6.3 Parametric Equations and Motion

parametric equation – *x* and *y* are functions of a third variable *t* (called the parameter)

Rectangular/ Cartesian Equation ⟶ $y = f(x)$

Parametric Equation ⟶ $y = f(t)$, $x = g(t)$

Parametric Interval ⟶ $a \le t \le b$

Initial Point ⟶ $(g(a), f(a))$

Terminal Point ⟶ $(g(b), f(b))$

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Graph the parametric equations. Check your answer in the graphing calculator.

$x = t + 1 \quad y = t^2 + 2t; \quad -3 \le t \le 2$

$x - 1 = t$

$y = (x-1)^2 + 2(x-1)$

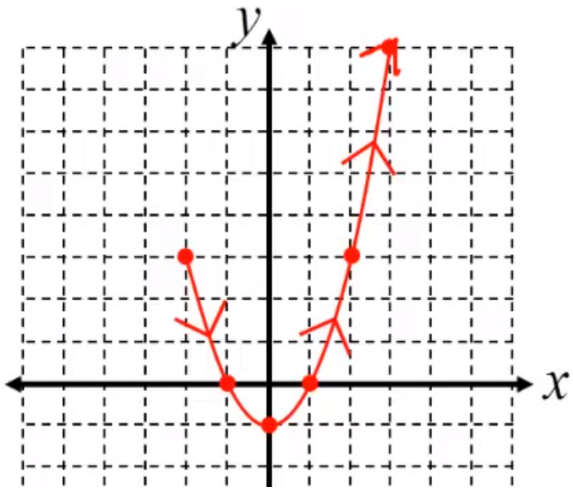
$y = x^2 - 2x + 1 + 2x - 2$

$y = x^2 - 1$

Domain: $[-2, 3]$

t	x	y
-3		
-2		
-1		
0		
1		
2		

t	x	y
−3	−2	3
−2	−1	0
−1	0	−1
0	1	0
1	2	3
2	3	8



Eliminate the parameter. Identify the domain.

1.) $x = 1 - 2t \quad y = 2 - t; \quad -\infty \le t \le \infty$

$2t = 1 - x$

$t = \frac{1}{2} - \frac{1}{2}x$

$y = 2 - \left(\frac{1}{2} - \frac{1}{2}x\right)$

$y = \frac{1}{2}x + 3/2$

$D: (-\infty, \infty)$

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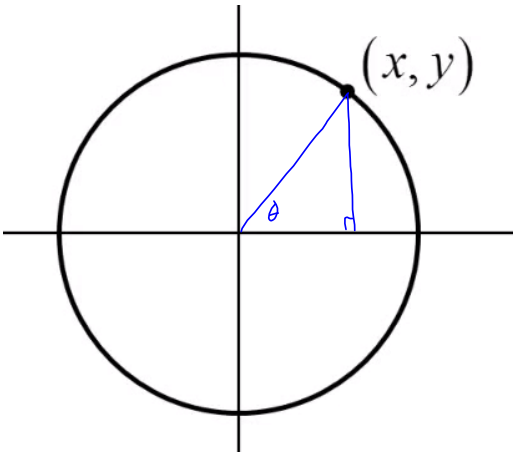
The Unit Circle



Parametric Form:

$x = \cos\theta$

$y = \sin\theta$

Cartesian Form: (Rectangular Form):

$x^2 + y^2 = r^2$

$\cos^2\theta + \sin^2\theta = 1$

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Eliminate the parameter and graph.

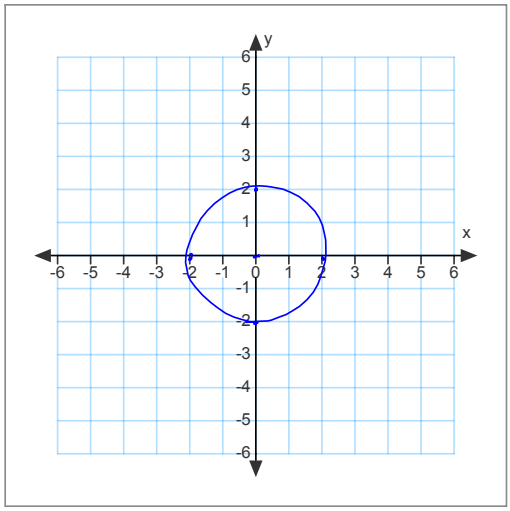
3.) $x = 2\cos t \quad y = 2\sin t; \quad 0 \le t \le 2\pi$

$\left(\frac{x}{2}\right) = \cos t \qquad \left(\frac{y}{2}\right) = \sin t$

$D: [-2, 2]$

$\frac{x^2}{4} + \frac{y^2}{4} = 1$

$x^2 + y^2 = 4$

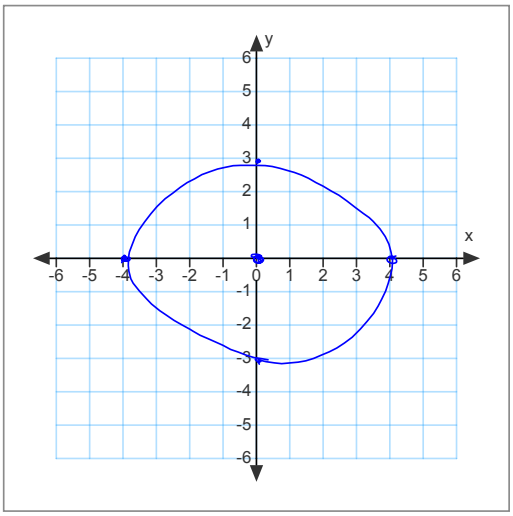


4.) $x = 4\cos t \quad y = 3\sin t; \quad 0 \le t \le 2\pi$

$\frac{x^2}{16} + \frac{y^2}{9} = 1$

Ellipse

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

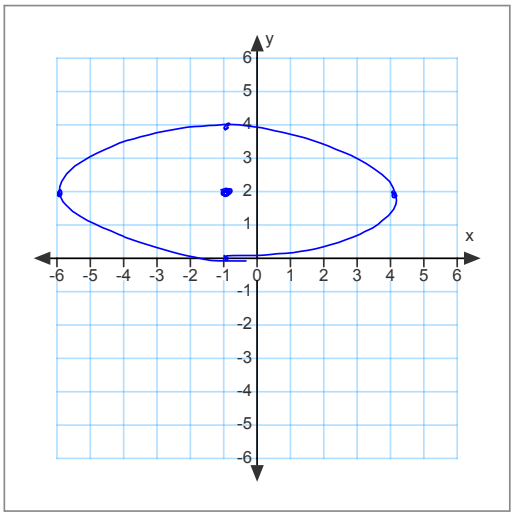


Eliminate the parameter and graph.

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

5.) $x=-1+5\cos t \quad y=2+2\sin t; \quad 0\le t\le 2\pi$

$$\frac{(x+1)^2}{25}+\frac{(y-2)^2}{4}=1$$



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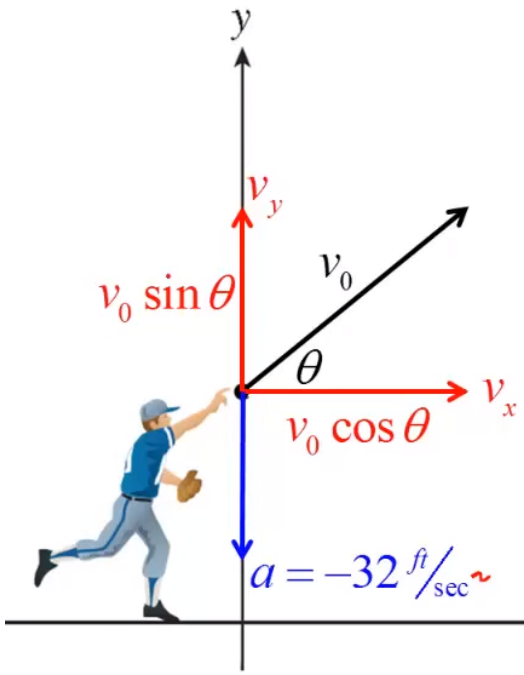
Parametric Equations and Projectile Motion



$$x = \frac{1}{2}a_x t^2 + v_x t + x_0$$

$$x(t) = (v_0 \cos\theta)t + x_0$$

$$y = \frac{1}{2}a_y t^2 + v_y t + y_0$$

$$y(t) = -16t^2 + (v_0 \sin\theta)t + y_0$$

Feb 16-12:32 PM

Jebediah hits a ball at 4 feet above the ground with an initial speed of 135 ft/sec at an angle of 25 degrees with the horizontal.

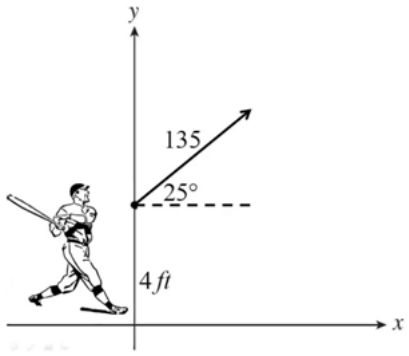
a.) Find the parametric equations that describe the position of the ball as a function of time.

b.) What is the height of the ball after 3 seconds?

c.) How far as the ball traveled after 3 seconds?

d.) What is the maximum height of the ball?

e.) How long is the ball in the air?



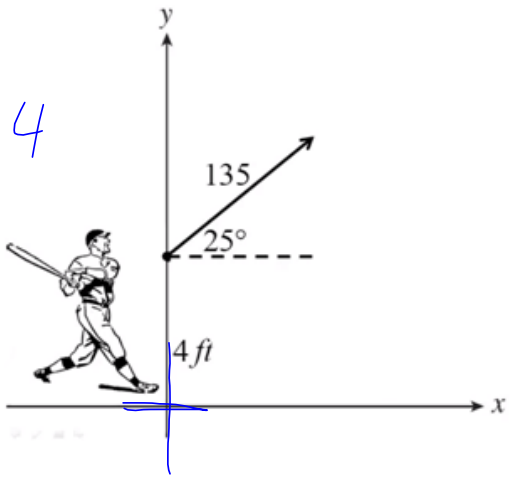
f.) A 20-foot wall stands 400 feet from the batter. Will the ball make it over the wall?

Feb 10-11:16 AM

a.) Find the parametric equations that describe the position of the ball as a function of time.

$x = (135\cos 25)t$

$y = -16t^2 + (135\sin 25)t + 4$



b.) What is the height of the ball after 3 seconds?

$y(3) = -16(3)^2 + (135\sin 25)(3) + 4$

$y(3) = 31.2 \text{ ft}$

c.) How far has the ball traveled after 3 seconds?

$x(3) = 135\cos 25(3)$

$x(3) = 367.1 \text{ ft}$

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d.) What is the maximum height of the ball?

$t = \frac{-135\sin 25}{-32}$ $y(\text{ANS}) = 54.86\text{ ft}$

$t = 1.7829\text{ sec}$

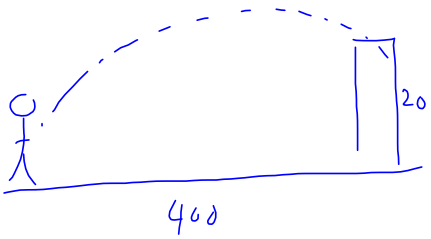
$t = (\text{ANS})$

e.) How long is the ball in the air?

$t = \frac{-135\sin 25 \pm \sqrt{(135\sin 25)^2 - 4(-16)(4)}}{-32}$

$t = 3.6346\text{ seconds}$

f.) A 20-foot wall stands 400 feet from the batter. Will the ball make it over the wall?



$400 = (135\cos 25)t$

$t = 3.2693\ (\text{ANS})$

$y(\text{ANS}) = 19.51\text{ ft}$

NO!!

Feb 6-9:45 AM