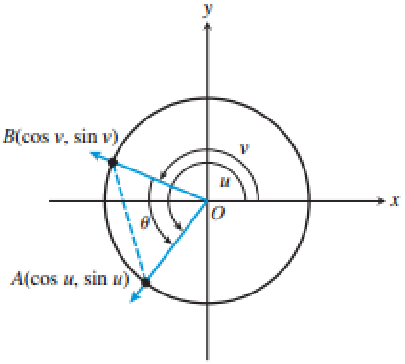
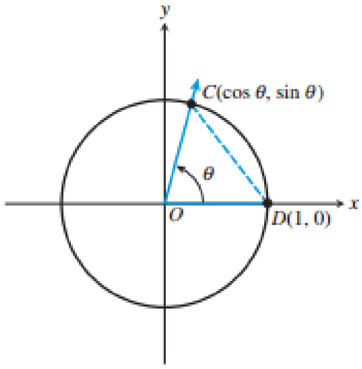
5.3 Sum and Difference Identities





$$AB = CD$$

$$\sqrt{(\cos v - \cos u)^2 + (\sin v - \sin u)^2} = \sqrt{(\cos\theta - 1)^2 + (\sin\theta - 0)^2}$$

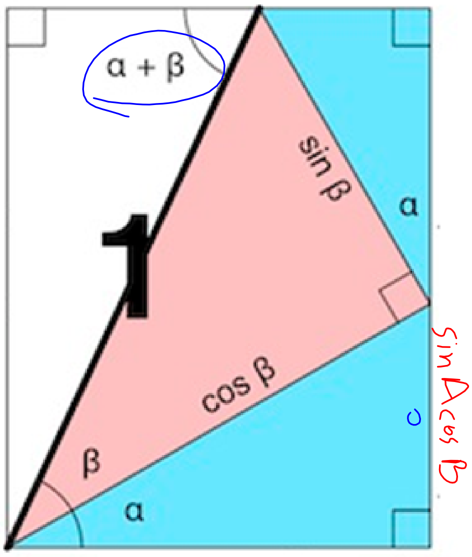
Square both sides to eliminate the radical and expand the binomials to get

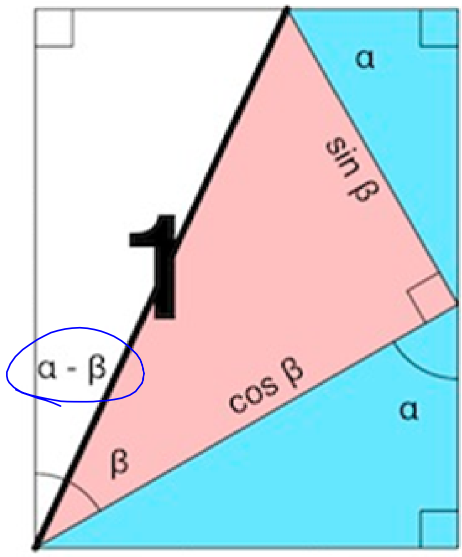
$$\cos^2 u - 2\cos u\cos v + \cos^2 v + \sin^2 u - 2\sin u\sin v + \sin^2 v$$
$$= \cos^2\theta - 2\cos\theta + 1 + \sin^2\theta$$
$$(\cos^2 u + \sin^2 u) + (\cos^2 v + \sin^2 v) - 2\cos u\cos v - 2\sin u\sin v$$
$$= (\cos^2\theta + \sin^2\theta) + 1 - 2\cos\theta$$
$$2 - 2\cos u\cos v - 2\sin u\sin v = 2 - 2\cos\theta$$
$$\cos u\cos v + \sin u\sin v = \cos\theta$$

Finally, since $\theta = u - v$, we can write

$$\cos(u - v) = \cos u\cos v + \sin u\sin v.$$

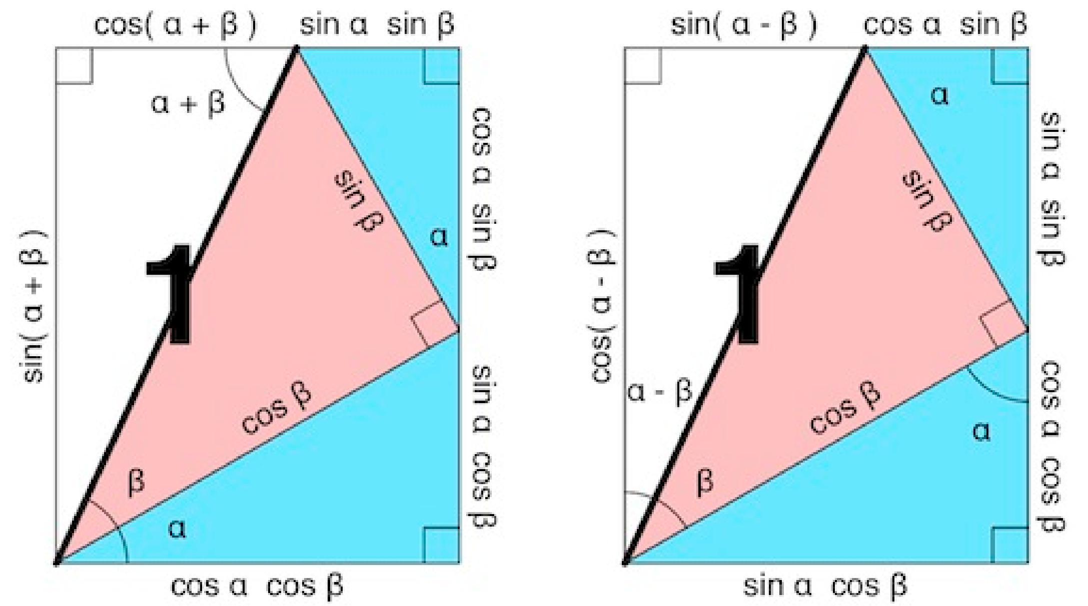
Jan 9-7:27 AM





$\sin\alpha = \frac{o}{\cos\beta}$

$\sin\alpha\cos\beta = o$



$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

Sum and Difference Formulas

$$\sin(u+v) = \sin u \cos v + \cos u \sin v$$

$$\sin(u-v) = \sin u \cos v - \cos u \sin v$$

$$\cos(u+v) = \cos u \cos v - \sin u \sin v$$

$$\cos(u-v) = \cos u \cos v + \sin u \sin v$$

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v} \qquad \tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

Find the exact value using sum and difference formulas.

1.) $\cos 75^\circ$

$$= \cos(30+45) = \cos 30 \cos 45 - \sin 30 \sin 45$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$$

Jan 10-8:59 AM

Find the exact value using sum and difference formulas.

$$2.)\ \sin\left(\frac{\pi}{12}\right)$$

Find the exact value using sum and difference formulas.

$$2.)\ \sin\left(\frac{\pi}{12}\right) = 15^\circ$$

$$\sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \sin\frac{\pi}{3}\cos\frac{\pi}{4} - \cos\frac{\pi}{3}\sin\frac{\pi}{4}$$

$(45-30)$ $\quad \frac{4\pi}{12} - \frac{3\pi}{12}$ $\quad \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2}\frac{\sqrt{2}}{2}$

$\frac{\pi}{4} - \frac{\pi}{6}$ $\quad \frac{\sqrt{6} - \sqrt{2}}{4}$

$\frac{3\pi}{12} - \frac{2\pi}{12}$

Find the exact value.

3.) $\dfrac{\tan 80 + \tan 55}{1 - \tan 80 \tan 55} = \tan(80+55)$

$= \tan(135)$

$= -1$

Prove the cofunction identity.

4.) $\cos\left(\frac{\pi}{2} - x\right) = \sin x$

$= \cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x$

$= 0 \cdot \cos x + 1 \cdot \sin x$

$= 0 + \sin x = \sin x$ ✓

Verify the identity.

5.) $\sin(x+y)+\sin(x-y)=2\sin x\cos y$

6.) $\sin(x+y)+\sin(x-y)=\boxed{2\sin x\cos y}$

$= \sin x\cos y + \cancel{\cos x\sin y} + \sin x\cos y - \cancel{\cos x\sin y}$

$= 2\sin x\cos y$

Verify the identity.

6.) $\cos 3x = \cos^3 x - 3\sin^2 x \cos x$

$= \cos((x+x)+x)$

$= \cos(x+x)\cos x - \sin(x+x)\sin x$

$= (\cos x\cos x - \sin x\sin x)\cos x - (\sin x\cos x + \cos x\sin x)\sin x$

$= (\cos^2 x - \sin^2 x)\cos x - (2\sin x\cos x)\sin x$

$= \cos^3 x - \sin^2 x\cos x - 2\sin^2 x\cos x$

$= \cos^3 x - 3\sin^2 x\cos x \checkmark$

Find all solutions on the interval $[0, 2\pi)$.

$$7.)\ \sin\left(x+\frac{\pi}{3}\right)+\sin\left(x-\frac{\pi}{3}\right)=1$$

$$\sin x\cos\frac{\pi}{3} + \cancel{\cos x\sin\frac{\pi}{3}} + \sin x\cos\frac{\pi}{3} - \cancel{\cos x\sin\frac{\pi}{3}} = 1$$

$$2\sin x\cos{}^{\pi}/_{3} = 1$$

$$2\sin x\left(\tfrac{1}{2}\right) = 1$$

$$\sin x = 1$$

$$x = \pi/2$$

Jan 10-9:57 AM

Find all solutions on the interval $[0, 2\pi)$.

$$8.)\ \sin\left(x+\frac{\pi}{4}\right)-\sin\left(\frac{\pi}{4}-x\right)=-1$$

Find all solutions on the interval $[0, 2\pi)$.

$$9.)\ \sin\left(x+\frac{\pi}{4}\right)-\sin\left(\frac{\pi}{4}-x\right)=-1$$

$$\sin x\cos\frac{\pi}{4}+\cancel{\cos x\sin\frac{\pi}{4}}-\left(\cancel{\sin\frac{\pi}{4}\cos x}-\cos\frac{\pi}{4}\sin x\right)=-1$$

$$2\cos\frac{\pi}{4}\sin x=-1$$

$$\cancel{2}\left(\frac{\sqrt{2}}{\cancel{2}}\right)\sin x=-1$$

$$\sin x=\frac{-1}{\sqrt{2}}=\frac{-\sqrt{2}}{2}$$

$$x=\frac{5\pi}{4},\ \frac{7\pi}{4}$$

Find all solutions on the interval $[0, 2\pi)$.

9.) $2\sin\left(x+\frac{\pi}{2}\right)+3\tan(\pi - x)=0$

$$2\left(\overset{\sin x\cdot 0}{\cancel{\sin x\cos\frac{\pi}{2}}} + \overset{\cos x\cdot(1)}{\cos x\sin\frac{\pi}{2}}\right) + 3\left(\frac{\overset{0}{\tan\pi} - \tan x}{1+\overset{0}{\tan\pi}\tan x}\right) = 0$$

$$2\cos x - 3\tan x = 0$$

$$2\cos x = 3\tan x$$

$$\frac{2\cos x}{1} = \frac{3\sin x}{\cos x}$$

$$2\cos^2 x = 3\sin x$$

$$2(1-\sin^2 x) = 3\sin x$$

$$2 - 2\sin^2 x = 3\sin x$$

$$2\sin^2 x + 3\sin x - 2 = 0$$

$$(2\sin x - 1)(\sin x + 2) = 0$$

$$\sin x = \frac{1}{2} \qquad \cancel{\sin x = -2}$$

$$\boxed{x = \pi/6,\ 5\pi/6}$$

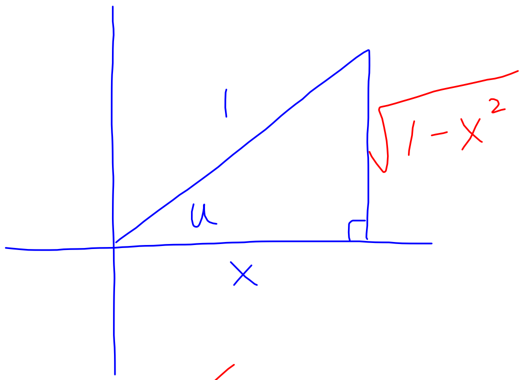
Write the trigonometric expression as an algebraic expression.

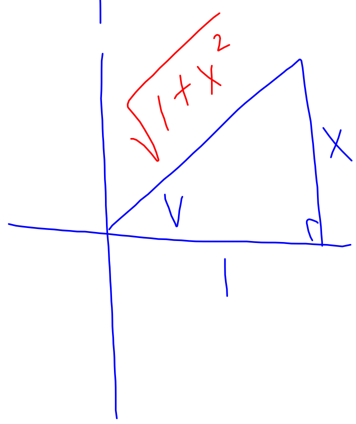
10.) $\cos\left(\overset{u}{\arccos \tfrac{x}{1}} - \overset{v}{\arctan \tfrac{x}{1}}\right)$

$\cos u \cos v + \sin u \sin v$

$$= x \cdot \frac{1}{\sqrt{1+x^2}} + \sqrt{1-x^2} \cdot \frac{x}{\sqrt{1+x^2}}$$

$$\boxed{= \frac{x + x\sqrt{1-x^2}}{\sqrt{1+x^2}}}$$





$\tan^2 x \cot^2 x$

$$\frac{\sin^2 x}{\cos^2 x} \cdot \frac{\cos^2 x}{\sin^2 x}$$

1

1

$$\tan^2 x \cdot \frac{1}{\tan^2 x}$$

1

Jan 9-8:50 AM

Jan 10-10:17 AM