Pre-Calc
5.4 – 5.5 Test Review

Name Answer Key
Date 3/9/18 **Period** ____

Find the exact value.

1.) $\cos 25^\circ \cos 20^\circ - \sin 25^\circ \sin 20^\circ$

$= \cos 45^\circ$

$= \boxed{\frac{\sqrt{2}}{2}}$

2.) $\cos\frac{\pi}{16}\cos\frac{3\pi}{16} - \sin\frac{\pi}{16}\sin\frac{3\pi}{16}$

$= \cos\frac{\pi}{4} = \boxed{\frac{\sqrt{2}}{2}}$

Find the exact value of the sine, cosine, and tangent of the angle by using the sum or difference formulas.

3.) 285° $(240+45)$

$\sin(240+45) = \sin 240 \cos 45 + \cos 240 \sin 45$

$= -\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + -\frac{1}{2}\cdot\frac{\sqrt{2}}{2} = \frac{-\sqrt{6}}{4} + \frac{-\sqrt{2}}{4}$

$= \boxed{\frac{-\sqrt{6}-\sqrt{2}}{4}}$

$\cos(240+45) = \cos 240 \cos 45 - \sin 240 \sin 45$

$= -\frac{1}{2}\cdot\frac{\sqrt{2}}{2} - -\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{-\sqrt{2}}{4} - \frac{-\sqrt{6}}{4}$

$= \boxed{\frac{-2+\sqrt{6}}{4}}$

$\tan(240+45) = \frac{\tan 240 + \tan 45}{1 - \tan 240 \tan 45} = \frac{\sqrt{3}+1}{1-\sqrt{3}} \cdot \frac{1+\sqrt{3}}{1+\sqrt{3}} = \frac{\sqrt{3}+1+\sqrt{3}+3}{1-3}$

$= \frac{4+2\sqrt{3}}{-2} = \boxed{-2-\sqrt{3}}$

4.) $\frac{17\pi}{12}$ Hint: $\frac{3\pi}{4}+\frac{2\pi}{3}$

$\sin\frac{17\pi}{12} = \sin\frac{3\pi}{4}\cos\frac{2\pi}{3} + \cos\frac{3\pi}{4}\sin\frac{2\pi}{3}$

$= \frac{\sqrt{2}}{2}\cdot -\frac{1}{2} + -\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} = \frac{-\sqrt{2}}{4} + \frac{-\sqrt{6}}{4} = \boxed{\frac{-\sqrt{2}-\sqrt{6}}{4}}$

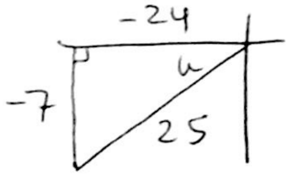
$\cos\frac{17\pi}{12} = \cos\frac{3\pi}{4}\cos\frac{2\pi}{3} - \sin\frac{3\pi}{4}\sin\frac{2\pi}{3} =$

$= -\frac{\sqrt{2}}{2}\cdot -\frac{1}{2} - \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} = \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$

$\tan\frac{17\pi}{12} = \frac{\tan\frac{3\pi}{4} + \tan\frac{2\pi}{3}}{1 - \tan\frac{3\pi}{4}\tan\frac{2\pi}{3}} = \frac{-1-\sqrt{3}}{1-\sqrt{3}} \cdot \frac{1+\sqrt{3}}{1+\sqrt{3}} = \frac{-1-\sqrt{3}-\sqrt{3}-3}{1-3} = \frac{-4-2\sqrt{3}}{-2}$

$= \boxed{2+\sqrt{3}}$

Find the exact value of trigonometric function given that $\sin u = -\frac{7}{25}$ and $\cos v = -\frac{4}{5}$. (Both are in Quadrant III.)



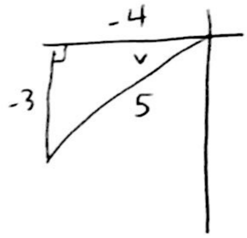
5.) $\cos(u+v)$

$\cos u \cos v - \sin u \sin v$

$\frac{-24}{25} \cdot \frac{-4}{5} - \frac{-3}{5} \cdot \frac{-7}{25}$

$\frac{96}{125} - \frac{21}{125}$

$\frac{75}{125} = \boxed{\frac{3}{5}}$



6.) $\cos(u-v)$

$\cos u \cos v + \sin u \sin v$

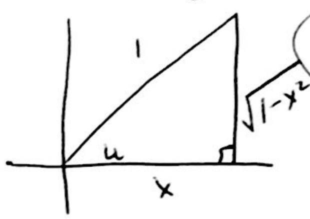
$\frac{-24}{25} \cdot \frac{-4}{5} + \frac{-7}{25} \cdot \frac{-3}{5}$

$\frac{96}{125} + \frac{21}{125}$

$\boxed{\frac{117}{125}}$

Write the trigonometric expression as an algebraic expression.

7.) $\cos(\overset{u}{\arccos x} + \overset{v}{\arcsin x})$

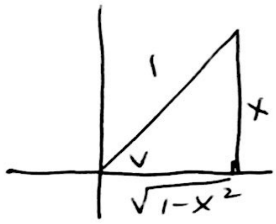


$\cos u \cos v - \sin u \sin v$

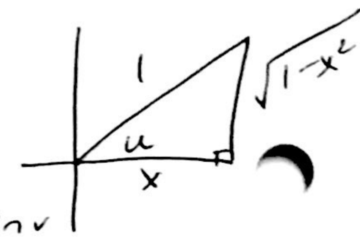
$x \cdot \sqrt{1-x^2} - \sqrt{1-x^2} \cdot x$

$x\sqrt{1-x^2} - x\sqrt{1-x^2}$

$= 0$



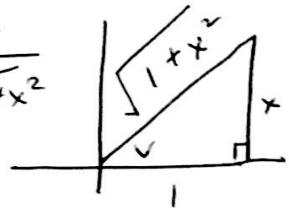
8.) $\cos(\overset{u}{\arccos x} - \overset{v}{\arctan x})$



$\cos u \cos v + \sin u \sin v$

$x \cdot \frac{1}{\sqrt{1+x^2}} + \sqrt{1-x^2} \cdot \frac{x}{\sqrt{1+x^2}}$

$= \boxed{\frac{x + x\sqrt{1-x^2}}{\sqrt{1+x^2}}}$



Simplify the expression.

9.) $\sin\left(\frac{3\pi}{2} + \theta\right)$

$= \underset{(-1)}{\sin \frac{3\pi}{2}} \cos\theta + \cancel{\underset{0}{\cos \frac{3\pi}{2}} \sin\theta}$

$= \boxed{-\cos\theta}$

$\tan \pi = 0$

10.) $\tan(\pi + \theta)$

$= \frac{\cancel{\tan \pi} + \tan\theta}{1 - \cancel{\tan\pi \tan\theta}}$

$= \boxed{\tan\theta}$

Find all solutions of the equation in the interval $[0, 2\pi)$.

11.) $\sin\left(x+\frac{\pi}{6}\right)-\sin\left(x-\frac{\pi}{6}\right)=\frac{1}{2}$

$$\sin x \cancel{\cos\frac{\pi}{6}} + \cos x \sin\frac{\pi}{6} - \left(\cancel{\sin x \cos\frac{\pi}{6}} - \cos x \sin\frac{\pi}{6}\right) = \frac{1}{2}$$

$$\cos x \sin\frac{\pi}{6} + \cos x \sin\frac{\pi}{6} = 1/2$$

$$2\cos x \sin\frac{\pi}{6} = 1/2$$

$$\cancel{2}\cos x \cdot \cancel{\tfrac{1}{2}} = 1/2$$

$$\cos x = 1/2$$

$$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$$

12.) $\tan(x+\pi)-\cos\left(x-\frac{\pi}{2}\right)=0$

$$\frac{\tan x + \cancel{\tan\pi}}{1-\cancel{\tan x \tan\pi}} - \left(\cancel{\cos x \cos\frac{\pi}{2}} + \sin x \sin\frac{\pi}{2}\right) = 0$$

$$\tan x - \sin x = 0$$

$$\cancel{\frac{\sin x}{\cos x} = \frac{\sin x}{1}}$$

$$\sin x \cos x = \sin x$$

$$\sin x \cos x - \sin x = 0$$

$$\sin x (\cos x - 1) = 0$$

$$\sin x = 0 \qquad \cos x = 1$$

$$x = 0, \pi \qquad x = 0$$

13.) $\sin 2x - \sin x = 0$

$$2\sin x \cos x - \sin x = 0$$

$$\sin x (2\cos x - 1) = 0$$

$$\sin x = 0 \qquad 2\cos x = 1$$

$$x = 0, \pi \qquad \cos x = 1/2$$

$$x = \frac{\pi}{3}, 5\pi/3$$

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Use a double-angle formula to rewrite the expression.

14.) $6\sin x\cos x$

$3(2\sin x\cos x)$

$3\sin 2x$

15.) $4-8\sin^2 x$

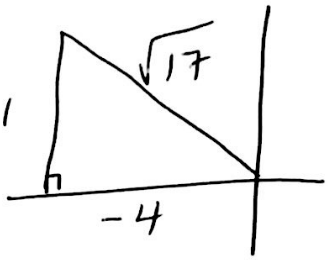
$4(1-2\sin^2 x)$

$4\cos 2x$

Using double-angle formulas, find the exact values of $\sin 2u$, $\cos 2u$, ***and*** $\tan 2u$.

16.) $\cot u=-4;\quad \frac{\pi}{2}<u<\pi$

$\frac{A}{O}=-\frac{4}{1}$



$\sin 2u = 2\left(\frac{-4}{\sqrt{17}}\right)\left(\frac{1}{\sqrt{17}}\right)=\boxed{\frac{-8}{17}}$

$\cos 2u = \left(-\frac{4}{\sqrt{17}}\right)^2-\left(\frac{1}{\sqrt{17}}\right)^2=\boxed{\frac{15}{17}}$

$\tan 2u = \frac{\frac{-8}{17}}{15/17} = -\frac{8}{17}\cdot\frac{17}{15}=\boxed{\frac{-8}{15}}$

Using half-angle formulas, find the exact values of $\sin\frac{u}{2}$, $\cos\frac{u}{2}$, ***and*** $\tan\frac{u}{2}$.

$u=135^\circ$

17.) $\theta=67.5^\circ$

$\sin\frac{135}{2}=\sqrt{\frac{1-(-\sqrt{2}/2)}{2}}=\sqrt{\frac{1+\sqrt{2}/2}{2}}=\sqrt{\frac{2+\sqrt{2}}{2}\cdot\frac{1}{2}}=\boxed{\frac{\sqrt{2+\sqrt{2}}}{2}}$

$\cos\frac{135}{2}=\sqrt{\frac{1+-\sqrt{2}/2}{2}}=\sqrt{\frac{1-\sqrt{2}/2}{2}}=\sqrt{\frac{2-\sqrt{2}}{2}\cdot\frac{1}{2}}=\boxed{\frac{\sqrt{2-\sqrt{2}}}{2}}$

$\tan\frac{135}{2}=\frac{\frac{\sqrt{2}}{2}}{1+-\sqrt{2}/2}=\frac{\frac{\sqrt{2}}{2}}{1-\frac{\sqrt{2}}{2}}=\frac{\sqrt{2}/2}{\frac{2-\sqrt{2}}{2}}=\frac{\sqrt{2}}{2}\cdot\frac{2}{2-\sqrt{2}}$

$=\frac{\sqrt{2}}{2-\sqrt{2}}\cdot\frac{2+\sqrt{2}}{2+\sqrt{2}}=\frac{2\sqrt{2}+2}{4-2}=\frac{2\sqrt{2}+2}{2}=\boxed{\sqrt{2}+1}$

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d all solutions of the equation in the interval $[0, 2\pi)$.

18.) $\sin\frac{x}{2} + \cos x = 0$

$$\left(\sqrt{\frac{1-\cos x}{2}}\right)^2 = (-\cos x)^2$$

$$\frac{1-\cos x}{2} = \frac{\cos^2 x}{1}$$

$$2\cos^2 x + \cos x - 1 = 0$$

$$(2\cos x - 1)(\cos x + 1) = 0$$

$\cos x = \frac{1}{2}$ $\qquad$ $\cos x = -1$

$\cancel{x = \frac{\pi}{3}, \frac{5\pi}{3}}$ $\qquad$ $x = \pi$

↑ extraneous

Use the product-to-sum formulas to write each product as a sum or difference.

19.) $\cos 4\theta \sin 6\theta$

$$= \frac{1}{2}\left[\sin(10\theta) - \sin(-2\theta)\right]$$

$$= \frac{1}{2}\left[\sin 10\theta + \sin 2\theta\right]$$

$= \frac{1}{2} \sin 10\theta$

Use the sum-to-product formulas to write each sum or difference as a product.

20.) $\sin(x+y) - \sin(x-y)$

$$= 2\cos\left(\frac{x+y+x-y}{2}\right)\sin\left(\frac{\cancel{x}+y-(\cancel{x}-y)}{2}\right)$$

$$= 2\cos\left(\frac{2x}{2}\right)\sin\left(\frac{2y}{2}\right)$$

$$= 2\cos x \sin y$$

Find all solutions of the equation in the interval $[0, 2\pi)$.

21.) $\cos 4x + \cos 6x = 0$

$2\cos 5x \cos(-x) = 0$

$2\cos 5x \cos x = 0$

$\cos 5x = 0$

$\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\frac{5x}{5} = \frac{\frac{\pi}{2} + 2\pi n}{5}$

$x = \frac{\pi}{10} + \frac{2\pi}{5}n$

$x = \frac{\pi}{10}, \frac{5\pi}{10}, \frac{9\pi}{10}, \frac{13\pi}{10}, \frac{17\pi}{10}$

$\frac{3\pi}{10}, \frac{7\pi}{10}, \frac{11\pi}{10}, \frac{15\pi}{10}, \frac{19\pi}{10}$

$\frac{5x}{5} = \frac{\frac{3\pi}{2} + 2\pi n}{5}$

$x = \frac{3\pi}{10} + \frac{2\pi}{5}n$

22.) $\sin x + \sin 3x = 0$

$= 2\sin(2x)\cos x = 0$

$\sin 2x = 0$

$\cos x = 0$

$x = \pi/2, \frac{3\pi}{2}$

$\frac{2x}{2} = \frac{0 + 2\pi n}{2}$

$x = 0 + \pi n$

$\frac{2x}{2} = \frac{\pi + 2\pi n}{2}$

$x = 0, \pi, \frac{\pi}{2}, \frac{3\pi}{2}$

$x = \frac{\pi}{2} + \pi n$

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