3.5 Logarithmic Models

1.) Bacterial Growth

The number N of bacteria in a culture is modeled by

$$N = 250e^{kt}$$

where t is the time in hours. If N = 280 when t = 10, estimate the time required for the population to double in size.

$$\frac{280}{250} = \frac{250e^{10k}}{250}$$

$$1.12 = e^{10k}$$

$$\frac{\ln 1.12}{10} = \frac{10k}{10}$$

$$\boxed{.0113 = k}$$

$$500 = 250e^{.0113T}$$

$$2 = e^{.0113T}$$

$$\ln 2 = .0113T$$

$$\boxed{61.3 \text{ hours} = T}$$

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Logarithmic Models

2.) Value of a Painting

The value V (in millions of dollars) of a famous painting can be modeled by:

$$V = 10e^{kt}$$

where t represents the year, with t = 0 corresponding to 1990. In 2004, the same painting was sold for \$65 million. Find the value of k, and use the results to predict the value of the painting in 2010.

$$65 = 10e^{14k}$$

$$6.5 = e^{14k}$$

$$\frac{\ln 6.5}{14} = \boxed{k = .1337}$$

$$V = 10e^{(.1337 \cdot 20)}$$

$$V = 144.98 \text{ million}$$

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Logarithmic Models

Newton's Law of Cooling

$$T(t) = T_m + (t_0 - T_m)e^{-kt}$$

T_m : the temperature of the surrounding medium

T_0 : initial temperature of the object

3.) A hard boiled egg at temperature $\overset{T_0}{96^\circ C}$ is placed in $\overset{T_m}{16^\circ C}$ water to cool. Four minutes later the temperature of the egg is $45^\circ C$. Use Newton's Law of Cooling to determine when the egg will be $20^\circ C$

$45 = 16 + (96-16)e^{-4k}$

$45 = 16 + 80e^{-4k}$

$29 = 80e^{-4k}$

$.3625 = e^{-4k}$

$\boxed{k = .2537}$

$20 = 16 + 80e^{-.2537T}$

$4 = 80e^{-.2537T}$

$.05 = e^{-.2537T}$

$\ln .05 = -.2537T$

$\boxed{11.8\text{ min} = T}$

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Newton's Law of Cooling

$$T(t) = T_m + (t_0 - T_m)e^{-kt}$$

T_m : the temperature of the surrounding medium

T_0 : initial temperature of the object

4.) A casserole is removed from a 375ºF oven, and it cools to 200ºF after 15 minutes in a 75ºF room. How long (from the time it is taken out of the oven) does it take to cool to 100ºF? Round k to four decimal places and your final answer to the nearest tenth.

$200 = 75 + (375-75)e^{-15k}$

$125 = 300e^{-15k}$

$\frac{125}{300} = e^{-15k}$

$\ln(125/300) = -15k$

$k = .0584$

$100 = 75 + (300)e^{-.0584T}$

$T = 42.5\text{min}$

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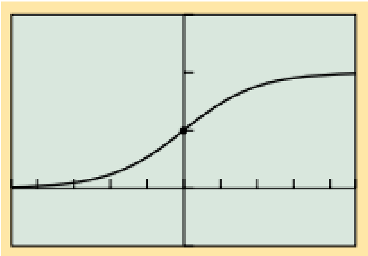
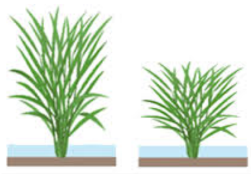
3.5 Logistic Growth Functions





$$f(x)=\frac{1}{1+e^{-x}}$$

DEFINITION Logistic Growth Functions

Let a, b, c, and k be positive constants, with $b < 1$. A **logistic growth function** in x is a function that can be written in the form

$$f(x) = \frac{c}{1 + a \cdot b^x} \quad \text{or} \quad f(x) = \frac{c}{1 + a \cdot e^{-kx}}$$

where the constant c is the **limit to growth**.

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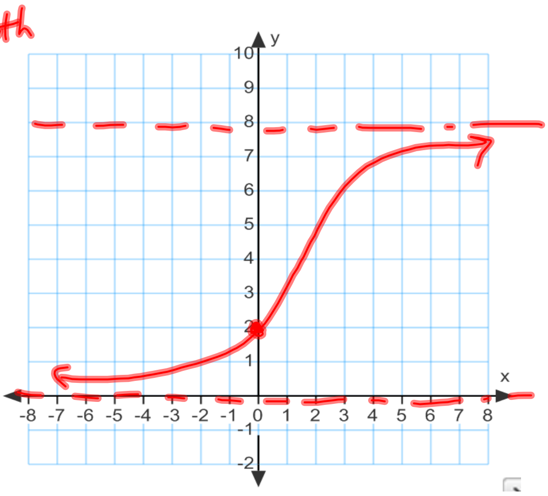
Graphing Logistic Growth Functions

Graph the function. Find the y-intercept and the horizontal asymptotes.

limit to growth

5.) $f(x)=\dfrac{8}{1+3\cdot 0.7^{x}}$

(0, 2)



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Equations of Logistic Functions

Find the logistic function that satisfies the given conditions.

6.) Initial Value: 10 → (0,10)

Limit to Growth: 40

✓ Passing Through: (1, 20)

$$f(x) = \frac{c}{1+ab^x}$$

$c = 40$

$$\frac{10}{1} = \frac{40}{1+ab^0}$$

$10 + 10a = 40$

$10a = 30$

$a = 3$

$$\frac{20}{1} = \frac{40}{1+3b^1}$$

$20 + 60b = 40$

$60b = 20$

$b = 1/3$

$$\boxed{f(x) = \frac{40}{1+3\left(\frac{1}{3}\right)^x}}$$

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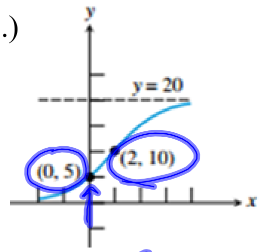
Find the logistic function that satisfies the given conditions.

7.)



$$y = \frac{c}{1+ab^x}$$

$c = 20$

$$5 = \frac{20}{1+a}$$

$5 + 5a = 20$

$5a = 15$

$a = 3$

$$10 = \frac{20}{1+3b^2}$$

$10 + 30b^2 = 20$

$30b^2 = 10$

$b^2 = 1/3$

$\boxed{b = .5774}$

$$\boxed{f(x) = \frac{20}{1+3(.5774)^x}}$$

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Find the logistic function that satisfies the given conditions.

8.) Initial Population: 16
Max Sustainable
Population: 128
Passing through (5,32)

$y = \frac{128}{1+ab^x}$

$c = 128$

$16 = \frac{128}{1+a}$

$16 + 16a = 128$

$16a = 112$

$a = 7$

$\frac{32}{1} = \frac{128}{1+7b^5}$

$32 + 224b^5 = 128$

$b^5 = .42857$

$.8441 = b$

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Campus Virus

9.) On a college campus of 5000 students, one student returns from vacation with a contagious and long-lasting flu virus. The spread of the virus is modeled by:

$$y = \frac{5000}{1+4999e^{-0.8t}}, t \geq 0$$

where y is the total number of students infected after t days. The college will cancel classes when 40% or more of the students are infected. After how many days will the college cancel classes?

$\frac{2000}{1} = \frac{5000}{1+4999e^{-.8t}}$

$\frac{2000(1+4999e^{-.8t})}{2000} = \frac{5000}{2000}$

$1+4999e^{-.8t} = 2.5$

$\frac{4999e^{-.8t}}{4999} = \frac{1.5}{4999}$

$e^{-.8t} = (1.5/4999)$

$\frac{-.8t}{-.8} = \frac{\ln(1.5/4999)}{-.8}$

$\boxed{t = 10.14 \text{ days}}$

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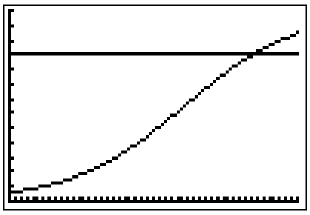
Dallas

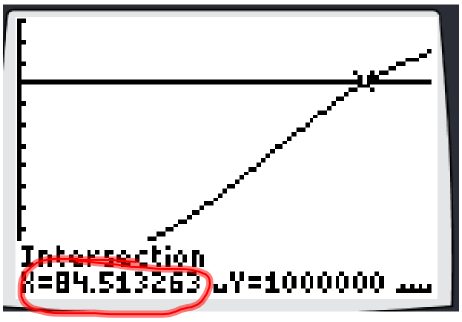
10.) Based on recent census data, a logistic model for the population of Dallas, t years after 1900, is as follows:

$$P(t) = \frac{1{,}301{,}642}{1 + 21.602e^{-0.05054t}}$$

According to this model, when was the population 1 million?



Graph



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