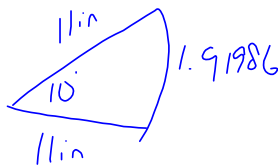35. To the nearest inch, find the perimeter of a 10-degree sector cut from a circular disc of radius 11 inches.

36. A 100-degree arc of a circle has a length of 7 cm. To the nearest centimeter, what is the radius of the circle?
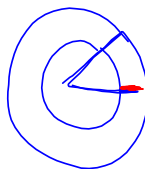
37. It takes ten identical pieces to form a circular track for a pair of toy racing cars. If the inside arc of each piece is 3.4 inches shorter than the outside arc, what is the width of the track?

38. The concentric circles on an archery target are 6 inches apart. The inner circle (red) has a perimeter of 37.7 inches. What is the perimeter of the next-largest (yellow) circle?

43. **Navigation** Two Coast Guard patrol boats leave Cape May at the same time. One travels with a bearing of 42°30′ and the other with a bearing of 52°12′. If they travel at the same speed, approximately how far apart will they be when they are 25 statute miles from Cape May?

52°12′

42°30′

Cape May
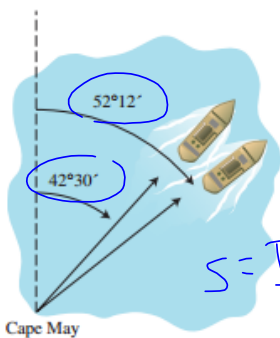
45. **Bicycle Racing** Cathy Nguyen races on a bicycle with 13-inch-radius wheels. When she is traveling at a speed of 44 ft/sec, how many revolutions per minute are her wheels making?

47. **Tool Design** A radial arm saw has a circular cutting blade with a diameter of 10 inches. It spins at 2000 rpm. If there are 12 cutting teeth per inch on the cutting blade, how many teeth cross the cutting surface each second?

11 in 10 1.91986
11 in

$S = r\theta$

$61° 24'$

$61 + \dfrac{24}{60} \cdot \dfrac{\pi}{180}$

$S_0 - S_i = 3.4$

$r_0\left(\dfrac{\pi}{5}\right) - r_i\left(\dfrac{\pi}{5}\right) = 3.4$

$S = r\theta$

$S = \dfrac{\pi r \theta}{180}$

$\dfrac{S}{\pi} \cdot \dfrac{\pi}{5}(r_0 - r_i) = 3.4 \cdot \dfrac{5}{\pi}$

$r_0 - r_i = 5.4 \text{ in}$

$\dfrac{2000 \text{ rev}}{\text{min}} \cdot \dfrac{1 \text{ min}}{60 \text{ sec}} \cdot \dfrac{10\pi \text{ in}}{1 \text{ rev}} = $

$\times 12$

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1

4.2 Trigonometric Functions of Acute Angles

Right Triangle Definitions of Trigonometric Functions

Let θ be an *acute* angle of a right triangle. The six trigonometric functions of the angle θ are defined as follows. (Note that the functions in the second row are the *reciprocals* of the corresponding functions in the first row.)
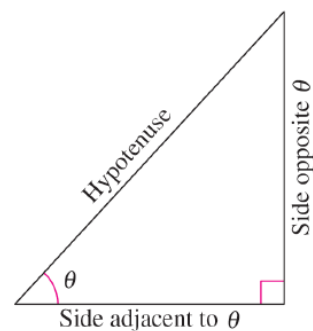
$$\sin \theta = \frac{O}{H} \qquad \cos \theta = \frac{A}{H} \qquad \tan \theta = \frac{O}{A}$$
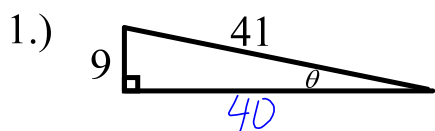
$$\csc \theta = \frac{H}{O} \qquad \sec \theta = \frac{H}{A} \qquad \cot \theta = \frac{A}{O}$$

Hypotenuse

Side opposite θ

θ

Side adjacent to θ

Find the exact values of the six trigonometric functions of the angle θ shown in the figure.

1.)

9 ⌐ ___41___

40

$9^2 + x^2 = 41^2$

3 4 5

5 12 13

7 24 25

9 40 41 8 15 17

$$\sin\theta = \frac{O}{H} = \frac{9}{41}$$

$$\cos\theta = \frac{A}{H} = \frac{40}{41}$$

$$\tan\theta = \frac{O}{A} = \frac{9}{40}$$
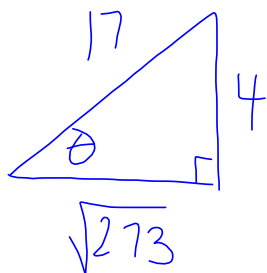
$$\csc\theta = \frac{H}{O} = \frac{41}{9}$$

$$\sec\theta = \frac{H}{A} = \frac{41}{40}$$

$$\cot\theta = \frac{A}{O} = \frac{40}{9}$$

Sketch a right triangle corresponding to the trig function of the acute angle θ. Use the Pythagorean Theorem to determine the third side of the triangle and then find the other 5 trig functions of θ.

2.) $\csc \theta = \dfrac{17}{4}$

17 /|
   / | 4
  /θ_|
 √273

$$\sin \theta = \frac{O}{H} = \frac{4}{17}$$

$$\cos \theta = \frac{A}{H} = \frac{\sqrt{273}}{17}$$

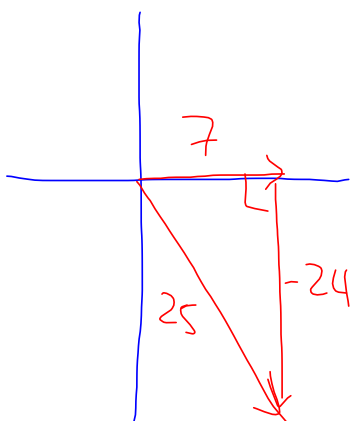$$\tan \theta = \frac{O}{A} = \frac{4\sqrt{273}}{273}$$

$$\csc \theta = \frac{H}{O} = \frac{17}{4}$$

$$\sec \theta = \frac{H}{A} = \frac{17\sqrt{273}}{273}$$

$$\cot \theta = \frac{A}{O} = \frac{\sqrt{273}}{4}$$

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The given point is on the terminal side of an angle in standard position. Determine the exact values of the six trigonometric functions.

3.) $(7, -24)$

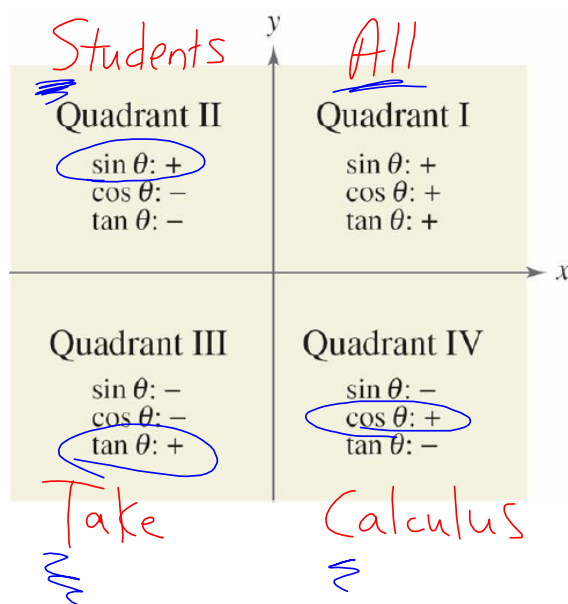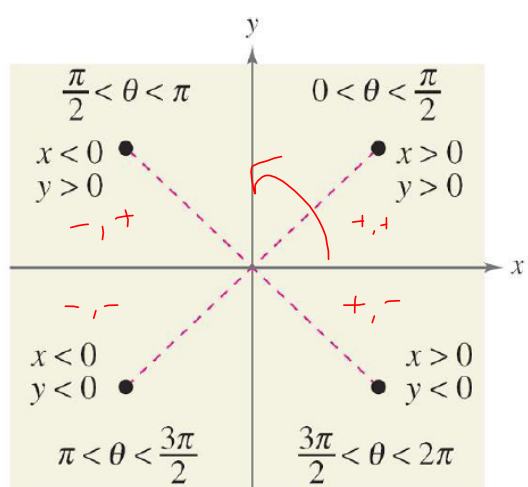$\sin \theta = \dfrac{O}{H} = \dfrac{-24}{25}$ $\csc \theta = \dfrac{H}{O} = \dfrac{-25}{24}$

$\cos \theta = \dfrac{A}{H} = \dfrac{7}{25}$ $\sec \theta = \dfrac{H}{A} = 25/7$
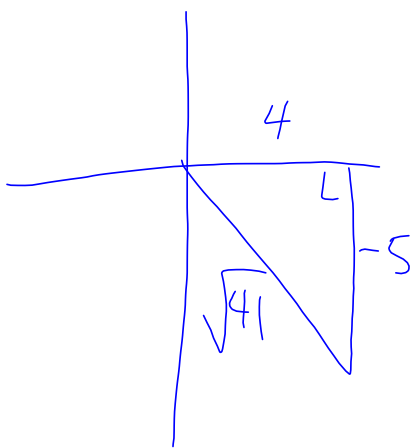
$\tan \theta = \dfrac{O}{A} = \dfrac{-24}{7}$ $\cot \theta = \dfrac{A}{O} = -7/24$

Determining the signs of the trigonometric functions.

$$\frac{\pi}{2} < \theta < \pi \qquad\qquad 0 < \theta < \frac{\pi}{2}$$

$x < 0$ ●   ● $x > 0$
$y > 0$ $y > 0$

$-,+$ $+,+$

$x < 0$ $x > 0$
$y < 0$ ●   ● $y < 0$

$-,-$ $+,-$

$$\pi < \theta < \frac{3\pi}{2} \qquad\qquad \frac{3\pi}{2} < \theta < 2\pi$$

Students · **All**

Quadrant II	Quadrant I
$\sin\theta$: +	$\sin\theta$: +
$\cos\theta$: −	$\cos\theta$: +
$\tan\theta$: −	$\tan\theta$: +

Quadrant III	Quadrant IV
$\sin\theta$: −	$\sin\theta$: −
$\cos\theta$: −	$\cos\theta$: +
$\tan\theta$: +	$\tan\theta$: −

Take   **Calculus**
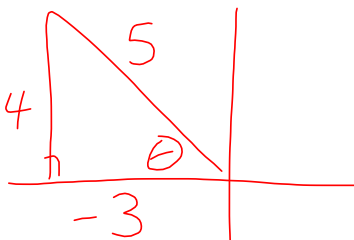
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4.) Given $\tan \theta = -\dfrac{5}{4}$ and $\cos \theta > 0$, find $\sin \theta$ and $\sec \theta$.

$$\sin \theta = \frac{-5}{\sqrt{41}} = \frac{-5\sqrt{41}}{41}$$

$$\sec \theta = \frac{\sqrt{41}}{4}$$

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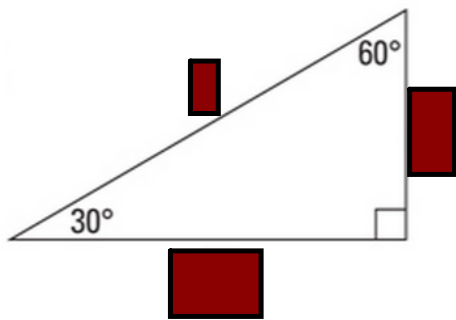5.) Given $\sin\theta = \dfrac{4}{5}$ and $\tan\theta < 0$, find $\cos\theta$ and $\tan\theta.$

$$5$$
$$4$$
$$h$$
$$\theta$$
$$-3$$

$\cos(\theta) = -3/5$

$\tan(\theta) = -4/3$

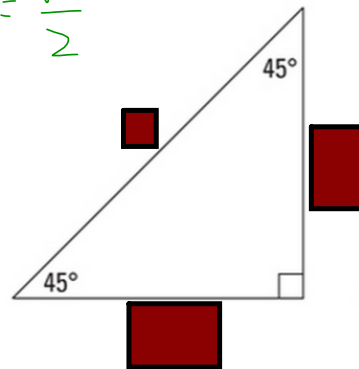*Geometry Quick Review: Special Right Triangles*

$30:60:90$

$X : x\sqrt{3} : 2x$
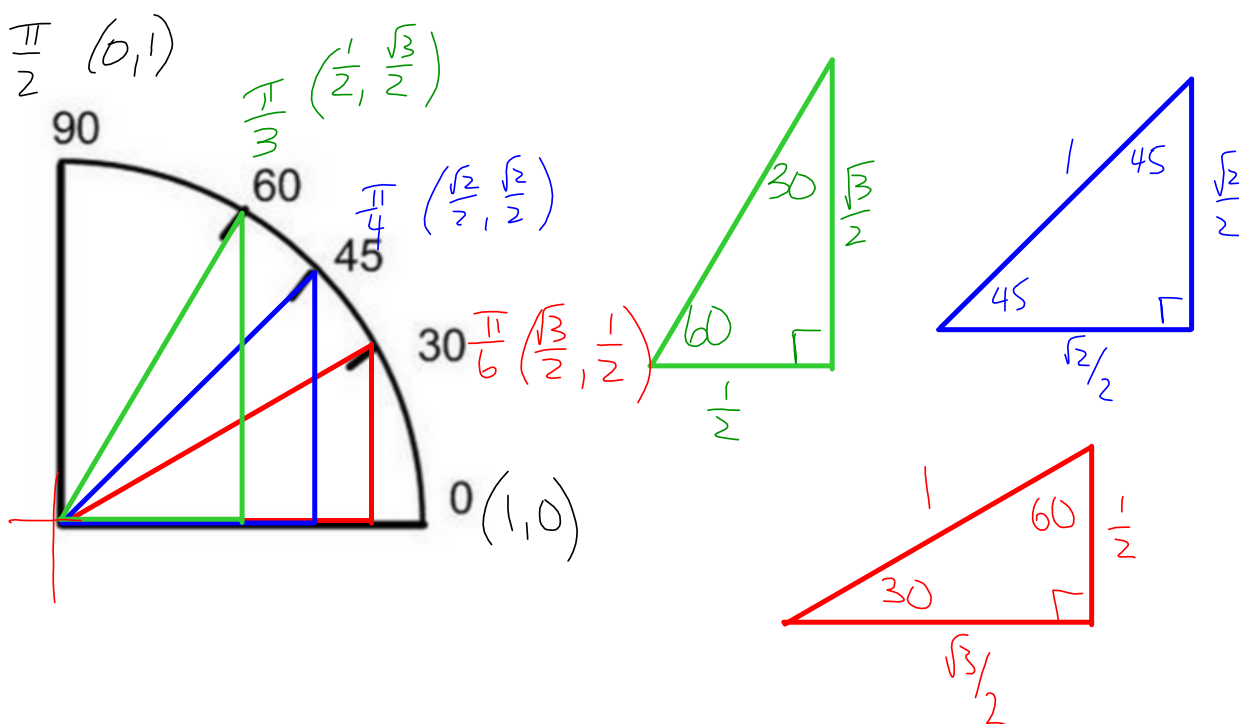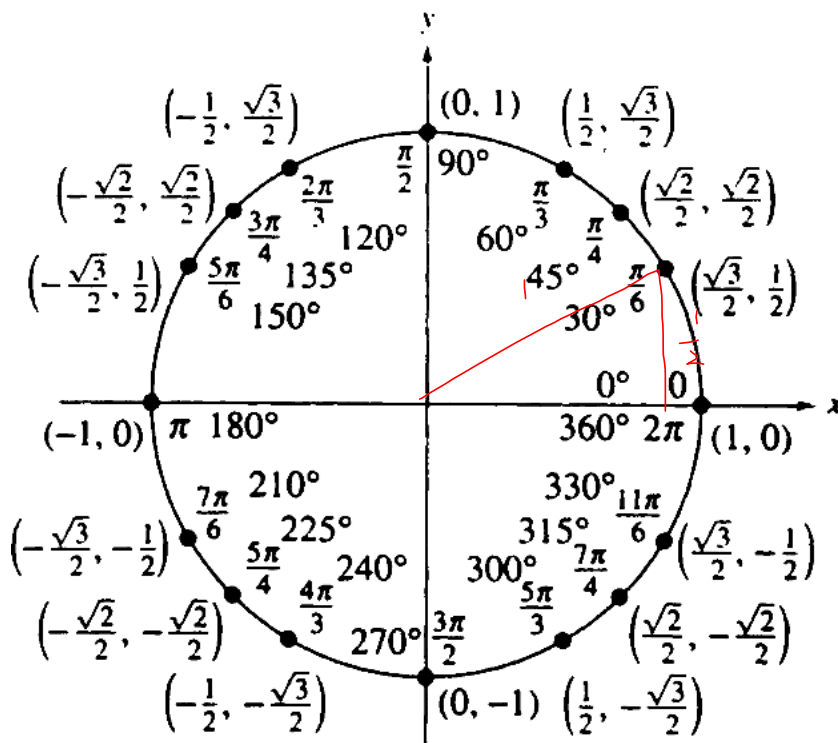
$45:45:90$

$X : X : x\sqrt{2}$

$$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

The Unit Circle: Quadrant I

$\frac{\pi}{2}$ $(0,1)$

90

$\frac{\pi}{3}$ $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

60

$\frac{\pi}{4}$ $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

45

30 $\frac{\pi}{6}$ $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

0 $(1,0)$

30 $\frac{\sqrt{3}}{2}$ 60 $\frac{1}{2}$

1 45 $\frac{\sqrt{2}}{2}$

45 $\frac{\sqrt{2}}{2}$

1 60 $\frac{1}{2}$

30 $\frac{\sqrt{3}}{2}$

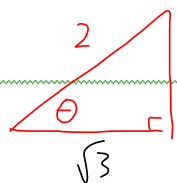$$\sin\theta = \frac{O}{H} = \frac{\frac{1}{2}}{1} = \frac{1}{2}$$

$$\cos\theta = \frac{A}{H} = \frac{\frac{\sqrt{3}}{2}}{1} = \frac{\sqrt{3}}{2}$$

Unit Circle Trig Functions

$\sin\theta = y$	$\cos\theta = x$	$\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{y}{x}$
$\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{y}$	$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{1}{x}$	$\cot\theta = \dfrac{\cos\theta}{\sin\theta} = \dfrac{x}{y}$

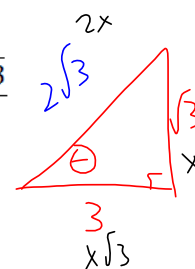### Find the acute angle that satisfies the given equation.

1.) $\sin\theta = \dfrac{1}{2}$
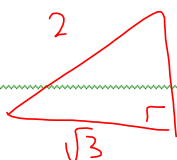
$\theta = 30, \dfrac{\pi}{6}$

2.) $\tan\theta = 1$

$\theta = 45, \dfrac{\pi}{4}$

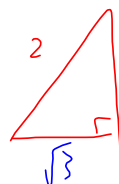3.) $\tan\theta = \dfrac{\sqrt{3}}{3}$

$\theta = 30, \pi/6$

4.) $\cos\theta = \dfrac{\sqrt{3}}{2}$

$\theta = 30, \pi/6$

5.) $\csc\theta = 2$

$\sin\theta = \dfrac{1}{2}$

$\theta = \dfrac{\pi}{6}, 30$

6.) $\sin\theta = \dfrac{\sqrt{3}}{2}$

$\theta = 60, \dfrac{\pi}{3}$

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7.) $\cot \theta = 1$

$$\frac{\pi}{4}, 45$$

8.) $\cos \theta = \dfrac{\sqrt{2}}{2}$

$$\frac{\pi}{4}, 45$$

Find the exact value.

9.) $\sec\left(\dfrac{\pi}{3}\right)$

10.) $\tan\left(\dfrac{\pi}{4}\right)$

11.) $\csc\left(\dfrac{\pi}{3}\right)$

Nov 15-11:15 AM

4.2 Mini-Lesson

Use the calculator to evaluate each function to four decimal places.

1.) $\sin 10°$ 2.) $\tan 23.5°$ 3.) $\cot 66.5°$

4.) $\csc 16.35°$ 5.) $\tan 11° \ 15'$ 6.) $\sec 56° \ 8' \ 10''$

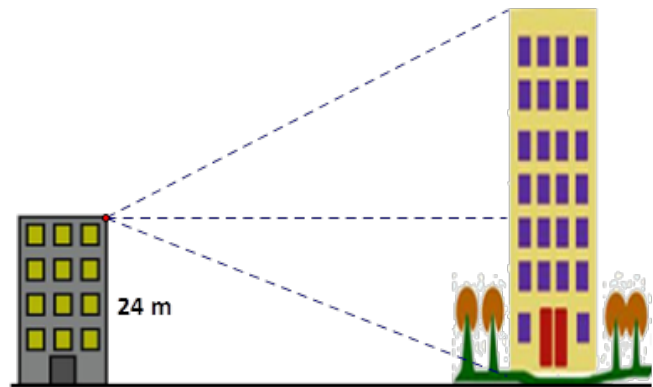*Use the calculator to evaluate each function to four decimal places.*

7.) $\sin \dfrac{\pi}{3}$ 8.) $\csc 1.3$ 9.) $\sec 22.8$ 10.) $\cos(-2.5)$

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11.) From an apartment window 24 m above the ground, the angle of depression of the base of a nearby building is 38 degrees and the angle of elevation of the top is 63 degrees. Find the height of the nearby building (to the nearest foot).

24 m

12.) From a lighthouse 1000 ft above sea level, the angle of depression to a boat (A) is 29 degrees. A little bit later the boat has moved closer to the shore (B) and the angle of depression measures 44 degrees. How far (to the nearest foot) has the boat moved in that time?

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