

Honors Pre-Calc  
Extra Practice 6.1/6.2

Name \_\_\_\_\_  
Date \_\_\_\_\_ Block \_\_\_\_\_

1. Given  $\mathbf{u} = -5\mathbf{i} + 3\mathbf{j}$  and  $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j}$ . Find the resultant vector and write in component and linear combination form.

a.)  $\mathbf{u} + \mathbf{v}$   
 $\langle -5, 3 \rangle + \langle 2, -3 \rangle$   
 $= \langle -3, 0 \rangle$   
 $= -3\mathbf{i}$

b.)  $\mathbf{u} - \mathbf{v}$   
 $\langle -5, 3 \rangle - \langle 2, -3 \rangle$   
 $= \langle -7, 6 \rangle$   
 $= -7\mathbf{i} + 6\mathbf{j}$

c.)  $2\mathbf{u} - 3\mathbf{v}$   
 $2\langle -5, 3 \rangle - 3\langle 2, -3 \rangle$   
 $= \langle -10, 6 \rangle - \langle 6, -9 \rangle$   
 $= \langle -16, 15 \rangle$   
 $= -16\mathbf{i} + 15\mathbf{j}$

2. Find the unit vector and the direction angle,  $\theta$ , of the vector given.

a.)  $\mathbf{v} = 6\mathbf{i} - 6\mathbf{j}$  ~~+~~  
 $\left\langle \frac{6}{\sqrt{72}}, \frac{-6}{\sqrt{72}} \right\rangle \theta = 315^\circ$   
 $= \left\langle \frac{6}{6\sqrt{2}}, \frac{-6}{6\sqrt{2}} \right\rangle$   
 $= \left\langle \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right\rangle$

b.)  $\mathbf{v} = \langle -5, 4 \rangle$   
~~+~~  
 $\left\langle \frac{-5}{\sqrt{41}}, \frac{4}{\sqrt{41}} \right\rangle$   
 $\theta = 141.34^\circ$

c.)  $\overrightarrow{PQ}$ , P:(-3, -5) Q:(5, 1)  
 $\langle 5, 1 \rangle - \langle -3, -5 \rangle$   
 $= \langle 8, 6 \rangle$   
 $\left\langle \frac{8}{10}, \frac{6}{10} \right\rangle = \left\langle \frac{4}{5}, \frac{3}{5} \right\rangle$   
 $\theta = 36.87^\circ$

3. Given two vectors, find the angle  $\theta$  (degrees) between them and then find the projection of  $p$  onto  $q$ . Determine if the vectors are orthogonal, parallel or neither.

a.)  $\mathbf{p} = \langle 3, 2 \rangle, \mathbf{q} = \langle 4, 0 \rangle$   
 $\cos \theta = \frac{12}{\sqrt{13} \cdot 4} = 33.69^\circ$   
 $|\mathbf{p}| \cos \theta (\mathbf{u} \cdot \mathbf{q})$   
 $= \frac{12}{4} \cdot \frac{\langle 4, 0 \rangle}{4}$   
 $= \langle 3, 0 \rangle$   
 Neither

b.)  $\mathbf{p} = 2\mathbf{i} - \mathbf{j}, \mathbf{q} = 6\mathbf{i} + 4\mathbf{j}$   
 $\cos \theta = \frac{8}{\sqrt{5} \sqrt{52}} = 60.26^\circ$   
 $|\mathbf{p}| \cos \theta (\mathbf{u} \cdot \mathbf{q})$   
 $= \frac{8}{\sqrt{52}} \cdot \frac{\langle 6, 4 \rangle}{\sqrt{52}}$   
 $= \langle \frac{12}{13}, \frac{8}{13} \rangle$  Neither

c.)  $\mathbf{p} = \langle 2, -3 \rangle, \mathbf{q} = \langle 6, 4 \rangle$   
 $\cos \theta = 0$   
 $= 90^\circ$   
 Orthogonal  
 $\text{proj}_{\mathbf{q}} \mathbf{p} = \langle 0, 0 \rangle$

4. Find the work done in moving a particle from  $P$  to  $Q$  if the magnitude & direction of the force are given by  $\mathbf{v}$ . (hint: remember that  $\theta$  is the angle between the force and direction vectors).

a.) P:(0,0), Q:(4,7),  $\mathbf{v} = \langle 1, 4 \rangle$

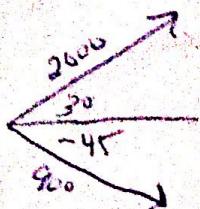
$W = Fd \cos \theta$   
 $F = \sqrt{17}$   
 $\theta = 15.7^\circ$   
 $d = \sqrt{65}$   
 $W = \sqrt{65} \cdot \sqrt{17} \cos 15.7^\circ = \boxed{32}$

b.) P:(1,3), Q:(-3,5),  $\mathbf{v} = -2\mathbf{i} + 3\mathbf{j}$   
 $\langle -4, 2 \rangle \quad \langle -2, 3 \rangle$

$F = \sqrt{13}$   
 $d = \sqrt{20}$   
 $\theta = 29.74^\circ$   
 $W = Fd \cos \theta = \sqrt{13} \cdot \sqrt{20} \cos 29.74^\circ = \boxed{14}$

## Application

5. Forces with magnitudes of 2000 newtons and 900 newtons act on a machine part at angles of  $30^\circ$  and  $-45^\circ$  respectively, with the horizontal. Find the direction and magnitude of the resultant of these forces.



$$\langle 2000 \cos 30, 2000 \sin 30 + 900 \cos -45 \rangle$$

$$\langle 2368.44, 363.60 \rangle$$

$$2396.19 \text{ N at } 8.73^\circ$$

6. An airplane is flying bearing  $148^\circ$ , with an air speed of 875 kph. Because of the wind, its ground speed and direction are 800 kpm and bearing  $140^\circ$ , respectively. Find the direction and speed of the wind.

Plane : 875 kph at  $-58^\circ$

ground : 800 kpm at  $-50^\circ$

Plane + Wind = Ground

$8.63^\circ =$   
21.37 bearing

$$w \cos \theta = 50.55$$

$$w \sin \theta = 129.21$$

$$\langle 875 \cos -58, 875 \sin -58 \rangle$$

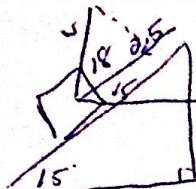
$$+ \langle w \cos \theta, w \sin \theta \rangle$$

$$\langle 800 \cos -50, 800 \sin -50 \rangle$$

$$w = 138.74 \text{ kph}$$

at bearing 21.37

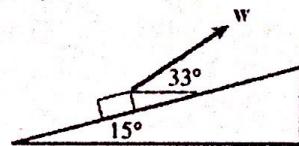
7. Suppose the box described below is being towed up the inclined plane. Find the force needed for the component of the force parallel to the inclined plane to be 2.5 lbs. Give the answer in component form.



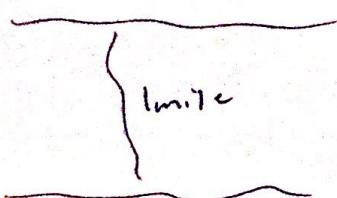
$$\cos 18 = \frac{2.5}{w}$$

$$(2.63 \cos 33, 2.63 \sin 33)$$

$$\langle 2.20, 1.43 \rangle$$



8. A motor boat capable of 20 mph keeps the bow of the boat pointed straight across a mile wide river. The current is flowing left to right at 8 mph. Find where the boat meets the opposite shore.



$$B \langle 0, 20 \rangle$$

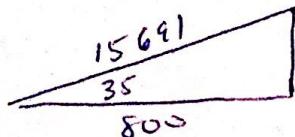
$$\langle 0, 20t \rangle = \langle 0, 17 \rangle$$

$$C \langle 8, 0 \rangle$$

$$t = \frac{1}{20}$$

$$8\left(\frac{1}{20}\right) = 0.4 \text{ miles}$$

9. A tractor pulls a log 800 meters, and the tension in the cable connecting the tractor and the log is approximately 1600 kg (15,691 newtons). The direction of the force is  $35^\circ$  above the horizontal. Approximate the work done in pulling the log.



$$W = F d \cos \theta$$

$$= 15691 \cdot 800 \cos 35$$

$$= 10282651.78 \text{ N-m}$$

10. The angle between a 75 lb force and  $\overrightarrow{AB}$  is  $60^\circ$ , where A: (-1,1) and B: (4,3). Find the work done by F in moving an object from A to B.

$$\theta = 60^\circ$$

$$F = 75$$

$$d = \sqrt{29}$$

$$W = F d \cos \theta$$

$$= 75 \cdot \sqrt{29} \cos 60^\circ$$

$$= 201.94$$