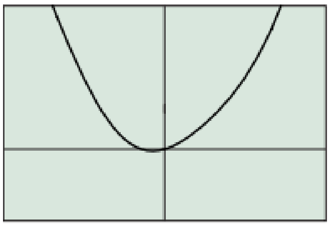
4.6 Graphs of Composite Trigonometric Functions

Which of the functions appear to be periodic?

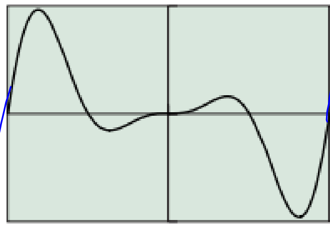
$y = \sin x + x^2$



$[-2\pi, 2\pi]$ by $[-10, 20]$

No.

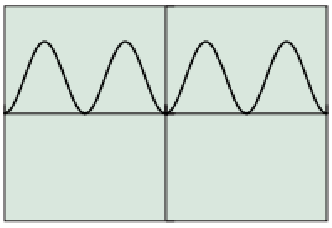
$y = x^2 \sin x$



$[-2\pi, 2\pi]$ by $[-25, 25]$

No.

$y = (\sin x)^2$



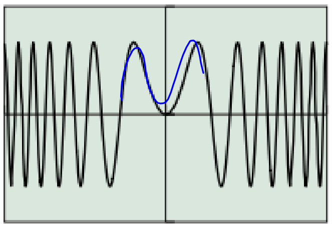
$[-2\pi, 2\pi]$ by $[-1.5, 1.5]$

Yes.

Periodic

Sinusoid

$y = \sin x^2$



$[-2\pi, 2\pi]$ by $[-1.5, 1.5]$

No

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Verifying Periodicity Algebraically

Prove the function is periodic algebraically.

1.) $f(x) = \sin^3 x$

$f(x) = f(x+2\pi)$

$$\begin{aligned} f(x+2\pi) &= \sin^3(x+2\pi) \\ &= (\sin(x+2\pi))^3 \\ &= (\sin x)^3 \\ &= \sin^3 x \\ &= f(x) \end{aligned}$$

$\therefore f(x) = f(x+2\pi)$

$\rightarrow$ periodic

Analyzing Nonnegative Periodic Functions

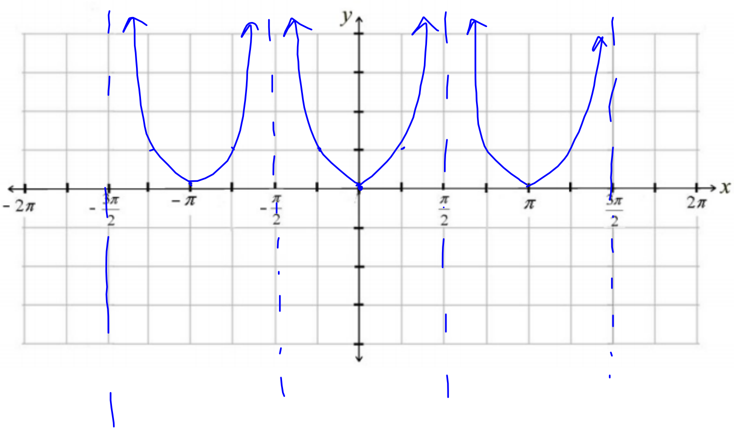
Find the domain, range, and period of the function. Sketch the graph.

2.) $y = |\tan x|$

D: $x \neq \frac{\pi}{2} + \pi n$

R: $[0, \infty)$

Period: π



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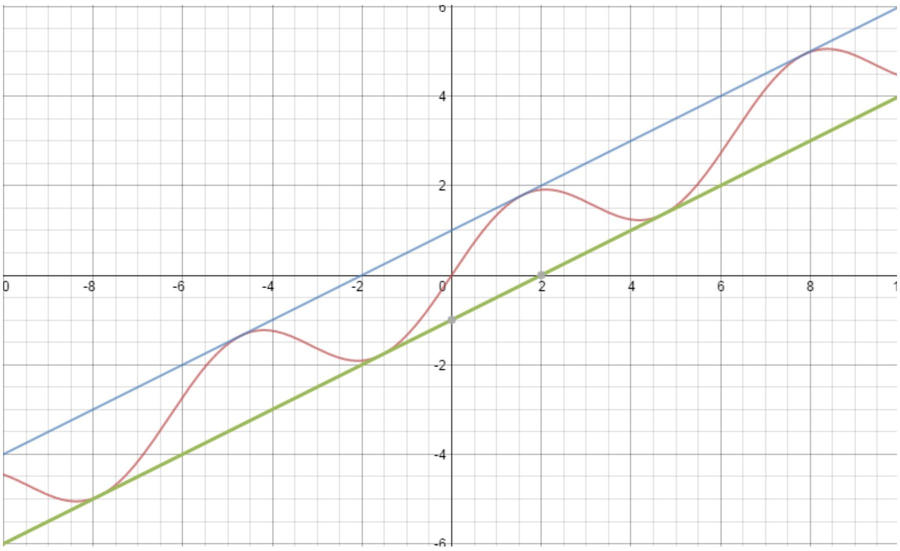
Adding a Sinusoid to a Linear Function

Graph: $y = 0.5x + \sin x$

$y = \frac{1}{2}x + 1$

$y = \frac{1}{2}x - 1$

$y = \frac{1}{2}x$



Identify the parallel lines in which the function oscillates.

Combination of Sinusoids

Music, like any other sound, is trasmitted by waves.

A "pure" musical note can be represented by a sine or cosine graph.

The frequency of the note is represented by the period of the graph, and the loudness of the note is represented by the amplitude of the graph.

Two musical notes with the same frequency played at the same time are shown in the figure.

$f(\theta) = 3\cos\theta$

$f(\theta) = 4\sin\theta$

$f(\theta) = 3\cos\theta + 4\sin\theta$

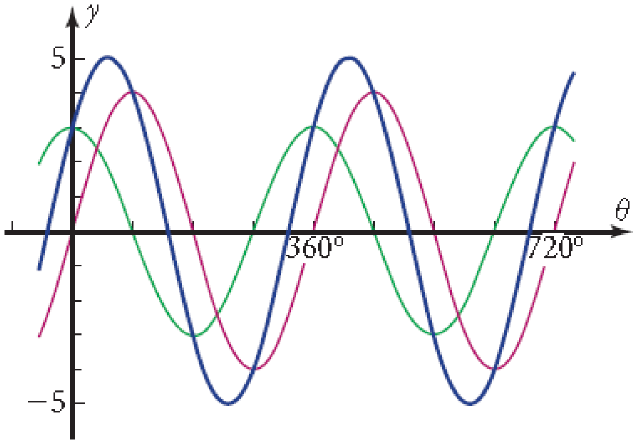


Figure 8-1a

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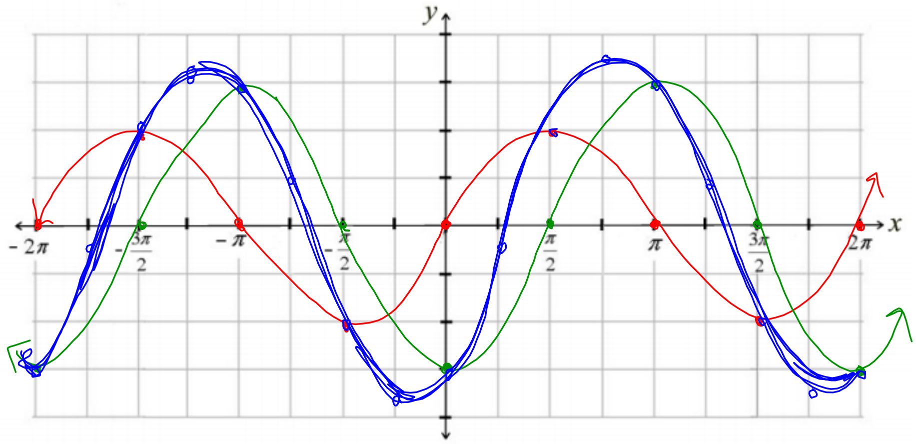
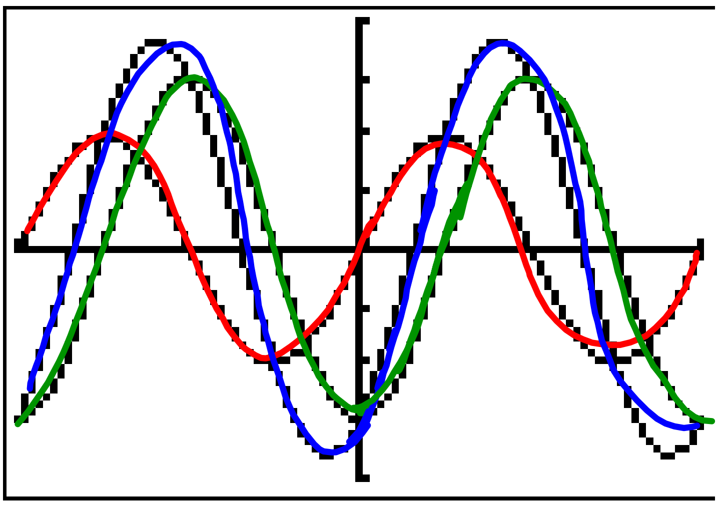
Investigating Sinusoids

Graph the functions. Then determine if the function is a sinusoid.

$y = 2\sin x$ (*red*)
$y = -3\cos x$ (*green*)
$y = 2\sin x - 3\cos x$ (*blue*)

periodic | sinusoid





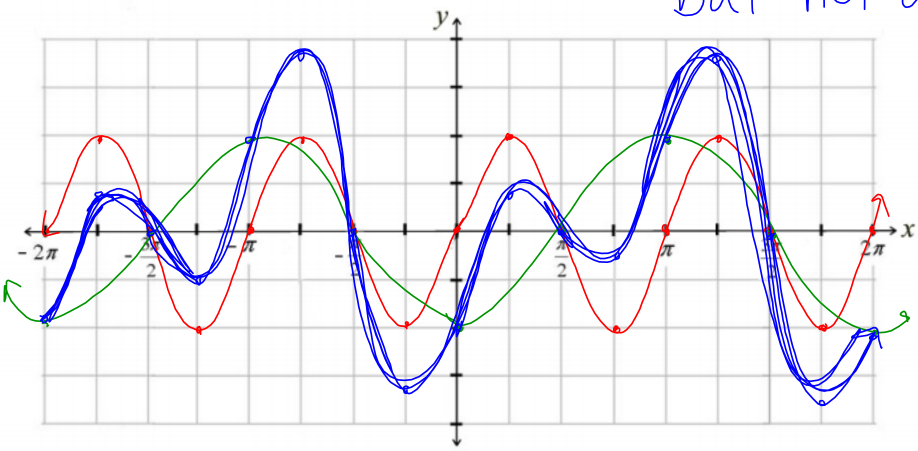
$y = 2\sin 2x$ (*red*)

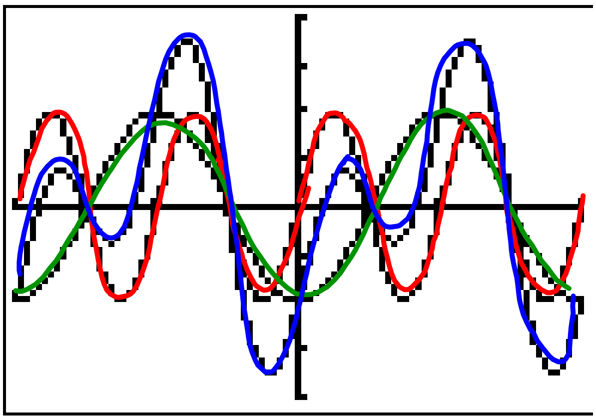
$y = -2\cos x$ (*green*)

$y = 2\sin 2x - 2\cos x$ (*blue*)

$\frac{2\pi}{2} = \pi$ I: $\pi/4$

Periodic
but not a sinusoid





Sums and Differences that are Sinusoid Functions:

Determine whether each of the following functions is or is not a sinusoid.

1.) $f(x) = 5\cos x + 3\sin x$ Yes : 2π

2.) $f(x) = \cos 5x + \sin 3x$ No $\frac{2\pi}{5} \neq \frac{2\pi}{3}$

3.) $f(x) = 2\cos 3x - 3\cos 2x$ No $\frac{2\pi}{3} \neq \pi$

4.) $f(x) = a\cos\frac{3x}{7} - b\cos\frac{3x}{7} + c\sin\frac{3x}{7}$ Yes $\frac{14\pi}{3}$

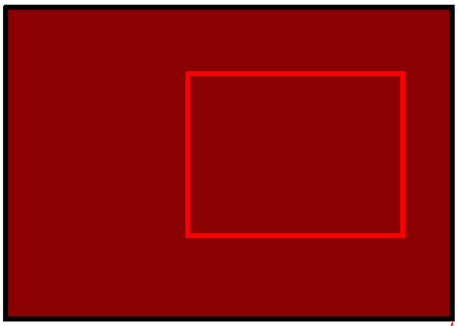
Finding Equations of Combined Sinusoids

Write the equation of the resulting sine and cosine function with the help of your calculator:

$y = a\sin(b(x-c))+d$

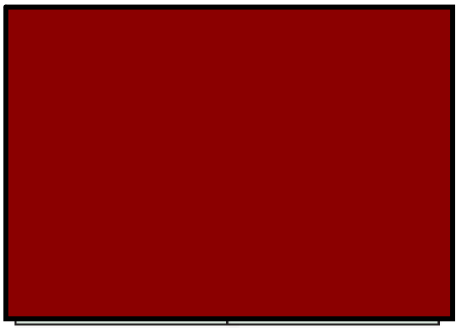
$$f(x) = 2\sin x + 5\cos x$$

$\sin x$



$y = 5.39\sin(x+1.19)$

$\cos x$



$y = 5.39\cos(x-.38)$

How can you easily manipulate a sine function into a cosine function and vice versa?

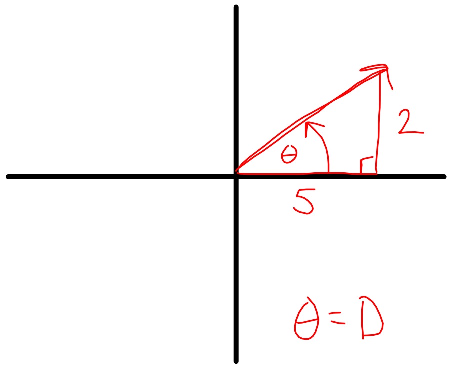
$\pm \pi/2$

Finding Equations of Combined Sinusoids

$f(x) = 2\sin x + 5\cos x$

Let u = coefficient of cos
Let v = coefficient of sin

Graph u and v on the coordinate plane to produce the terminal side of angle D. Find angle D.



Find A: (the hypotenuse of the reference triangle) $\sqrt{29} = 5.39 = A$

Use A and D and your answer on the previous slide to find a cosine function of the combined sinusoids.

$\tan^{-1}(2/5) = \theta = D = .38$

$\boxed{y = 5.39\cos(x - .38)}$

PROPERTY: Linear Combination of Cosine and Sine with Equal Periods

$$b\cos x + c\sin x = A\cos(x - D)$$

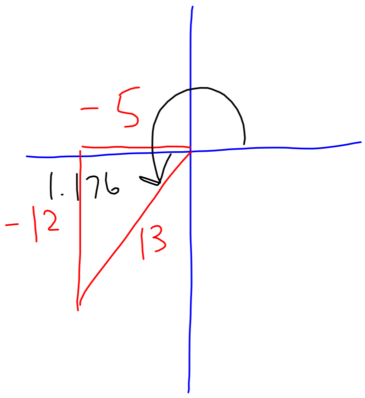
where

$$A = \sqrt{b^2 + c^2} \qquad \text{and} \qquad D = \arctan\frac{c}{b}$$

The quadrant for $D = \arctan\frac{c}{b}$ depends on the signs of b and c and may be determined by sketching angle D in standard position. The length of the hypotenuse of the reference triangle is A.

Practice: Express the following function as a single cosine function with a phase displacement:

$$y = -5\cos\theta - 12\sin\theta$$



$$y = 13\cos(x - 4.318)$$

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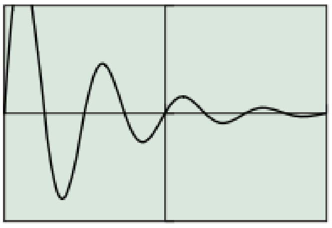
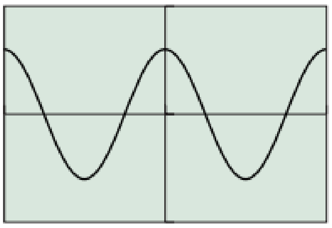
Damped Oscillation: $y = f(x)\cos bx \; OR \; y = f(x)\sin bx$

Graph the following functions:

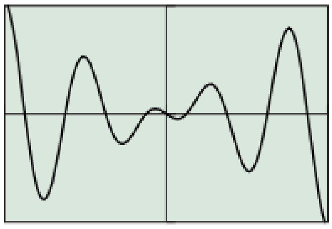
1.) $y = 2^{-x}\sin 4x$



2.) $y = 3\cos 2x$



3.) $y = -2x\cos 2x$



Damping Factor:

Make a sketch for the graph of $y = x^2 \cos 6x$ without using your calculator.

Linear Combination of Sine & Cosine with Equal Periods:

$$b\cos x + c\sin x = A\cos(x - D)$$

Where A = the hypotenuse of the reference triangle and D = the reference angle

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