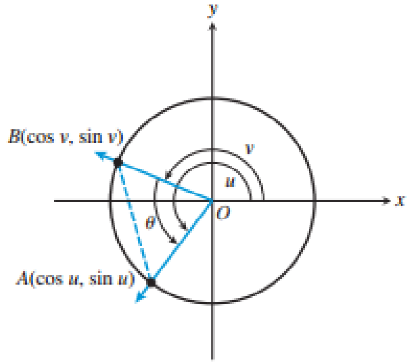
5.3 Sum and Difference Identities

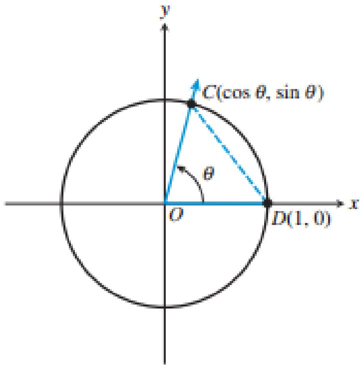


$$AB = CD$$

$$\sqrt{(\cos v - \cos u)^2 + (\sin v - \sin u)^2} = \sqrt{(\cos\theta - 1)^2 + (\sin\theta - 0)^2}$$

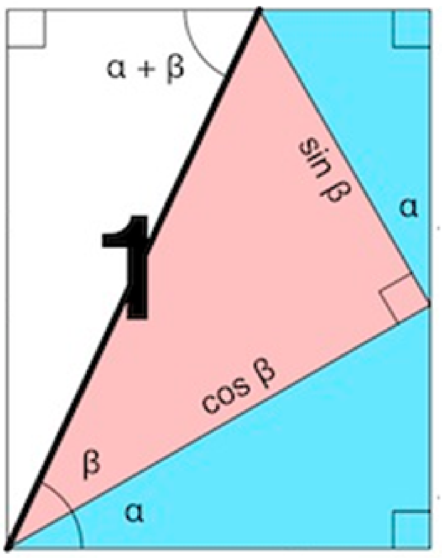
Square both sides to eliminate the radical and expand the binomials to get

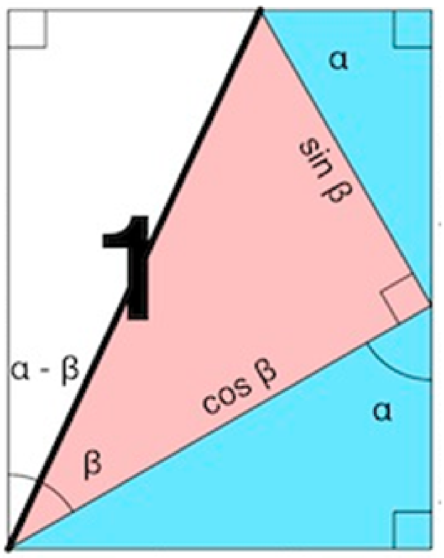
$$\cos^2 u - 2\cos u\cos v + \cos^2 v + \sin^2 u - 2\sin u\sin v + \sin^2 v$$
$$= \cos^2\theta - 2\cos\theta + 1 + \sin^2\theta$$
$$(\cos^2 u + \sin^2 u) + (\cos^2 v + \sin^2 v) - 2\cos u\cos v - 2\sin u\sin v$$
$$= (\cos^2\theta + \sin^2\theta) + 1 - 2\cos\theta$$
$$2 - 2\cos u\cos v - 2\sin u\sin v = 2 - 2\cos\theta$$
$$\cos u\cos v + \sin u\sin v = \cos\theta$$



Finally, since $\theta = u - v$, we can write

$$\cos(u - v) = \cos u\cos v + \sin u\sin v.$$





$\cos a \cos \beta$

$\cos(a+\beta) =$

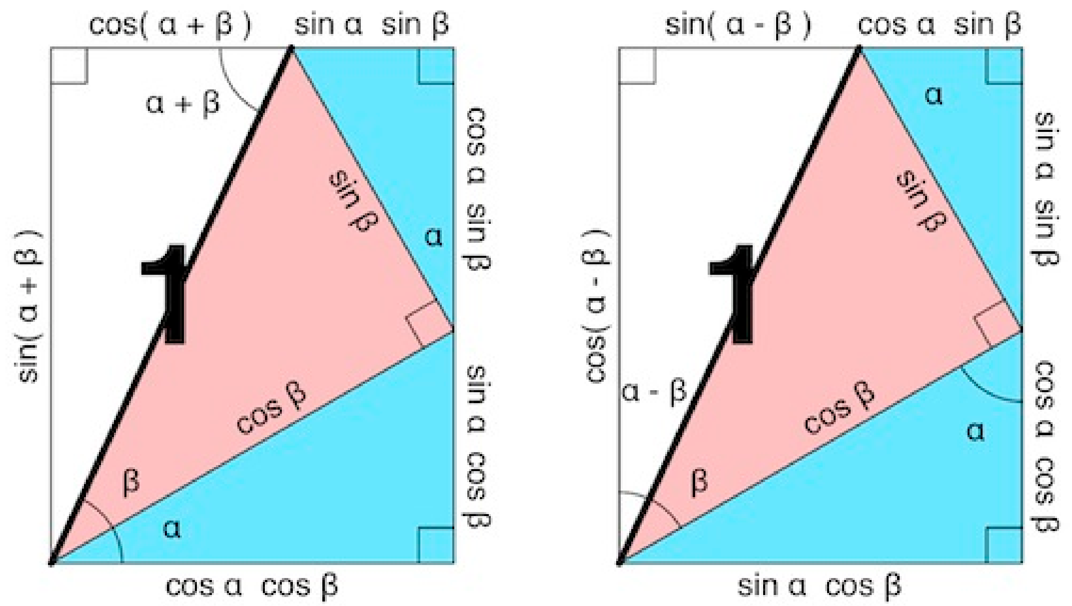
$\sin(a+b) =$

$\cos a = \dfrac{A}{\cos \beta}$

$\cos a \cos \beta = A$

$\cos(a-\beta) =$

$\sin(a-\beta) =$



$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

Sum and Difference Formulas

$$\sin(u + v) = \sin u \cos v + \cos u \sin v$$

$$\sin(u - v) = \sin u \cos v - \cos u \sin v$$

$$\cos(u + v) = \cos u \cos v - \sin u \sin v$$

$$\cos(u - v) = \cos u \cos v + \sin u \sin v$$

$$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v} \qquad \tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

Find the exact value using sum and difference formulas.

1.) $\cos 75^\circ = \cos(30^\circ + 45^\circ)$

$= \cos 30 \cos 45 - \sin 30 \sin 45$

$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$

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Find the exact value using sum and difference formulas.

$$2.)\ \sin\left(\frac{\pi}{12}\right)$$

Find the exact value.

3.) $\dfrac{\tan 80 + \tan 55}{1 - \tan 80 \tan 55} = \tan(80+55)$

$= \tan(135)$

$= -1$

Prove the cofunction identity.

4.) $\cos\left(\dfrac{\pi}{2}-x\right)=\sin x$

$= \cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x$

$= 0\cdot\cos x + 1\cdot\sin x$

$= 0 + \sin x$

$= \sin x$ ✓

Prove the reduction formula.

5.) $\cos\left(x - \frac{3\pi}{2}\right) = -\sin x$

Verify the identity.

6.) $\sin(x+y)+\sin(x-y)=2\sin x\cos y$

Verify the identity.

7.) $\cos 3x = \cos^3 x - 3\sin^2 x\cos x$

$= \cos((\overset{u}{x+x}) \overset{+v}{+x})$

$= \cos(x+x)\cos x - \sin(x+x)\sin x$

$= (\cos x\cos x - \sin x\sin x)\cos x - (\sin x\cos x + \cos x\sin x)\sin x$

$= (\cos^2 x - \sin^2 x)\cos x - (2\sin x\cos)\sin x$

$= \cos^3 x - \underline{\sin^2 x\cos x} - 2\underline{\sin^2 x\cos x}$

$= \cos^3 x - 3\sin^2 x\cos x$ ✓

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Find all solutions on the interval $[0, 2\pi)$.

$$8.)\ \sin\left(x+\frac{\pi}{3}\right)+\sin\left(x-\frac{\pi}{3}\right)=1$$

$$\sin x\cos\frac{\pi}{3}+\cancel{\cos x\sin\frac{\pi}{3}}+\sin x\cos\frac{\pi}{3}-\cancel{\cos x\sin\frac{\pi}{3}}=1$$

$$2\sin x\cos\frac{\pi}{3}=1$$

$$2\sin x\left(\frac{1}{2}\right)=1$$

$$\sin x=1$$

$$x=\frac{\pi}{2}$$

Find all solutions on the interval $[0, 2\pi)$.

9.) $\sin\left(x+\frac{\pi}{4}\right)-\sin\left(\frac{\pi}{4}-x\right)=-1$

$\sin x\cos\frac{\pi}{4}+\cancel{\cos x\sin\frac{\pi}{4}}-\left(\cancel{\sin\frac{\pi}{4}\cos x}-\cos\frac{\pi}{4}\sin x\right)=-1$

$2\cos\frac{\pi}{4}\sin x=-1$

$\cancel{2}\left(\frac{\sqrt{2}}{\cancel{2}}\right)\sin x=-1$

$\sin x=\frac{-1}{\sqrt{2}}=\frac{-\sqrt{2}}{2}$

$x=\frac{5\pi}{4},\ \frac{7\pi}{4}$

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Find all solutions on the interval $[0, 2\pi)$.

10.) $2\sin\left(x+\frac{\pi}{2}\right)+3\tan(\pi - x)=0$

$$2\left(\cancel{\sin x\cos\tfrac{\pi}{2}} + \cos x\sin\tfrac{\pi}{2}\right)+3\left(\frac{\cancel{\tan\pi} - \tan x}{1+\cancel{\tan\pi\tan x}}\right)=0$$

$$2\cos x - 3\tan x = 0$$

$$2\cos x = 3\frac{\sin x}{\cos x}$$

$$2\cos^2 x = 3\sin x$$

$$2(1-\sin^2 x) = 3\sin x$$

$$2-2\sin^2 x = 3\sin x$$

$$2\sin^2 x + 3\sin x - 2 = 0$$

$$(2\sin x - 1)(\sin x + 2) = 0$$

$\sin x = 1/2$ $\quad$ $\sin x = -2$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ $\quad$ DNE

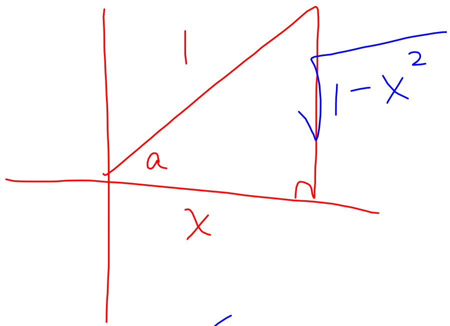
Write the trigonometric expression as an algebraic expression.

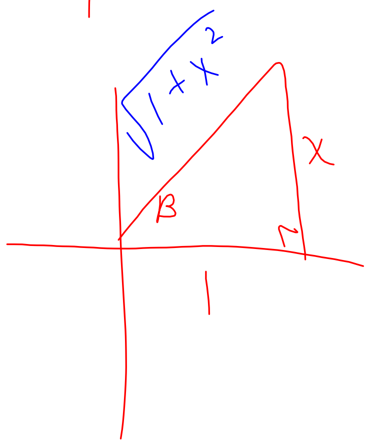
11.) $\cos(\overset{a}{\arccos \tfrac{x}{1}} - \overset{\beta}{\arctan \tfrac{x}{1}})$

$\cos a \cos \beta + \sin a \sin \beta$

$x \cdot \frac{1}{\sqrt{1+x^2}} + \sqrt{1-x^2} \cdot \frac{x}{\sqrt{1+x^2}}$

$= \boxed{\frac{x + x\sqrt{1-x^2}}{\sqrt{1+x^2}}}$





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