4.1 Applications - Arc Length

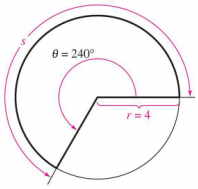
Arc Length $s = r\theta$

r = radius

θ = central angle (in radians)

1. A circle has a radius of 4 inches. Find the length intercepted by a central angle of $240°$.

$240° \left| \frac{\pi \text{ rad}}{180°} \right| = \frac{4\pi}{3} \text{ rad}$



$s = r\theta$

$s = (4 \text{ in})\left(\frac{4\pi}{3}\right) = \frac{16\pi}{3} \text{ in}$

$s = 16.755 \text{ in}$

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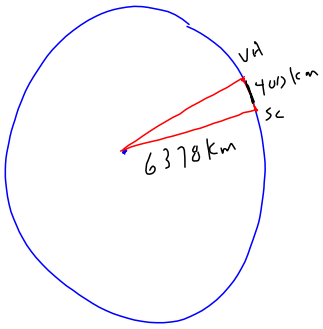
Ex.) Assuming the Earth is a sphere of radius 6378 kilometers, what is the difference in latitudes of Lynchburg, VA and Myrtle Beach, SC, where Lynchburg is 400 kilometers due north of Myrtle Beach?



$s = r\theta$

$\frac{400}{6378} = \frac{6378\theta}{6378}$

$.0627 = \theta$

$.0627 \left| \frac{180}{\pi} \right| = 3.593°$

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4.1 Applications - Linear and Angular Speed

distance/time

Linear Speed = $\frac{\text{arc length}}{\text{time}} = \frac{s}{t}$ **how fast the particle moves**

* 1 rev = C

$\frac{1\text{ rev}}{C}$ or $\frac{C}{1\text{ rev}}$

$C = 2\pi r = \pi d$

Angular Speed = $\frac{\text{central angle}}{\text{time}} = \frac{\theta}{t}$ **how fast the angle changes**

1 rev = 2π rad $\frac{1\text{ rev}}{2\pi\text{ rad}}$ or $\frac{2\pi\text{ rad}}{1\text{ rev}}$

2. A lawn roller with a 10-inch radius makes 1.2 revolutions per second.

$C = 20\pi$ in

a.) Find the angular speed of the roller in radians per second.

$1.2\frac{\text{rev}}{\text{sec}} \cdot \frac{2\pi\text{ rad}}{1\text{ rev}} = 2.4\pi\text{ rad/sec} = 7.540\text{ rad/sec}$

b.) Find the speed of the tractor that is pulling the roller. (miles/hour)

$1.2\frac{\text{rev}}{\text{sec}} \cdot \frac{20\pi\text{ in}}{1\text{ rev}} \cdot \frac{1\text{ ft}}{12\text{ in}} \cdot \frac{1\text{ mile}}{5280\text{ ft}} \cdot \frac{60\text{ sec}}{1\text{ min}} \cdot \frac{60\text{ min}}{1\text{ hr}}$

4.284 mph

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Additional Examples

$C = 5\pi$ in

3. An electric winch is used to pull a boat out of the water onto a trailer. The winch winds the cable around a circular drum of diameter 5 inches. Approximately how many times will the winch have to rotate in order to roll in 5 feet of cable?

* revolutions

$\frac{5\text{ ft}}{1} \cdot \frac{12\text{ in}}{1\text{ ft}} \cdot \frac{1\text{ rev}}{5\pi\text{ in}} = 3.820\text{ rev}$

$C = 80\pi$ cm

4. The wheel of a machine rotates at the rate of 300 rpm (revolutions per minute). If the diameter of the wheel is 80 cm, what are the angular (in radian per second) and linear speed (in cm per second) of a point on the wheel?

* Angular

$300\frac{\text{rev}}{\text{min}} \cdot \frac{2\pi\text{ rad}}{1\text{ rev}} \cdot \frac{1\text{ min}}{60\text{ sec}} = 31.416\text{ rad/sec}$

* Linear

$300\frac{\text{rev}}{\text{min}} \cdot \frac{80\pi\text{ cm}}{1\text{ rev}} \cdot \frac{1\text{ min}}{60\text{ sec}} = 1256.637\text{ cm/sec}$

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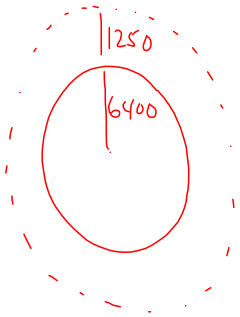
5. A satellite in a circular orbit 1250 kilometers above Earth makes one complete revolution every 110 minutes. What is its angular (radians per minute) and linear speed (kilometers per min)? Assume that Earth is a sphere of radius 6400 kilometers.

$C = 15300\pi$ km

$\frac{1\text{ rev}}{110\text{ min}} \cdot \frac{2\pi\text{ rad}}{1\text{ rev}} = .057\text{ rad/min}$



$r = 7650$km

$\frac{1\text{ rev}}{110\text{ min}} \cdot \frac{15300\pi\text{ km}}{1\text{ rev}} = 436.967\ \frac{\text{km}}{\text{min}}$

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