5.5 Multiple-Angle and Product to Sum Formulas

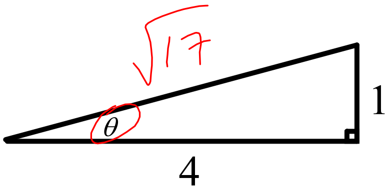
Double-Angle Formulas

$$\sin 2u = 2 \sin u \cos u$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

$$\begin{aligned} \cos 2u &= \cos^2 u - \sin^2 u \\ &= 2\cos^2 u - 1 \\ &= 1 - 2\sin^2 u \end{aligned}$$

Use the figure to find the exact value of the trigonometric function.



1.) $\sin\theta$

$$\frac{1}{\sqrt{17}}\cdot\frac{\sqrt{17}}{\sqrt{17}}$$

$$\frac{\sqrt{17}}{17}$$

2.) $\tan\theta$

$$\frac{1}{4}$$

3.) $\cos 2\theta$

$$= 1-2\sin^2 u$$

$$= 1-2\left(\frac{1}{\sqrt{17}}\right)^2$$

$$= 1-\frac{2}{17} = \frac{15}{17}$$

4.) $\sin 2\theta$

$$= 2\sin u\cos u$$

$$= 2\left(\frac{1}{\sqrt{17}}\right)\left(\frac{4}{\sqrt{17}}\right)$$

$$= \frac{8}{17}$$

5.) $\tan 2\theta$

$$\frac{\frac{8}{\cancel{17}}}{\frac{15}{\cancel{17}}} = \frac{8}{15}$$

$$= \frac{2\tan u}{1-\tan^2 u}$$

$$= \frac{2\left(\frac{1}{4}\right)}{1-\frac{1}{16}} = \frac{\frac{1}{\cancel{2}}}{\frac{15}{16}} \cdot \frac{\cancel{16}^{8}}{15}$$

$$= 8/15$$

6.) $\sec 2\theta$

$$\frac{17}{15}$$

7.) $\csc 2\theta$

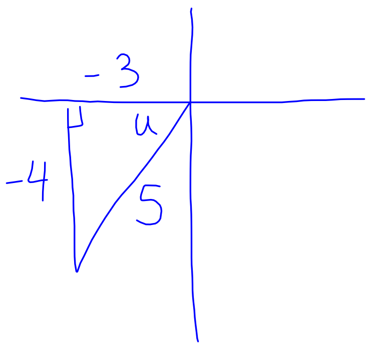
$$\frac{17}{8}$$

8.) $\cot 2\theta$

$$\frac{15}{8}$$

Find the exact values of $\sin 2u, \cos 2u,$ *and* $\tan 2u$ ***using the double-angle formulas.***

9.) $\sin u = -\frac{4}{5}; \pi < u < \frac{3\pi}{2}$



$\sin 2u = 2\sin u \cos u$

$= 2\left(-\frac{4}{5}\right)\left(-\frac{3}{5}\right) = \frac{24}{25}$

$\cos 2u = \cos^2 u - \sin^2 u$

$= \left(-\frac{3}{5}\right)^2 - \left(-\frac{4}{5}\right)^2 = \frac{9}{25} - \frac{16}{25} = \frac{-7}{25}$

$\tan 2u = \frac{\frac{24}{25}}{\frac{-7}{25}} = \frac{24}{-7}$

Use a double-angle formula to rewrite each expression.

10.) $8\sin x\cos x$

$4\cdot(2\sin x\cos x)$

$4\sin 2x$

11.) $6-12\sin^2 x$

$6(1-2\sin^2 x)$

$6\cos 2x$

Find the solutions of the equation in the interval $[0, 2\pi)$.

12.) $2\cos x + \boxed{\sin 2x} = 0$

$2\cos x + 2\sin x \cos x = 0$

$2\cos x(1 + \sin x) = 0$

$2\cos x = 0$

$\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$1 + \sin x = 0$

$\sin x = -1$

$x = \frac{3\pi}{2}$

Find the solutions of the equation in the interval $[0, 2\pi)$.

13.) $4\sin x \cos x = 1$

$2 \cdot \underline{2\sin x \cos x} = 1$

$2\sin 2x = 1$

$\sin 2x = \frac{1}{2}$

$\frac{2x}{2} = \frac{\frac{\pi}{6} + 2\pi n}{2}$

$\boxed{x = \frac{\pi}{12} + \pi n}$

$\frac{2x}{2} = \frac{\frac{5\pi}{6} + 2\pi n}{2}$

$\boxed{x = \frac{5\pi}{12} + \pi n}$

$\boxed{[0, 2\pi): \frac{\pi}{12}, \frac{13\pi}{12}, \frac{5\pi}{12}, \frac{17\pi}{12}}$