2}{T} + B\right)$$

$$= \log\frac{a_1}{T} \cancel{+B} - \log\frac{a_2}{T} \cancel{-B}$$

$$= \log\frac{a_1}{T} - \log\frac{a_2}{T}$$

$$= \log a_1 - \log T - (\log a_2 - \log T)$$

$$= \log a_1 \cancel{-\log T} - \log a_2 \cancel{+\log T}$$

$$= \log a_1 - \log a_2$$

$$R_1 - R_2 = \log\frac{a_1}{a_2}$$

$$10^{R_1 - R_2} = \frac{a_1}{a_2}$$

$10^2 = 100\text{x}$ as severe

Chemistry

The acidity of a water-based solution is measured by the concentration of hydrogen ions in the solution (in moles per liter) given by:

$$pH = -\log\left[H^+\right]$$

Vinegar has a pH of 2.4 and Baking Soda has a pH of 8.4.

a.) What are their Hydrogen Ion concentrations?

Vinegar: $2.4 = -\log[H^+]$

$-2.4 = \log[H^+]$

$10^{-2.4} = H^+$

$\boxed{H^+ = 3.98 \times 10^{-3} \text{ moles/liter}}$

Baking Soda:

$8.4 = -\log[H^+]$

$\boxed{H^+ = 3.98 \times 10^{-9} \text{ moles/liter}}$

b.) How many times greater is the hydrogen-ion concentration of the vinegar than that of the baking soda?

$$\frac{3.98 \times 10^{-3}}{3.98 \times 10^{-9}} = \boxed{1000000}$$

c.) By how many orders of magnitude do the concentrations differ?

6

Newton's Law of Cooling

$$T(t) = T_m + (t_0 - T_m)e^{-kt}$$

T_m : the temperature of the surrounding medium

T_0 : initial temperature of the object

A hard boiled egg at temperature $\overset{T_0}{96^\circ C}$ is placed in $\overset{T_m}{16^\circ C}$ water to cool. Four minutes later the temperature of the egg is $45^\circ C$. Use Newton's Law of Cooling to determine when the egg will be $20^\circ C$.

$$45 = 16 + (96-16)e^{-4k}$$

$$\frac{29}{80} = \frac{80e^{-4k}}{80}$$

$$\frac{29}{80} = e^{-4k}$$

$$\frac{\ln(29/80)}{-4} = \frac{-4k}{-4}$$

$$\boxed{k = .2537}$$

$$20 = 16 + (96-16)e^{-.2537t}$$

$$4 = 80e^{-.2537t}$$

$$\frac{4}{80} = e^{-.2537t}$$

$$\frac{\ln(4/80)}{-.2537} = \frac{-.2537t}{-.2537}$$

$$\boxed{t = 11.81 \text{ min}}$$

DEFINITION Decibels

The level of sound intensity in **decibels** (dB) is

$$\beta = 10 \log(I/I_0),$$

where β (beta) is the number of decibels, I is the sound intensity in W/m^2, and $I_0 = 10^{-12}$ W/m^2 is the threshold of human hearing (the quietest audible sound intensity).

Table 3.17 Approximate Intensities of Selected Sounds

Sound	Intensity (W/m^2)
Hearing threshold	10^{-12}
Soft whisper at 5 m	10^{-11}
City traffic	10^{-5}
Subway train	10^{-2}
Pain threshold	10^{0}
Jet at takeoff	10^{3}

Source: Adapted from R. W. Reading, Exploring Physics: Concepts and Applications. Belmont, CA: Wadsworth, 1984.

How loud is the train in the subway?

$$\beta = 10 \log\left(\frac{10^{-2}}{10^{-12}}\right)$$

$\cancel{\log_{10} 10^{10}}$

$$= 10 \log\left(10^{10}\right)$$

$$= 10 \cdot 10 = 100 \text{ decibels}$$

Oct 25-7:34 AM