

## 5.5 Multiple-Angle and Product to Sum Formulas

### Double-Angle Formulas

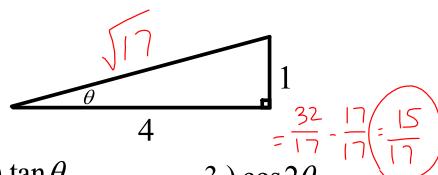
$$\sin 2u = 2 \sin u \cos u$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

$$\begin{aligned}\cos 2u &= \cos^2 u - \sin^2 u \\ &= 2 \cos^2 u - 1 \\ &= 1 - 2 \sin^2 u\end{aligned}$$

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**Use the figure to find the exact value of the trigonometric function.**



1.)  $\sin \theta$

$$= \frac{1}{\sqrt{17}} = \frac{\sqrt{17}}{17}$$

2.)  $\tan \theta$

$$= \frac{1}{4}$$

3.)  $\cos 2\theta$

$$\begin{aligned}&= 2 \cos^2 u - 1 \\ &= 2 \left(\frac{4}{\sqrt{17}}\right)^2 - 1 \\ &= 2 \left(\frac{16}{17}\right) - 1\end{aligned}$$

4.)  $\sin 2\theta$

$$\begin{aligned}&= 2 \sin u \cos u \\ &= 2 \left(\frac{1}{\sqrt{17}}\right) \cdot \left(\frac{4}{\sqrt{17}}\right) \\ &= \frac{8}{17}\end{aligned}$$

5.)  $\tan 2\theta$

$$= \frac{2 \tan u}{1 - \tan^2 u}$$

$$= \frac{2 \left(\frac{1}{4}\right)}{1 - \left(\frac{1}{4}\right)^2} = \frac{1}{2} \cdot \frac{16}{15}$$

$$= \frac{8}{15}$$

6.)  $\sec 2\theta$

$$\left(\frac{17}{15}\right)$$

7.)  $\csc 2\theta$

$$\left(\frac{17}{8}\right)$$

8.)  $\cot 2\theta$

$$\left(\frac{15}{8}\right)$$

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**Find the exact values of  $\sin 2u$ ,  $\cos 2u$ , and  $\tan 2u$  using the double-angle formulas.**

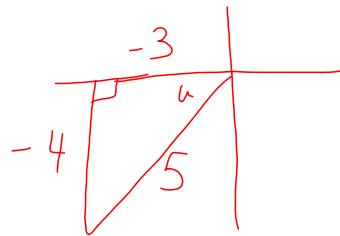
$$9.) \sin u = -\frac{4}{5}; \pi < u < \frac{3\pi}{2}$$

$$\sin 2u = 2 \sin u \cos u$$

$$= 2 \left(-\frac{4}{5}\right) \left(-\frac{3}{5}\right) = \frac{24}{25}$$

$$\cos 2u = \cos^2 u - \sin^2 u$$

$$\frac{9}{25} - \frac{16}{25} = -\frac{7}{25}$$



$$\begin{aligned} \tan 2u &= \frac{2 \left(\frac{4}{3}\right)}{1 - \frac{16}{9}} = \frac{\frac{8}{3}}{-\frac{7}{9}} \\ &= \frac{8}{3} \cdot \frac{9}{-7} = \frac{24}{-7} \end{aligned}$$

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**Use a double-angle formula to rewrite each expression.**

$$10.) 8 \sin x \cos x$$

$$4(2 \sin x \cos x)$$

$$\boxed{4 \sin 2x}$$

$$11.) 6 - 12 \sin^2 x$$

$$6(1 - 2 \sin^2 x)$$

$$\boxed{6 \cos 2x}$$

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**Find the solutions of the equation in the interval  $[0, 2\pi]$ .**

12.)  $2\cos x + \boxed{\sin 2x} = 0$

$$2\cos x + 2\sin x \cos x = 0$$

$$2\cos x (1 + \sin x) = 0$$

$$2\cos x = 0$$

$$\sin x = -1$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$x = \frac{3\pi}{2}$$

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**Find the solutions of the equation in the interval  $[0, 2\pi]$ .**

13.)  $4\sin x \cos x = 1$

$$2(2\sin x \cos x) = 1$$

$$x = \frac{\pi}{12}, \frac{13\pi}{12}, \frac{5\pi}{12}, \frac{17\pi}{12}$$

$$2(\sin 2x) = 1$$

$$\sin 2x = \frac{1}{2}$$

$$\frac{2x}{2} = \frac{\pi}{6} + 2\pi n$$

$$x = \frac{\pi}{12} + \pi n$$

$$\frac{2x}{2} = \frac{5\pi}{6} + 2\pi n$$

$$x = \frac{5\pi}{12} + \pi n$$

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