4.7 Inverse Functions

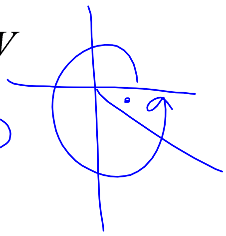
Find the angle in the interval [0, 2pi), given the reference angle and quadrant in which the terminal side of the angle lies.

1.) $\theta = .763; II$

2.379

2.) $\theta = .763; IV$

5.520



3.) $\theta = .763; III$

3.905

4.) $\theta = .415; IV$

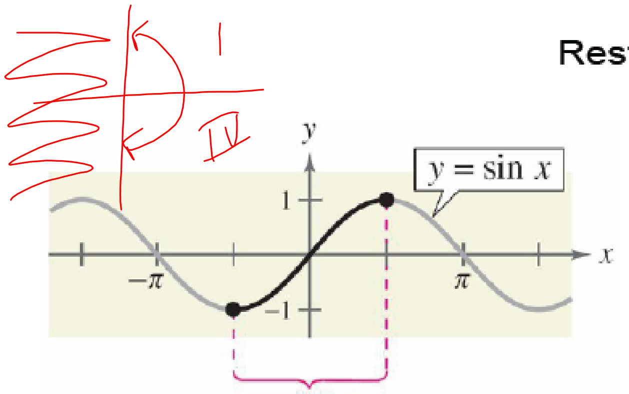
5.868

5.) $\theta = .415; III$

3.557

6.) $\theta = .415; II$

2.727



Restrict the domain to: $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

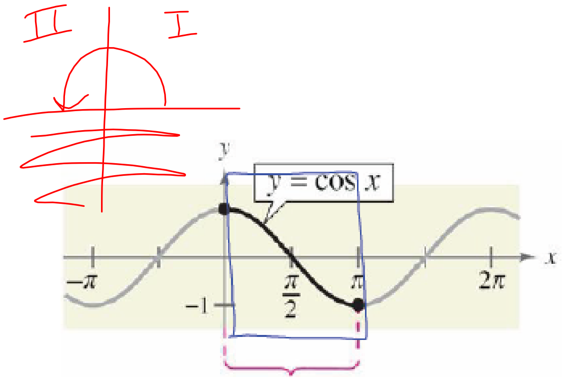
On this interval:

1.) y = sin x is increasing

2.) Range: $[-1, 1]$

3.) y = sin x is one-to-one

Sin x has an inverse on this interval



Restrict the domain to: $[0, \pi]$

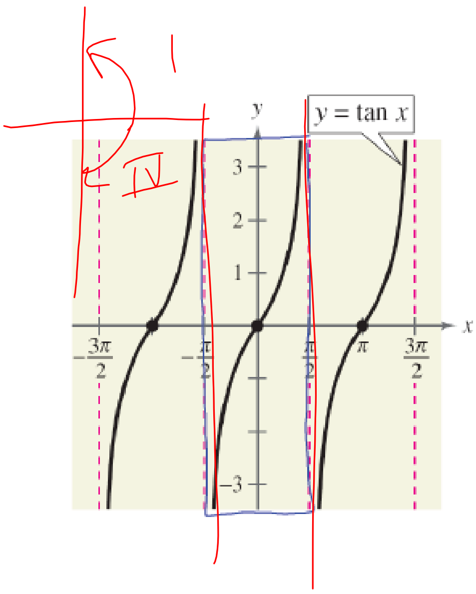
On this interval:

1.) y = cos x is decreasing

2.) Range: $[-1, 1]$

3.) y = cos x is one-to-one

Cos x has an inverse on this interval



Restrict the domain to: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

On this interval:

1.) y = tan x is increasing

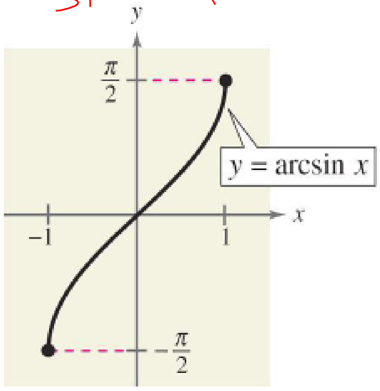
2.) Range: $[-\infty, \infty]$

3.) y = tan x is one-to-one

Dec 5-12:20 PM

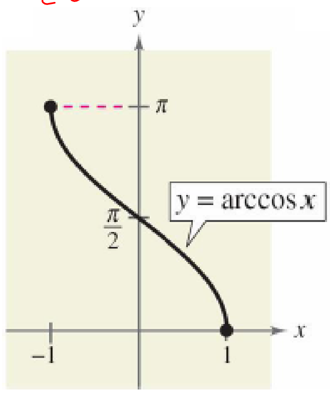
4.7 Inverse Trigonometric Functions

sine up

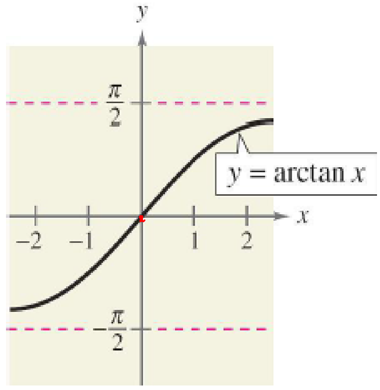


DOMAIN: [–1,1]
RANGE: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

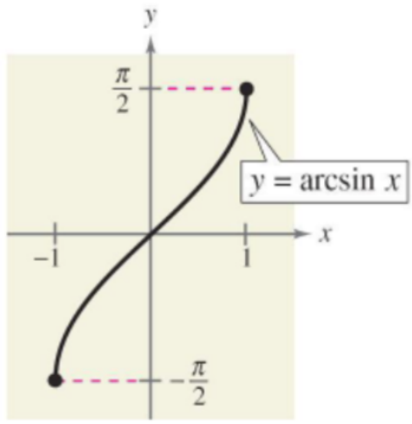
cos down



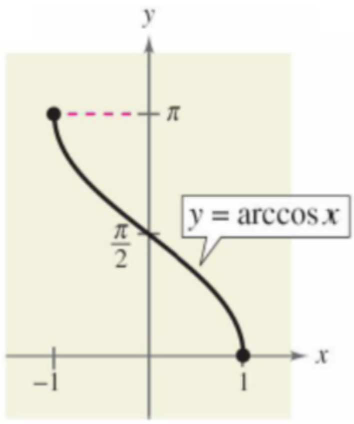
DOMAIN: [–1,1]
RANGE: $[0, \pi]$



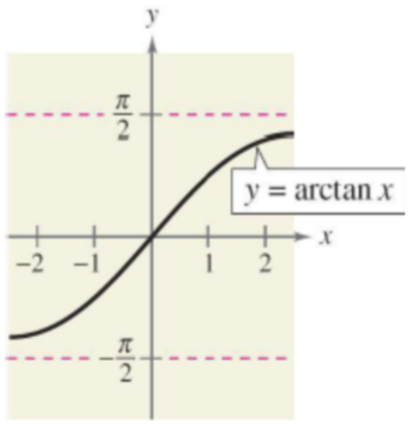
DOMAIN: $(-\infty, \infty)$
RANGE: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



DOMAIN: [–1,1]
RANGE: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

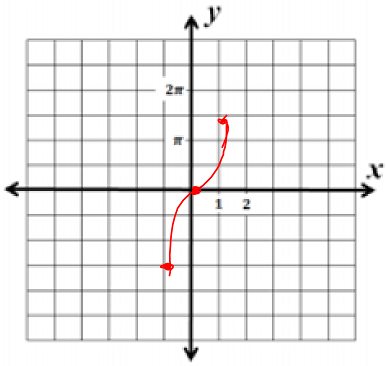


DOMAIN: [–1,1]
RANGE: [0, π]

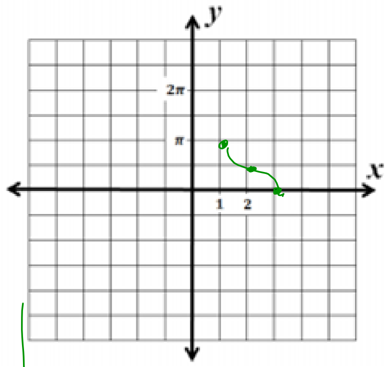


DOMAIN: $(-\infty, \infty)$
RANGE: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

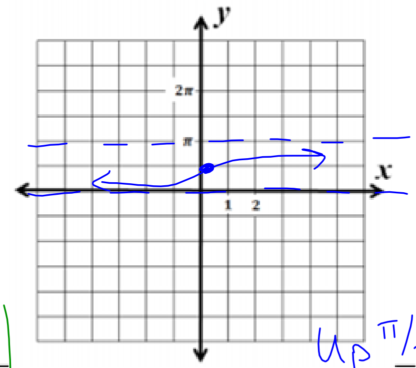
1) $y = 3\sin^{-1} x$



2) $y = \cos^{-1}(x - 2)$



3) $y = \tan^{-1} x + \frac{\pi}{2}$



up $\pi/2$

x	y	$3y$
-1	$-\pi/2$	$-3\pi/2$
0	0	0
1	$\pi/2$	$3\pi/2$

x	$x+2$	y
-1	1	π
0	2	$\pi/2$
1	3	0

$y = \pi/2$ $\quad$ $y = \pi$

$(0,0)$ $\quad$ $(0, \pi/2)$

$y = -\pi/2$ $\quad$ $y = 0$

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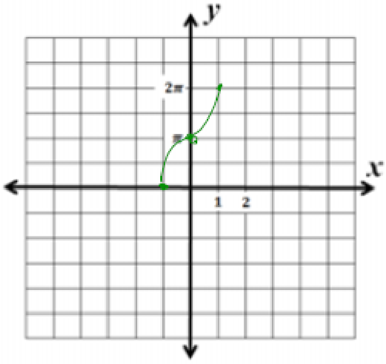
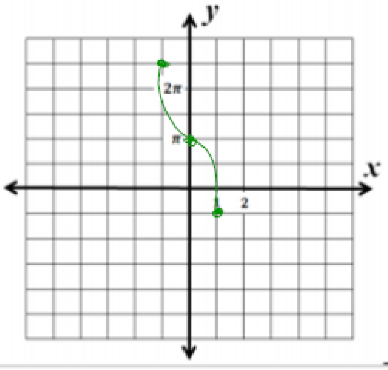
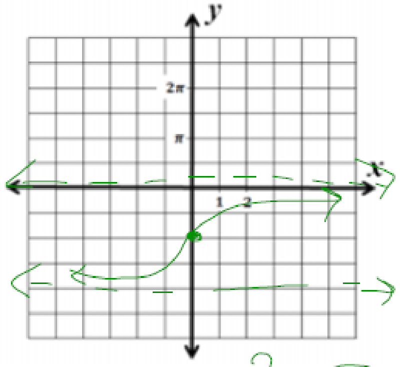
4) $y = 2\sin^{-1}x + \pi$



5) $y = 3\cos^{-1}x - \frac{\pi}{2}$



6) $y = 2\tan^{-1}x - \pi$



x	y	$2y+\pi$
-1	$-\frac{\pi}{2}$	0
0	0	π
1	$\frac{\pi}{2}$	2π

x	y	$3y-\frac{\pi}{2}$
-1	π	$\frac{5\pi}{2}$
0	$\pi/2$	π
1	0	$-\pi/2$

	$2y-\pi$
* $y=\frac{\pi}{2}$	0
$(0,0)$	$(0,-\pi)$
* $y=-\pi/2$	-2π

If possible, find the exact value.

1.) $\tan\left[\arctan(-5)\right]$

θ

-5

2.) $\arcsin\left(\sin\frac{5\pi}{3}\right)$

$\left(-\frac{\sqrt{3}}{2}\right)$

$-\frac{\pi}{3}$

3.) $\cos\left(\cos^{-1}\pi\right)$

$[-1,1]$

$\cos\theta=\pi$

does not exist

4.) $\tan\left[\arctan(-14)\right]$

θ

-14

5.) $\sin(\arcsin\pi)$

$\sin\theta=\pi$

does not exist

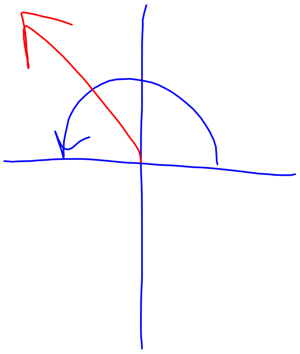
6.) $\cos\left[\arccos(0.54)\right]$

$.54$

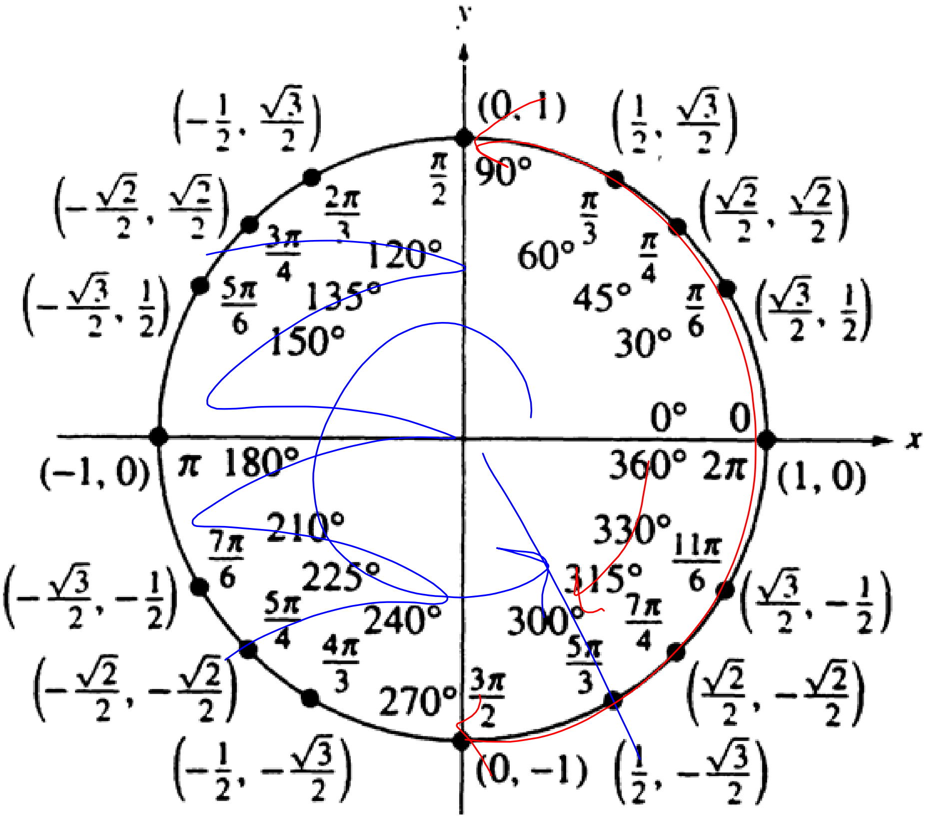
Mar 27-9:52 AM

$$\cos^{-1}\left(\underbrace{\cos\frac{5\pi}{4}}_{}\right)$$

$-\frac{\sqrt{2}}{2}$

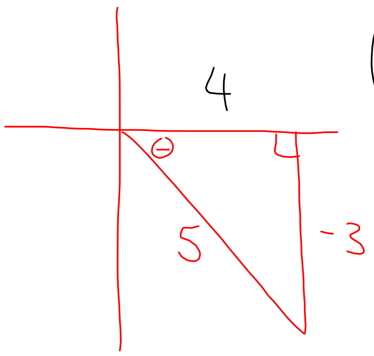


$$\boxed{\frac{3\pi}{4}}$$


y
x
(0, 1)
$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
(1, 0)
$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
$\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
(0, −1)
$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
(−1, 0)
$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\frac{\pi}{2}$ 90°
$\frac{\pi}{3}$ 60°
$\frac{\pi}{4}$ 45°
$\frac{\pi}{6}$ 30°
0° 0
360° 2π
330° $\frac{11\pi}{6}$
315° $\frac{7\pi}{4}$
300° $\frac{5\pi}{3}$
270° $\frac{3\pi}{2}$
240° $\frac{4\pi}{3}$
225° $\frac{5\pi}{4}$
210° $\frac{7\pi}{6}$
π 180°
150° $\frac{5\pi}{6}$
135° $\frac{3\pi}{4}$
120° $\frac{2\pi}{3}$

If possible, find the exact value.

$$7.)\ \cos\left(\overset{\theta}{\arcsin}\left(-\frac{3}{5}\right)\right)$$

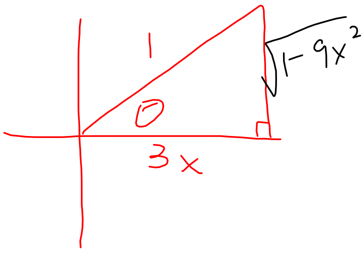


$$\frac{4}{5}$$

Write an algebraic expression that is equivalent to the expression.

8.) $\sin(\arccos 3x); \quad 0 \le x \le \frac{1}{3}$

θ



$(3x)^2 + b^2 = 1^2$

$9x^2 + b^2 = 1$

$b^2 = 1 - 9x^2$

$\boxed{b = \sqrt{1-9x^2}}$

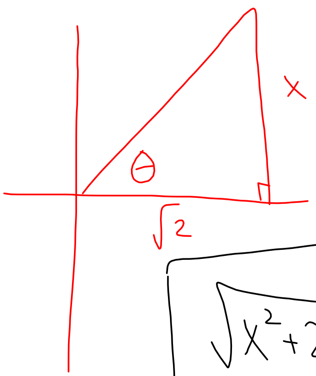
$\boxed{\sqrt{1-9x^2}}$

9.) $\cot(\arccos 3x); \quad 0 \le x \le \frac{1}{3}$

$\frac{A}{O} = \boxed{\frac{3x}{\sqrt{1-9x^2}}}$

Write an algebraic expression that is equivalent to the expression.

10.) $\csc\left(\arctan\frac{x}{\sqrt{2}}\right)$ $\frac{H}{O}$



$$x^2 + \sqrt{2}^2 = c^2$$

$$x^2 + 2 = c^2$$

$$\sqrt{x^2+2} = c$$

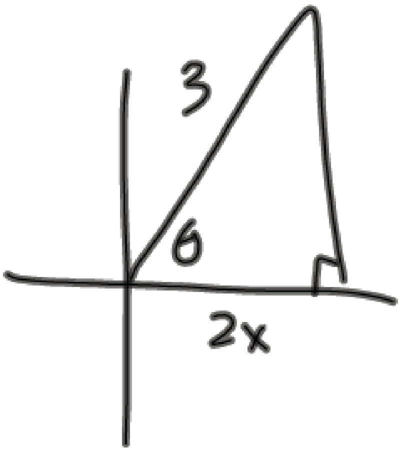
$$\boxed{\frac{\sqrt{x^2+2}}{x}}$$

Mar 27-10:07 AM

Write an algebraic expression that is equivalent to the expression.

θ

11.) $\tan\left(\arccos\frac{2x}{3}\right)$



$(2x)^2+b^2=9$

$b^2=9-4x^2$

$b=\sqrt{9-4x^2}$

$=\dfrac{\sqrt{9-4x^2}}{2x}$

Find the general solution. Then identify all solutions of the equation in the interval $[0, 2\pi)$

12.) $\sin x + \sqrt{2} = -\sin x$

$x + \sqrt{2} = -x$

$\sqrt{25}$

5

$x^2 = 25$

$x = \pm 5$

$2\sin x = -\sqrt{2}$

$\sin x = -\frac{\sqrt{2}}{2}$

$\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = x$

$\boxed{x = \frac{5\pi}{4}, \frac{7\pi}{4}}$

Dec 14-1:38 PM

Find the general solution. Then identify all solutions of the equation in the interval $[0, 2\pi)$

$w = \sin x$

13.) $2\sin^2 x - \sin x - 1 = 0$

$2w^2 - w - 1 = 0$

$(2w+1)(w-1) = 0$

$(2\sin x + 1)(\sin x - 1) = 0$

$\sin x = -1/2$ $\quad$ $\sin x = 1$

$\sin^{-1}(-1/2) = x$ $\quad$ $\sin^{-1}(1) = x$

$\boxed{x = \frac{7\pi}{6}, \frac{11\pi}{6}}$ $\quad$ $\boxed{x = \frac{\pi}{2}}$

Find the general solution. Then identify all solutions of the equation in the interval $[0, 2\pi)$

14.) $\cot x \cos^2 x = 2\cot x$

$\cot x \cos^2 x - 2\cot x = 0$

$\cot x(\cos^2 x - 2) = 0$

$\cot x = 0$

$\frac{x}{y} = \frac{0}{\#}$

$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}}$

$\cos^2 x = 2$

$\cos x = \pm\sqrt{2}$

$x^{\cancel{2}} = 5\cancel{x}$

$x = 5$

$x^2 - 5x = 0$

$x(x-5) = 0$

$x = 0 \quad x = 5$

Dec 15-9:41 AM