5.2

Pg. 418 #3-51 mult. of 3

(3) $\frac{x^2-4}{x-2} - \frac{x^2-9}{x+3} = 5$

$\frac{(x+2)(x-2)}{x-2} - \frac{(x+3)(x-3)}{x+3} = 5$

$x+2 - x+3 = 5$

$5 = 5$

(6) $\frac{\tan x}{\sec x} = \frac{\sin x}{\cos x} \cdot \frac{\cos x}{1} = \sin x$ ✓ yes

(9) $\frac{\sin^3 x}{1+\cot^2 x} = \frac{\sin^3 x}{\csc^2 x} = \frac{\sin^3 x}{\frac{1}{\sin^2 x}} = \sin^3 x \cdot \sin^2 x$

$\sin 5x$ NO

$\sin^3 x(1+\cot^2 x) = \sin^3 x(\csc^2 x)$

$= \sin^3 x(\csc^2 x) = \sin^3 x\left(\frac{1}{\sin^2 x}\right) = \sin x$ ✓ Yes

(12) $(\sin x)(\cot x + \cos x \tan x) = \cos x + \sin^2 x$

$= \sin x\left(\frac{\cos x}{\sin x} + \frac{\cos x}{\cos x} \cdot \frac{\sin x}{\cos x}\right)$

$= \frac{\sin x \cos x}{\sin x} + \sin^2 x = \cos x + \sin^2 x$ ✓

(15) $\frac{(1-\cos x)(1+\cos x)}{\cos^2 x} = \tan^2 x$

$= \frac{1-\cos^2 x}{\cos^2 x}$

$= \frac{\sin^2 x}{\cos^2 x} = \tan^2 x$ ✓

(18) $\dfrac{\sec^2 x - 1}{\sin x} = \dfrac{\sin x}{1 - \sin^2 x}$

$\dfrac{\tan^2 x}{\sin x} = \dfrac{\dfrac{\sin^2 x}{\cos^2 x}}{\sin x} = \dfrac{\sin^2 x}{\cos^2 x} \cdot \dfrac{1}{\sin x}$

$= \dfrac{\sin x}{\cos^2 x} = \dfrac{\sin x}{1 - \sin^2 x}$ ✓

(21) $(\cos x - \sin x)^2 + (\cos x + \sin x)^2 = 2$

$= \cos^2 x - 2\sin x \cos x + \sin^2 x$

$+ \cos^2 x + 2\sin x \cos x + \sin^2 x$

$= 2(\cos^2 x + \sin^2 x)$

$= 2(1) = 2$ ✓

(24) $\dfrac{1}{\tan x} + \tan x = \sec x \csc x$

$= \dfrac{1 + \tan^2 x}{\tan x} = \dfrac{\sec^2 x}{\tan x} = \dfrac{1}{\dfrac{\cos^2 x}{\sin x}}$

$= \dfrac{1}{\cos x}$

$= \dfrac{1}{\cos^2 x} \cdot \dfrac{\cos x}{\sin x} = \dfrac{1}{\cos x} \cdot \dfrac{1}{\sin x} = \sec x \csc x$ ✓

(27) $\frac{\tan^2 x}{\sec x + 1} = \frac{1 - \cos x}{\cos x}$

$= \frac{\sin^2 x}{\cos^2 x}$ (illegible)

$= \frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos x} + 1} = \frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{1 + \cos x}{\cos x}}$

$= \frac{\sin^2 x}{\cos^2 x} \cdot \frac{\cos x}{1 + \cos x} = \frac{\cos x (1 - \cos^2 x)}{\cos^2 x (1 + \cos x)}$

$= \frac{\cos x (1 + \cos x)(1 - \cos x)}{\cos^2 x (1 + \cos x)} = \frac{1 - \cos x}{\cos x}$ ✓

(30) $\tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$

$\frac{\sin^2 x}{\cos^2 x} - \frac{\sin^2 x}{1} = \frac{\sin^2 x - \sin^2 x \cos^2 x}{\cos^2 x}$

$= \frac{\sin^2 x (1 - \cos^2 x)}{\cos^2 x} = \frac{\sin^2 x \cdot \sin^2 x}{\cos^2 x} = \frac{\sin^2 x}{\cos^2 x} \cdot \sin^2 x$

$= \tan^2 x \sin^2 x$ ✓

(33) $(x \sin\alpha + y \cos\alpha)^2 + (x \cos\alpha - y \sin\alpha)^2 = x^2 + y^2$

$= x^2 \sin^2\alpha + 2xy \sin\alpha \cos\alpha + y^2 \cos^2\alpha$
$+ x^2 \cos^2\alpha - 2xy \sin\alpha \cos\alpha + y^2 \sin^2\alpha$

$= x^2 \sin^2\alpha + y^2 \cos^2\alpha + x^2 \cos^2\alpha + y^2 \sin^2\alpha$

$= x^2 (\sin^2\alpha + \cos^2\alpha) + y^2 (\sin^2\alpha + \cos^2\alpha)$

$= x^2 (1) + y^2 (1)$

$= x^2 + y^2$ ✓

(36) $\frac{\sin x}{1+\cos x} + \frac{1+\cos x}{\sin x} = 2\csc x$

$= \frac{\sin^2 x + 1 + 2\cos x + \cos^2 x}{\sin x(1+\cos x)}$

$= \frac{2+2\cos x}{\sin x(1+\cos x)} = \frac{2(1+\cos x)}{\sin x(1+\cos x)} = \frac{2}{\sin x}$

$= 2\csc x$ ✓

(39) $\frac{\sin x}{1-\cos x} + \frac{1+\cos x}{\sin x} = \frac{2(1+\cos x)}{\sin x}$

$= \frac{\sin^2 x + (1-\cos^2 x)}{\sin x(1-\cos x)}$

$= \frac{\sin^2 x + (1+\cos x)(1-\cos x)}{\sin x(1-\cos x)}$

$= \frac{\sin^2 x + 1 - \cos^2 x}{\sin x(1-\cos x)} = \frac{1-\cos^2 x + 1 - \cos^2 x}{\sin x(1-\cos x)}$

$= \frac{2 - 2\cos^2 x}{\sin x(1-\cos x)} = \frac{2(1-\cos^2 x)}{\sin x(1-\cos x)}$

$= \frac{2(1+\cos x)(1-\cos x)}{\sin x(1-\cos x)} = \frac{2(1+\cos x)}{\sin x}$ ✓

(42) $\sin^5 x \cos^2 x = (\cos^2 x - 2\cos^4 x + \cos^6 x)(\sin x)$

$= (\cos^2 x)(1 - 2\cos^2 x + \cos^4 x)(\sin x)$

$= (\cos^2 x)(1 - \cos^2 x)^2 (\sin x)$

$= \cos^2 x \ (\sin^2 x)^2 (\sin x)$

$= \cos^2 x \ (\sin^4 x)(\sin x)$

$= \cos^2 x \ (\sin^5 x)$

$= \sin^5 x \cos^2 x$ ✓

(45) $\dfrac{\tan x}{1 - \cot x} + \dfrac{\cot x}{1 - \tan x} = 1 + \sec x \csc x$

$\dfrac{\tan x(1 + \tan x) + \cot x(1 - \cot x)}{(1 - \cot x)(1 - \tan x)}$

$= \dfrac{\tan x - \tan^2 x + \cot x - \cot^2 x}{(1 - \cot x)(1 - \tan x)}$

$\dfrac{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}}{1 - \frac{\cos x}{\sin x}\left(1 - \frac{\sin x}{\cos x}\right)}$

$= \dfrac{\sin^2 x + \cos^2 x}{\sin x \cos x}$

$\dfrac{\tan x(1 - \tan x) + \cot x(1 - \cot x)}{(1 - \cot x)(1 - \tan x)}$

$= \dfrac{\frac{\sin x}{\cos x} - \frac{\sin^2 x}{\cos^2 x} + \frac{\cos x}{\sin x} - \frac{\cos^2 x}{\sin^2 x}}{\left(1 - \frac{\cos x}{\sin x}\right)\left(1 - \frac{\sin x}{\cos x}\right)}$

$= \dfrac{\frac{\sin x}{\cos x} - \frac{\sin^2 x}{\cos^2 x} + \frac{\cos x}{\sin x} - \frac{\cos^2 x}{\sin^2 x}}{\frac{\sin x - \cos x}{\sin x}\left(\frac{\cos x - \sin x}{\cos x}\right)}$

$\sin x \cos x - \sin^2 x +$

$=$

(45) $\frac{\tan x}{1-\cot x} + \frac{\cot x}{1-\tan x} = \csc x \sec x + 1$

$\frac{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}}{\left(1-\frac{\cos x}{\sin x}\right)\left(1-\frac{\sin x}{\cos x}\right)} = \frac{\sin^2 x + \cos^2 x}{\cos x \sin x}$

$\frac{\sin^2 x + \cos^2 x}{\cos x \sin x}$

$\frac{\cos x \sin x}{\cos x \sin x} - \frac{\cos^2 x}{\sin x \cos x} - \frac{\sin^2 x}{\cos x \sin x} - \frac{\cos x \sin x}{\cos x \sin x}$

$\frac{\sin^2 x + \cos^2 x}{\cos x \sin x}$

$-\frac{(\cos^2 x + \sin^2 x)}{\sin x \cos x}$

$=$

$\frac{\cos x \sin x}{-1}$

$\sin x$

aahh!!

(45) $\frac{\tan x}{1-\cot x} + \frac{\cot x}{1-\tan x} = \csc x \sec x + 1$

$= \frac{\tan x}{1 - \frac{\cos x}{\sin x}} + \frac{\cot x}{1 - \frac{\sin x}{\cos x}} =$

$= \frac{\frac{\sin x}{\cos x}}{\frac{\sin x - \cos x}{\sin x}} + \frac{\frac{\cos x}{\sin x}}{\frac{\cos x - \sin x}{\cos x}} =$

$= \frac{\sin x}{\cos x} \cdot \frac{\sin x}{\sin x - \cos x} + \frac{\cos x}{\sin x} \cdot \frac{\cos x}{\cos x - \sin x} =$

$= \frac{\sin^2 x}{\cos x(\sin x - \cos x)} + \frac{\cos^2 x}{\sin x(\cos x - \sin x)} = \text{(neg)}$

$= \frac{\sin^2 x}{\cos x(\sin x - \cos x)} - \frac{\cos^2 x}{\sin x(\sin x - \cos x)} =$

$= \frac{\sin^3 x - \cos^3 x}{(\cos x \sin x)(\sin x - \cos x)} = \frac{(\sin x - \cos x)(\sin^2 x + \sin x \cos x + \cos^2 x)}{\cos x \sin x (\sin x - \cos x)}$

$= \frac{1 + \sin x \cos x}{\cos x \sin x} = \frac{1}{\cos x \sin x} + \frac{\sin x \cos x}{\sin x \cos x}$

$= \frac{1}{\cos x} \cdot \frac{1}{\sin x} + 1 = \sec x \csc x + 1$

$= \csc x \sec x + 1$ ✓

$1 - 4x + x - 4x^2$ $\quad -4x + x$

$(1-4x) + x(1-4x^2$

~~(48) $\frac{\tan^2 x - 1}{1+\tan x} = \frac{(\tan x + 1)(\tan x - 1)}{\tan x + 1} = \tan x - 1$~~

(49) $\frac{1 - 3\cos x - 4\cos^2 x}{\sin^2 x} = \frac{1 - 4\cos x}{1 - \cos x}$

$= \frac{(1-4\cos x)(1+\cos x)}{1-\cos^2 x} = \frac{(1-4\cos x)(1+\cos x)}{(1+\cos x)(1-\cos x)}$

$= \frac{1-4\cos x}{1-\cos x}$ ✓

(51) $\sin^5 x = (1 - 2\cos^2 x + \cos^4 x)(\sin x)$

$= (1 - \cos^2 x)^2 (\sin x)$

$= (\sin^2 x)^2 (\sin x)$

$= (\sin^4 x)(\sin x)$

$= \sin^5 x$ ✓