

### 3.5 Extension - Future/Present Values

Future Value of an Annuity

$$FV = R \frac{(1+r)^t - 1}{r}$$

*R = Dollar amount of equal payments*

$$FV = R \frac{\left(1 + \frac{r}{n}\right)^{nt} - 1}{\left(\frac{r}{n}\right)}$$

To account for a rate that is compounded more frequently than annually,  $n$  is added.  
 $n$  = number of compounds per year.

1.) At the end of each quarter year, Emily makes a \$500 payment into the Lanaghan Mutual Fund. If her investments earn 8% annual interest compounded quarterly, what will be the value of Emily's annuity in 20 years?

$$FV = 500 \frac{\left(1 + \frac{.08}{4}\right)^{4 \cdot 20} - 1}{\left(\frac{.08}{4}\right)}$$

$$\boxed{\$96,885.98}$$

\* How long until her investment = \$250,000?

$$250000 = 500 \frac{\left(1 + \frac{.08}{4}\right)^{4t} - 1}{\left(\frac{.08}{4}\right)}$$

$$10 = \left(1 + \frac{.08}{4}\right)^{4t} - 1$$

$$11 = \left(1 + \frac{.08}{4}\right)^{4t}$$

$$\log \left(1 + \frac{.08}{4}\right) 11 = 4t$$

$$\boxed{t = 30.27 \text{ years}}$$

**The PRESENT VALUE** is the net amount of money put into an annuity. The periodic and equal payments made on a loan or mortgage actually constitute an annuity.

Present Value of an Annuity	$PV = R \frac{1-(1+i)^{-n}}{i}$
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2.) Hannah obtains a 30-year fixed mortgage of \$250,000 with an APR of 4.5%. What is her monthly payment?

$$250000 = R \cdot \frac{1 - \left(1 + \frac{.045}{12}\right)^{-12(30)}}{\left(\frac{.045}{12}\right)}$$

$$R = \$1266.71$$

How much does the loan actually cost Hannah?

$$\begin{array}{r} 1266.71 \\ \times \quad 12 \\ \times \quad 30 \\ \hline \end{array}$$

$$\boxed{\$456,015.60}$$

Hannah: \$1500.00

$$250000 = 1500 \cdot \frac{1 - \left(1 + \frac{.045}{12}\right)^{-12t}}{\left(\frac{.045}{12}\right)}$$

$$.375 = \left(1 + \frac{.045}{12}\right)^{-12t}$$

$$\log\left(1 + \frac{.045}{12}\right)^{-12t} = \log .375$$

$$t = 21.84 \text{ years}$$

$$\begin{array}{r} 21.84 \\ \times \quad 12 \\ \times 1500 \end{array}$$

$$\boxed{\$393,120.00}$$

