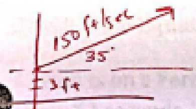
Parametric Equations and Projectile Motion

$x(t) = (V_0 \cos\theta)t + X_0$	v_0 = *initial velocity*
$y(t) = -16t^2 + (V_0 \sin\theta)t + Y_0$	y_0 = *initial height*

a = -32 ft/sec² or -9.8 m/sec² depending on units given for initial velocity.

1.) A baseball was hit with an initial velocity of 150 ft/sec at an angle of 35 degrees with the horizontal. The ball was hit a height of 3 feet off the ground.

a.) Find the parametric equations that describe the position of the ball as a function of time.



$x(t) = (150\cos 35)t$

$y(t) = -16t^2 + (150\sin 35)t + 3$

b.) Describe the ball's position after 1 second, 2 seconds, and 3 seconds.

1: (122.87, 73.036) 2: (245.75, 111.07) 3 (368.62, 117.11)

c.) How long is the ball in flight? What is the horizontal distance it travels before landing?

$$t = \frac{-150\sin 35 \pm \sqrt{(150\sin 35)^2 - 4(-16)(3)}}{-32}$$

$t = 5.4119$ seconds

$x = (150\cos 35)(5.4119)$

$x = 664.975$ ft

d.) How high does the ball go?

$t = \frac{-b}{2a} = \frac{-150\sin 35}{-32} = 2.6886$ sec

$y = -16(\text{Ans})^2 + 150\sin 35(\text{Ans}) + 3 = 118.66$ ft

e.) Will the ball clear the stadium if the back wall is 40 feet high and 600 feet from home plate?

$\frac{600}{150\cos 35} = \frac{150\cos 35 t}{150\cos 35}$

$t = 4.8831$

$y = -16(\text{Ans})^2 + 150\sin(\text{Ans}) + 3$

$y = 41.61$ ft.

YES!

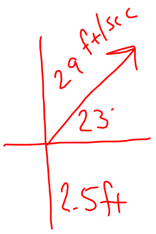
d.) How high does the ball go?

e.) Will the ball clear the stadium if the back wall is 40 feet high and 600 feet from home plate?

$$x(t) = (v_0 \cos\theta)t + x_0$$
$$y(t) = -16t^2 + (v_0 \sin\theta)t + y_0$$

2.) At a medieval festival, some garbage is catapulted with an initial velocity of 29 ft/sec at an angle of 23 degrees. The garbage is 30 inches from the ground when it is released from the catapult.

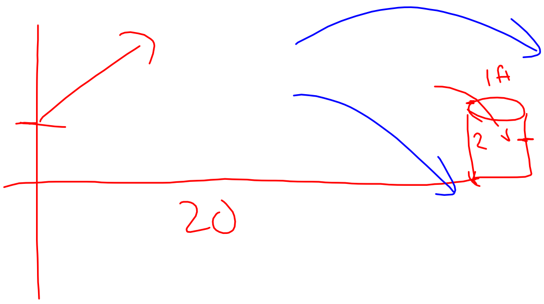
a.) Find the parametric equations that describe the position of the garbage as a function of time.



$x(t) = 29\cos(23)t$

$y(t) = -16t^2 + 29\sin(23)t + 2.5$

b.) Does the garbage land in a trash can that is 20 feet away and 2 feet high and a foot in diameter wide? Prove your answer algebraically.



$20 = 29\cos(23)t$

$t = .7492$ sec.

$y(\text{ANS}) = 2.00835$ ft.

$21 = 29\cos(23)t$

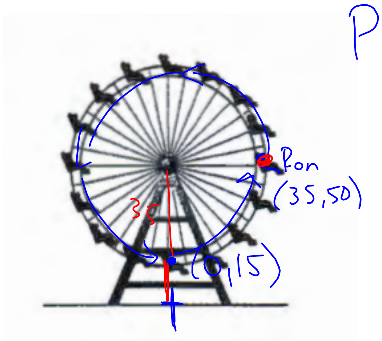
$t = .7867$ sec

$y(\text{ANS}) = 1.512$ ft.

Yes, just barely.

Parametric Equations and Circular Motion

3.) Ron is on a Ferris wheel of radius 35 feet that turns counterclockwise at the rate of one revolution every 12 seconds. The lowest point on the Ferris wheel is 15 feet above ground level at the point (0, 15) on a rectangular coordinate system. Find the parametric equation for the position of Ron as a function of time, t (in seconds), if the Ferris wheel starts with Ron at the point (35, 50).



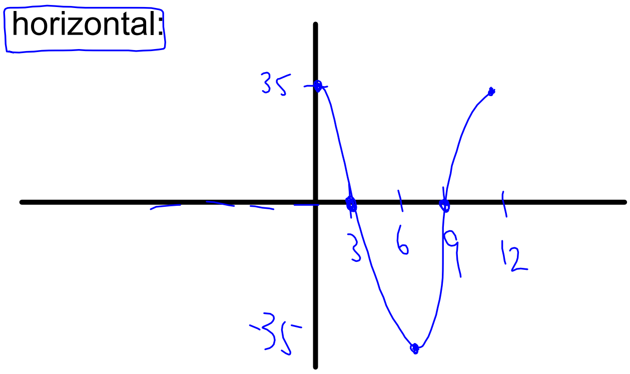
P=12seconds

$12 = \frac{2\pi}{b}$

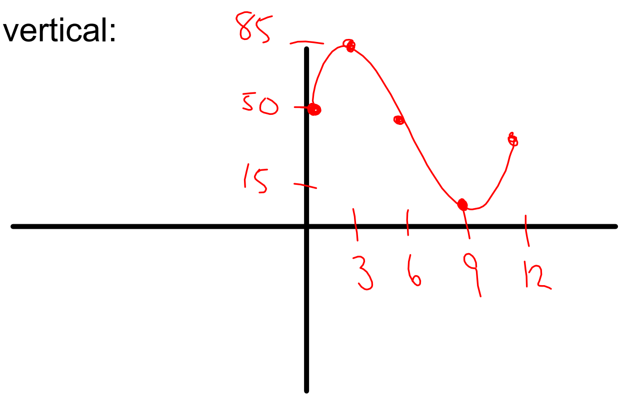
$x(t) = 35\cos\left(\frac{\pi}{6}t\right)$

$y(t) = 35\sin\left(\frac{\pi}{6}t\right) + 50$

$\boxed{\text{horizontal:}}$



vertical:



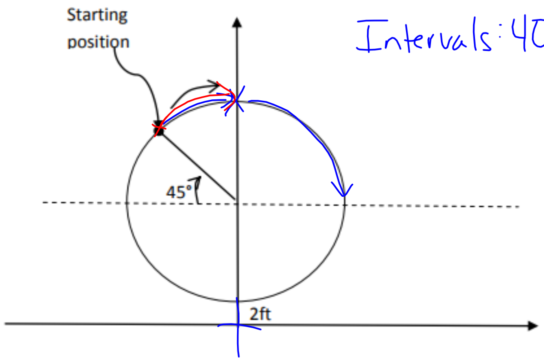
$$A\cos(B(x-C))+D$$

Parametric Equations and Circular Motion

4.) Find a set of parametric equations for the motion around a Ferris wheel with a radius of 50 feet, as pictured, if it takes 20 seconds to reach the top for the first time.

P= 160sec

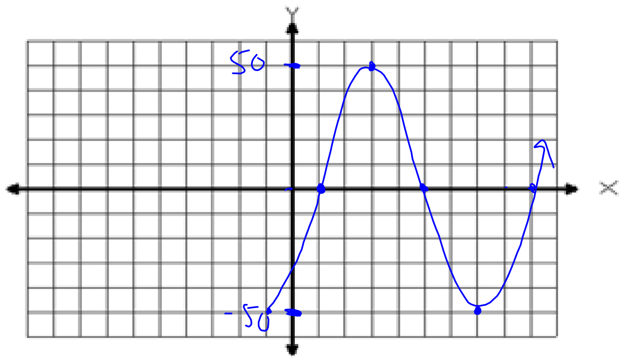
Intervals: 40sec



$$x(t) = 50\cos\left(\frac{\pi}{80}(t-60)\right)$$

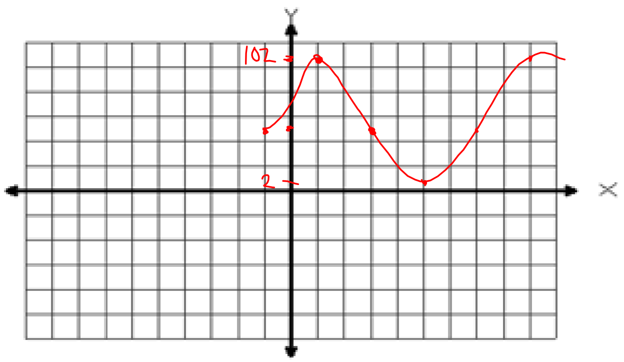
$$y(t) = 50\sin\left(\frac{\pi}{80}(t+20)\right)+52$$

Horizontal:



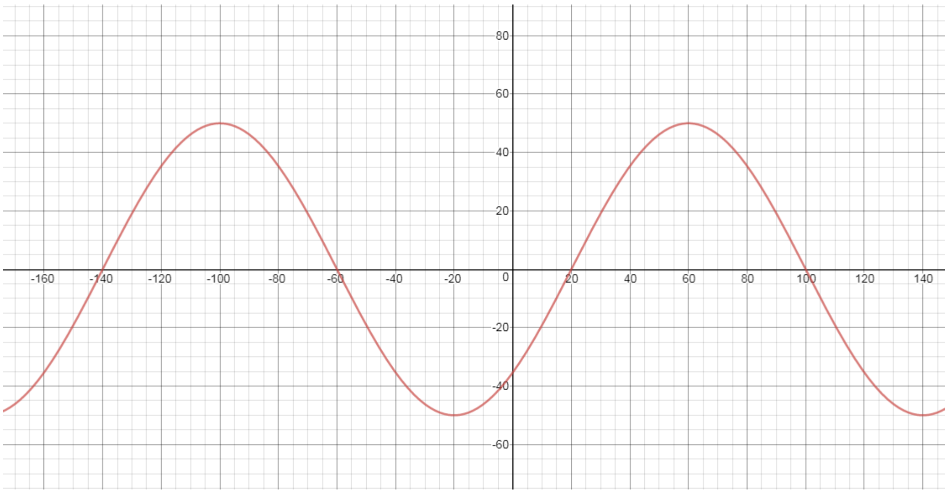
Intervals: 20sec

Vertical:

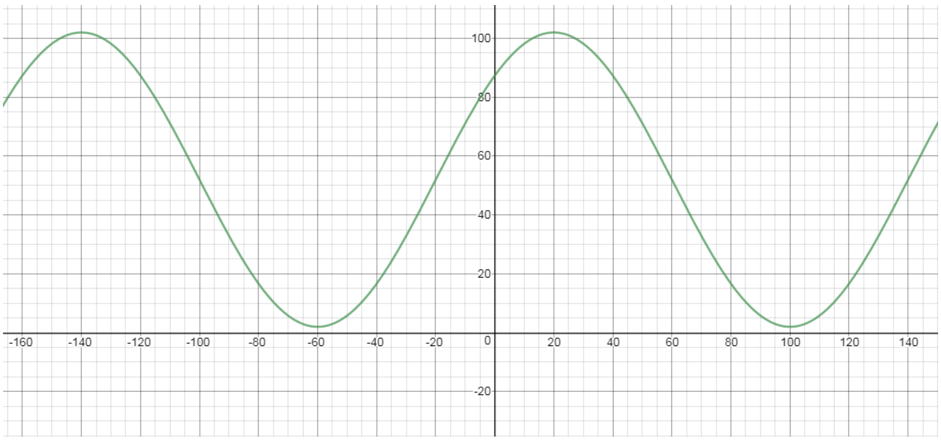


Intervals: 20

Horizontal:



Vertical:



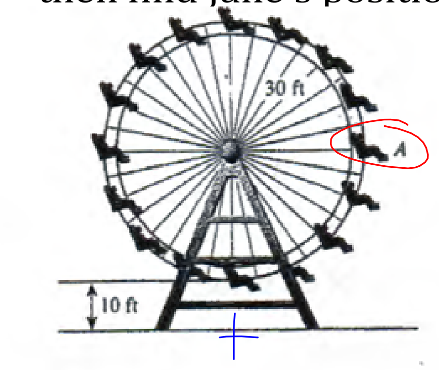
Feb 12-8:57 AM

Parametric Equations and Circular Motion

$$x(t) = R\cos(\theta_0 + \omega t) + \boxed{x_{center}} \qquad +\omega : counterclockwise$$
$$y(t) = R\sin(\theta_0 + \omega t) + \boxed{y_{center}} \qquad -\omega : clockwise$$

↳ center of ferris wheel

5.) Jane is riding on a Ferris wheel with a radius of 30 feet. The wheel is turning counterclockwise at the rate of one revolution every 10 seconds. Assume the lowest point on the Ferris wheel is 10 feet above the ground. At t = 0, Jane's seat is on an imaginary line that is parallel to the ground. Find parametric equations to model Jane's path and then find Jane's position after 22 seconds.



a.) Find parametric equations to model Jane's path and then find Jane's position after 22 seconds. * (9.27, 68.53)

$x(t) = 30\cos(\pi/5\, t)$

$y(t) = 30\sin(\pi/5\, t) + 40$

b.) If Jane's friend is on another ride at position (20, 15), how far is she from Jane? (22 seconds)

(9.27, 68.53)

(20, 15)

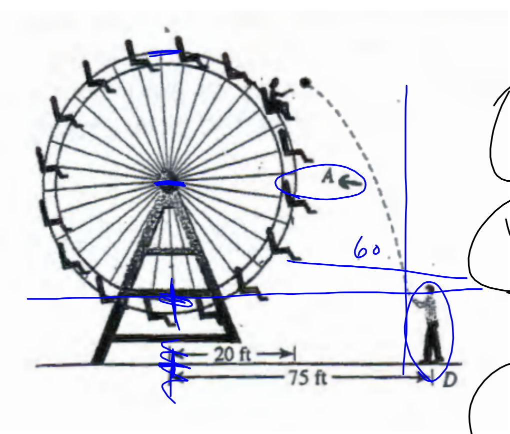
$d = \sqrt{(20-9.27)^2 + (15-68.53)^2}$

$\boxed{d = 54.59\text{ft}}$

Parametric Equations and Circular Motion

$$x(t) = R\cos(\theta_0 + \omega t) + x_{center} \qquad +\omega : counterclockwise$$
$$y(t) = R\sin(\theta_0 + \omega t) + y_{center} \qquad -\omega : clockwise$$

6.) A Ferris wheel with a 20 ft radius turns counterclockwise one revolution every 12 seconds. Eric stands at point D, 75 feet from the base of the wheel. At the instant Jane is at a point parallel to the ground (see picture), Eric throws a ball at the Ferris wheel, releasing it from the same height as the bottom of the wheel. If the ball's initial speed is 60 ft/sec and it is released at an angle of 120 degrees with the horizontal, does Jane have a chance to catch the ball?



a.) Find a set of parametric equations for the ball in terms of time.

Y_1 $x(t) = 75 - 60\cos(\frac{\pi}{3})t$

Y_2 $y(t) = -16t^2 + 60\sin(\pi/3)t$

b.) Find a set of parametric equations for Jane's position in terms of time.

Y_3 $x(t) = 20\cos(\pi/6\, t)$

Y_4 $y(t) = 20\sin(\pi/6\, t) + 20$

c.) Find a formula for the distance *d(t)* between Jane and the ball at any time *t*.

Y_5 $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

d.) Use $x_3 = t$ and $y_3 = d(t)$ to estimate the minimum distance between Jane and the ball and when it occurs. When will the ball and Jane be the closest? Can she catch it?

Feb 5-3:09 PM

Parametric Equations and Circular Motion

$$x(t) = R\cos\left(\theta_0 + \omega t\right) + x_{center} \qquad +\omega : counterclockwise$$
$$y(t) = R\sin\left(\theta_0 + \omega t\right) + y_{center} \qquad -\omega : clockwise$$

7.) Mrs. Lenhard is on a Ferris wheel with a center at (0, 25) and a radius of 20 ft turning counterclockwise at the rate of one revolution every 12 seconds. Ms. Carrigg is on a Ferris wheel with a center at (60, 20) and a radius of 15 ft turning counterclockwise at the rate of one revolution every 8 seconds.

a.) Find a set of parametric equations for Mrs. Lenhard's position in terms of time.

b.) Find a set of parametric equations for Ms. Carrigg's position in terms of time.

c.) Find the minimum distance between Mrs. Lenhard and Ms. Carrigg if both start out at a position parallel to the ground (3 o'clock).

Feb 7-1:41 PM