6.4 CW

2/24/16

① $r = 5$

$x^2 + y^2 = 25$

② $r = \frac{-4}{\cos\theta}$

$r\cos\theta = -4$

$x = -4$

③ $r = \frac{12}{-2\cos\theta + 5\sin\theta}$

$-2r\cos\theta + 5r\sin\theta = 12$

$-2x + 5y = 12$

$\frac{5y}{5} = \frac{2x + 12}{5}$

$y = \frac{2}{5}x + \frac{12}{5}$

④ $r^2\sin 2\theta = 2$

$r^2 \cdot 2\sin\theta\cos\theta = 2$

$2r\sin\theta r\cos\theta = 2$

$2yx = 2$

$y = \frac{2}{2x} = \frac{1}{x}$

⑤ $r = \frac{4}{-3 + 3\sin\theta}$

$-3r + 3r\sin\theta = 4$

$-3r + 3y = 4$

$(-3r)^2 = (4 - 3y)^2$

$9r^2 = 16 - 24y + 9y^2$

$9(x^2 + y^2) = 16 - 24y + 9y^2$

$9x^2 + 9y^2 = 16 - 24y + 9y^2$

$9x^2 + 24y - 16 = 0$

$9x^2 = 16 - 24y$

$\frac{9x^2 - 16}{-24} = \frac{-24y}{-24}$

$y = \frac{2}{3} - \frac{3x^2}{8}$

(6) $r = \frac{1}{1-\cos\theta}$

$r - r\cos\theta = 1$

$r - x = 1$

$r = x + 1$

$r^2 = x^2 + 2x + 1$

$x^2 + y^2 = x^2 + 2x + 1$

$y^2 = 2x + 1$

(7) $x = 7$

$r\cos\theta = 7$

$r = \frac{7}{\cos\theta}$

$r = 7\sec\theta$

(8) $2x - 3y = 5$

$2r\cos\theta - 3r\sin\theta = 5$

$r(2\cos\theta - 3\sin\theta) = 5$

$r = \frac{5}{2\cos\theta - 3\sin\theta}$

(9) $x + 5y = 8$

$r\cos\theta + 5r\sin\theta = 8$

$r(\cos\theta + 5\sin\theta) = 8$

$r = \frac{8}{\cos\theta + 5\sin\theta}$

(10) $y^2 = 6x$

$r^2\sin^2\theta = 6r\cos\theta$

$r^2\sin^2\theta - 6r\cos\theta = 0$

$r(r\sin^2\theta - 6\cos\theta) = 0$

$r = 0 \qquad \frac{r\sin^2\theta}{\sin^2\theta} = \frac{6\cos\theta}{\sin^2\theta}$

$r = 6\cot\theta\csc\theta$

(11) $(x-3)^2 + y^2 = 9$

$x^2 - 6x + 9 + y^2 = 9$

$x^2 - 6x + y^2 = 0$

$r^2 - 6r\cos\theta = 0$

$r(r - 6\cos\theta) = 0$

$r = 6\cos\theta$

(12) $(x-1)^2 + (y+4)^2 = 17$

$x^2 - 2x + 1 + y^2 + 8y + 16 = 17$

(12) $(x+3)^2 + (y+3)^2 = 18$

$x^2 + 6x + 9 + y^2 + 6y + 9 = 18$

$x^2 + 6x + y^2 + 6y = 0$

$r^2 + 6r\cos\theta + 6r\sin\theta = 0$

$r(r + 6\cos\theta + 6\sin\theta) = 0$

$r = -6\cos\theta - 6\sin\theta$

Honors Pre-Calc

Practice 6.4 Polar Coordinates

Name_____

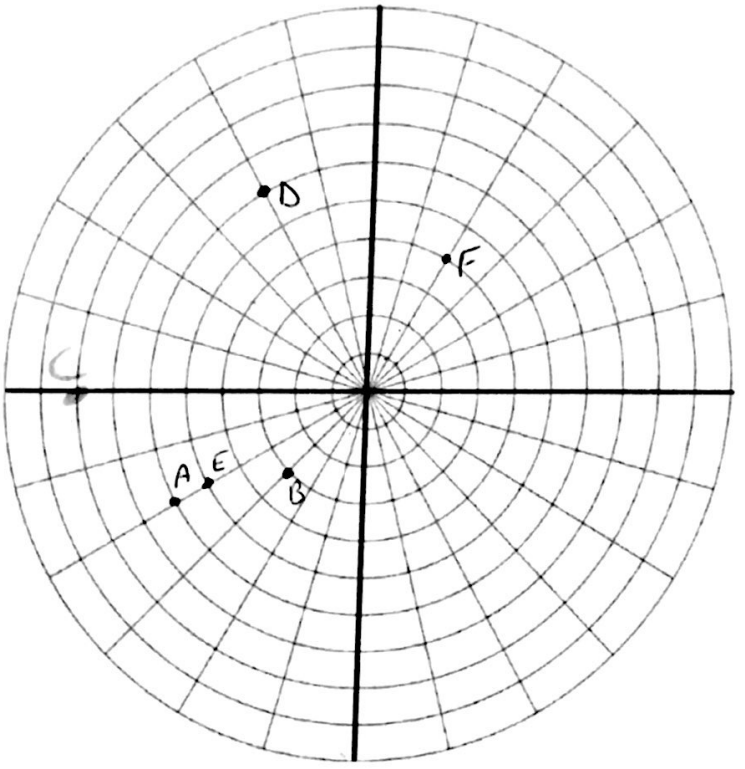
Date_____ **Block**____

1. ***Plot the following:***

A. $\left(6,\frac{7\pi}{6}\right)$ B. $\left(3,-\frac{3\pi}{4}\right)$

C. $(8,\pi)$ D. $\left(-6,\frac{5\pi}{3}\right)$

E. $\left(-5,-\frac{11\pi}{6}\right)$ F. $\left(-4,-\frac{2\pi}{3}\right)$



2. ***Find the rectangular coordinates for each of the following polar coordinates:***

A. $\left(4,\frac{5\pi}{6}\right)$ B. $\left(8,-\frac{2\pi}{3}\right)$ C. $(-2,-135^\circ)$ D. $\left(2,-\frac{14\pi}{5}\right)$

$(-2\sqrt{3}, 2)$ $(-4,-4\sqrt{3})$ $(\sqrt{2},\sqrt{2})$ $(-1.62, -1.18)$

3. ***Given the rectangular coordinates below, find the polar coordinates satisfying the conditions given:***

A. $(3,-3)$; $r \geq 0$ and $0 \leq \theta \leq 2\pi$

$r = 3\sqrt{2}$ $\left(3\sqrt{2}, \frac{7\pi}{4}\right)$

$\theta = 315, \frac{7\pi}{4}$

B. $\left(-5,-5\sqrt{3}\right)$; $r \leq 0$ and $0 \leq \theta \leq \pi$

$r = 10$ $\left(10, \frac{4\pi}{3}\right)$

$\theta = \frac{4\pi}{3}$ $\left(-10, \frac{\pi}{3}\right)$

C. $\left(4\sqrt{3},4\right)$; $r \leq 0$ and $0 \leq \theta \leq 2\pi$

$r = 8$ $\left(-8, \frac{4\pi}{3}\right)$

$\theta = \frac{\pi}{3}$

hange the following polar equations to rectangular equations:

4. $r = 3\sec\theta$

$r = \frac{3}{\cos\theta}$ → $x = 3$

$r\cos\theta = 3$

5. $r = -4\cos\theta$

$r^2 = -4r\cos\theta$

$x^2 + y^2 = -4x$

$(x+2)^2 + y^2 = 4$

6. $r = 4\cos\theta - 4\sin\theta$

$x^2 + y^2 = 4x - 4y$

$(x-2)^2 + (y+2)^2 = 8$

7. $r = 2\sin 3\theta$

$r = 2\sin(\theta + 2\theta)$

$r = 2(\sin\theta\cos$

8. $r = \frac{1}{1-\cos\theta}$

$r^2 =$

9. $r = \frac{6}{2\cos\theta - 3\sin\theta}$

2

Change the following rectangular equations to polar equations:

10. $x^2 + y^2 = 81$

$r^2 = 81$

$r = \pm 9$

$r = 9$

11. $y = -5$

$r\sin\theta = -5$

$r = \frac{-5}{\sin\theta}$

$r = -5\csc\theta$

12. $y^2 = 10x$

$(r\sin\theta)^2 = 10r\cos\theta$

$\frac{r^2\sin^2\theta}{r\sin^2\theta} = \frac{10r\cos\theta}{r\sin^2\theta}$

$r = 10\csc\theta\cot\theta$

13. $3x + 4y = 2$

$3r\cos\theta + 4r\sin\theta = 2$

$r(3\cos\theta + 4\sin\theta) = 2$

$r = \frac{2}{3\cos\theta + 4\sin\theta}$

14. $y^2 - 8x - 16 = 0$

$(r\sin\theta)^2 - 8r\cos\theta = 16$

$r^2\sin^2\theta - 8r\cos\theta = 16$

$r^2(1-\cos^2\theta) - 8r\cos\theta - 16 = 0$

$r^2\cos^2\theta$

ON Back.

15. $(x-1)^2 + (y+4)^2 = 17$

$x^2 - 2x + 1 + y^2 + 8y + 16 = 17$

$x^2 - 2x + y^2 + 8y = 0$

$r^2 - 2r\cos\theta + 8r\sin\theta$

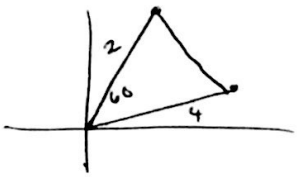
$r(r - 2\cos\theta + 8\sin\theta) = 0$

$r = 0$ $r = 2\cos\theta - 8\sin\theta$

16. The location, given in polar coordinates of two planes approaching the Vicksburg airport are (4 mi, 12°) and (2mi, 72°). Find the distance between the airplanes.

$x^2 = 2^2 + 4^2 - 2(2)(4)\cos 60$

$x = 3.46$ miles



17. The locations of two ships from Mays Landing Lighthouse, given in polar coordinates, are (3mi, 170°) and (5mi, 150°). Find the distance between the ships.

$x^2 = 5^2 + 3^2 - 2(5)(3)\cos 20$

$x = 2.41$ miles

$y^2 - 8x - 16 = 0$ $\qquad y^2 = 8(x+2)$

$(r\sin\theta)^2 - 8r\cos\theta - 16 = 0$

$\sin^2\theta\, r^2 - 8\cos\theta\, r - 16 = 0$

$$r = \frac{8\cos\theta \pm \sqrt{64\cos^2\theta - 2\sin^2\theta\cdot(-16)}}{2\sin^2\theta}$$

$$r = \frac{8\cos\theta \pm \sqrt{64(\cos^2\theta + \sin^2\theta)}}{2\sin^2\theta}$$

$$r = \frac{8\cos\theta \pm 8}{2\sin^2\theta}$$

$$r = \frac{8\cos\theta \pm 8}{2(1-\cos^2\theta)} = \frac{4\cos\theta \pm 4}{1-\cos^2\theta}$$

$$= \frac{4(\cos\theta - 1)}{(1-\cos\theta)(1+\cos\theta)}$$

$$= \frac{-4(\cos\theta - 1)(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$$

$$= \frac{-4}{1+\cos\theta}$$

Or $\dfrac{4(\cos\theta + 1)}{(1-\cos\theta)(1+\cos\theta)} = \dfrac{4}{1-\cos\theta}$

extra practice

$r = \dfrac{4}{-3+3\sin\theta}$

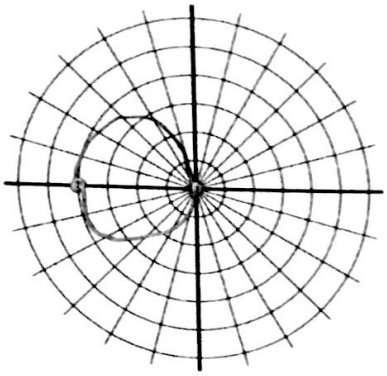
Answer: $9x^2 + 24y - 16 = 0$

Honors Pre-Calc
Classwork 6.5 Graphs of Polar Equations

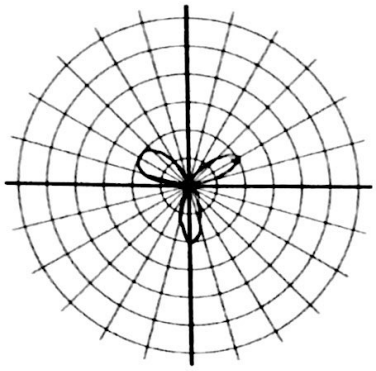
Name_____
Date_____ **Block**____

Classify and Graph the following polar equations:

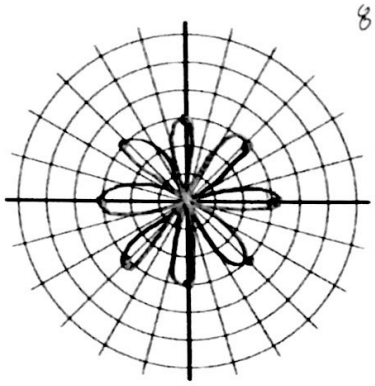
1. $r = -4\cos\theta$ Circle



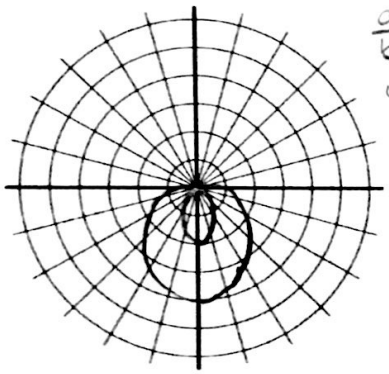
2. $r = 2\sin 3\theta$ Rose Curve



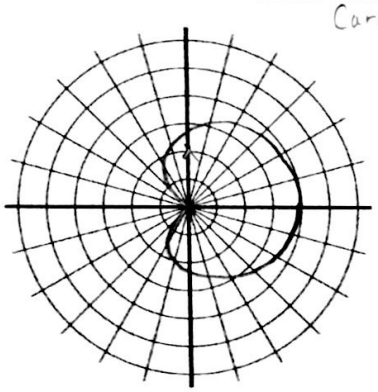
3. $r = 3\cos 4\theta$ Rose Curve
8 petals
$\frac{2\pi}{8} = \frac{\pi}{4}$



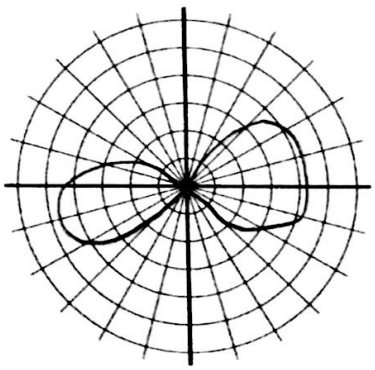
4. $r = 1 - 3\sin\theta$ Limacon w/ a loop
$\frac{a}{b} = \frac{1}{3}$
w/a loop



5. $r = 2 + 2\cos\theta$ Limacon
Cardioid



6. $r^2 = 9\cos 2\theta$ Lemniscate



Classify the polar graph, identify any symmetry and the location of the maximum r-value.

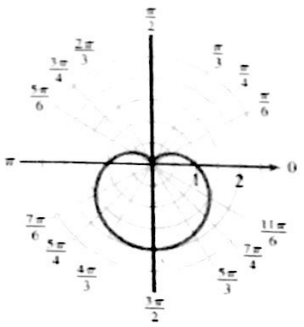
Polar Curve	Type	Symmetry	Max r-value point
7. $r = 3 + 3\sin\theta$	Cardiod Limacon	y-axis	~~(0,0)~~ ~~6~~ ? $(6, \pi/2)$
8. $r = 4 - 3\cos\theta$	Limacon w/ dimple	x-axis	$(-7, 0)$ / $(7, \pi)$
9. $r = 7\sin 3\theta$	Rose Curve	y-axis	$\left(7, \frac{\pi}{6} + \frac{2\pi n}{3}\right)$
10. $r = 9\sin 2\theta$	Rose Curve	~~x-axis~~ / y-axis all three. origin	$\left(9, \frac{\pi}{4} + \frac{\pi}{2}n\right)$

$\frac{\pi}{4}$ $\frac{2\pi}{4}$ $\frac{\pi}{2n}$

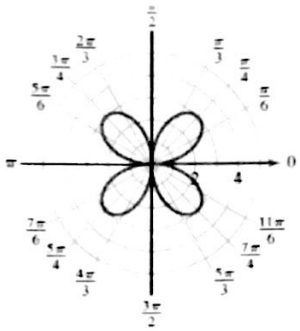
Careful about Radius values

Write the equation of the polar graph given:

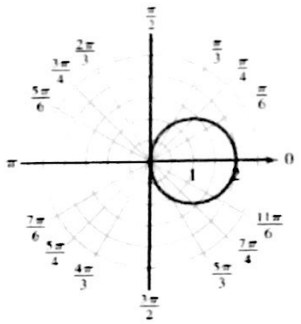
11. $1 - 1\sin\theta$



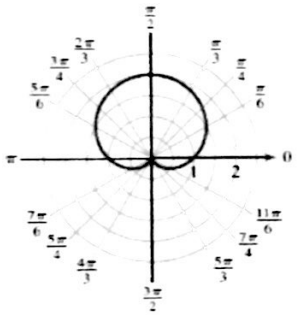
12. $r = 3\sin 2\theta$



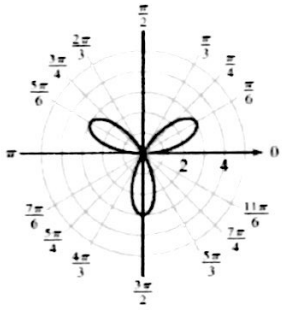
13. $r = 2\cos\theta$



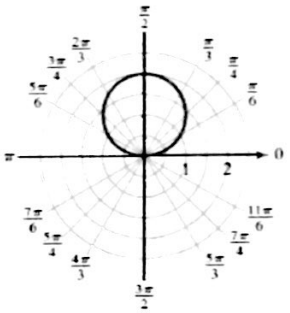
14. $1 + 1\sin\theta$



15. $r = 3\sin 3\theta$



16. $r = 2\sin\theta$

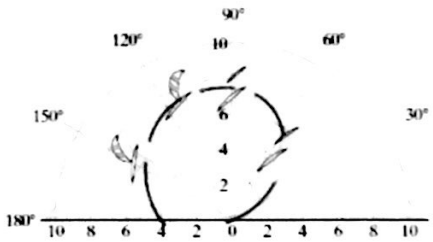


Application:

Sailors look for a sailing angle to a 10-knot wind that produces the maximum sailing speed. Each point (r,θ) on the graph gives the sailing speed, r, in knots, at an angle, θ, to the 10-knot wind. Use this information to solve.

17. What is the speed of a sailboat sailing at the following angles to the wind?

Angle	60°	90°	120°	180°
Speed	6	7.5	7	4



18. What angle to the wind produces the maximum sailing speed? What is the speed?

90° 7.5 knots