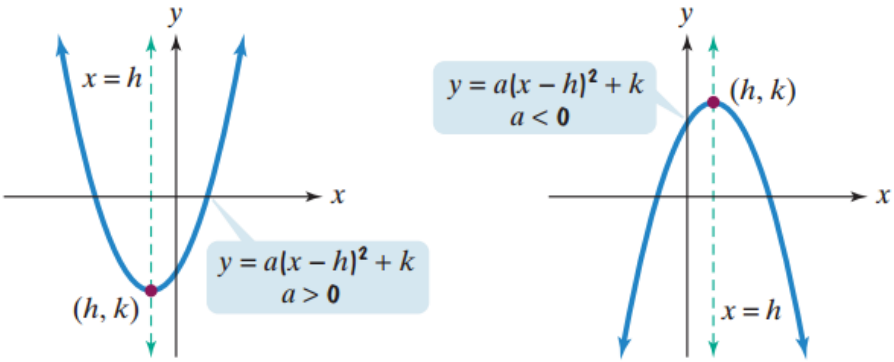
Parabolas: Quick Review

Here's a brief summary:

Graphing $y = a(x - h)^2 + k$ and $y = ax^2 + bx + c$

1. If $a > 0$, the graph opens upward. If $a < 0$, the graph opens downward.
2. The vertex of $y = a(x - h)^2 + k$ is (h, k).



3. The x-coordinate of the vertex of $y = ax^2 + bx + c$ is $x = -\dfrac{b}{2a}$.

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Definition of a Parabola

A **parabola** is the set of all points in a plane that are equidistant from a fixed line, the **directrix**, and a fixed point, the **focus**, that is not on the line (see **Figure 9.29**).

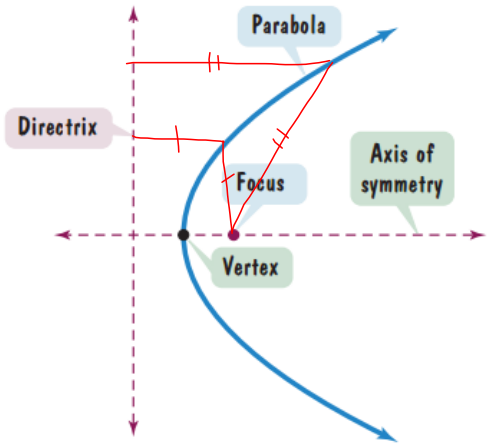

FIGURE 9.29

Axis of Symmetry: the line passing through the focus and perpendicular to the directrix.

Vertex: the point of intersection of the parabola with its axis of symmetry.

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Standard Equation of a Parabola

The **standard form of the equation of a parabola** with vertex at (h, k) is as follows.

$(x - h)^2 = 4p(y - k),\ p \neq 0$ — Vertical axis, directrix: $y = k - p$

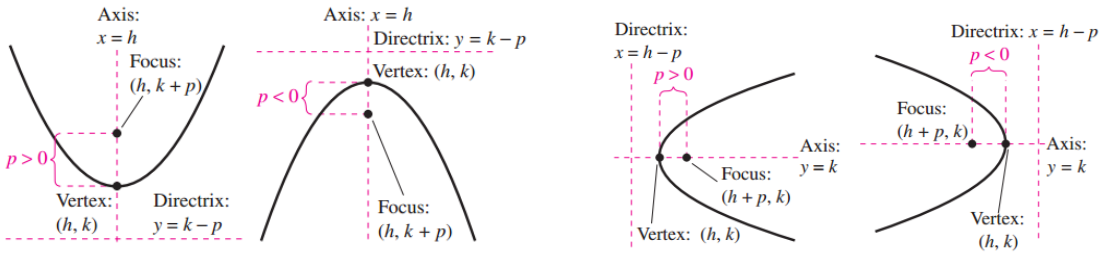
$(y - k)^2 = 4p(x - h),\ p \neq 0$ — Horizontal axis, directrix: $x = h - p$

The focus lies on the axis p units (*directed distance*) from the vertex. If the vertex is at the origin $(0, 0)$, the equation takes one of the following forms.

$x^2 = 4py$ — Vertical axis

$y^2 = 4px$ — Horizontal axis

See Figure 10.11.



(a) $(x - h)^2 = 4p(y - k)$ Vertical axis: $p > 0$
(b) $(x - h)^2 = 4p(y - k)$ Vertical axis: $p < 0$
(c) $(y - k)^2 = 4p(x - h)$ Horizontal axis: $p > 0$
(d) $(y - k)^2 = 4p(x - h)$ Horizontal axis: $p < 0$

FIGURE 10.11

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1.) Find the vertex, focus, and directrix of the parabola given by:

$$(x-3)^2 = 8(y+1)$$

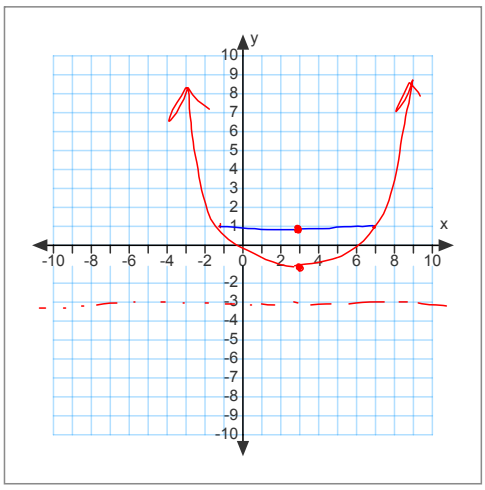
$(x - h)^2 = 4p(y - k),\ p \neq 0$

$(y - k)^2 = 4p(x - h),\ p \neq 0$

Then graph the parabola.

$4p = 8$

$p = 2$



Coordinate of Vertex: $(3, -1)$

Direction it Opens: Up

Axis of Symmetry: $x = 3$

Coordinate of Focus: $(3, 1)$

Equation of Directrix: $y = -3$

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2.) Find the vertex, focus, and directrix of the parabola given by:

$$(y-4)^2 = 20(x+2)$$

$(x-h)^2 = 4p(y-k),\ p \neq 0$

$(y-k)^2 = 4p(x-h),\ p \neq 0$

Then graph the parabola.

$4p = 20$

$p = 5$

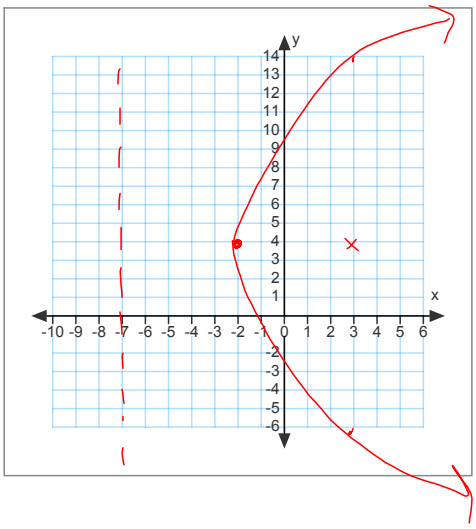
Coordinate of Vertex: $(-2, 4)$

Direction it Opens: Right

Axis of Symmetry: $y = 4$

Coordinate of Focus: $(3, 4)$

Equation of Directrix: $x = -7$



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3.) Find the vertex, focus, and directrix of the parabola given by:

Then graph the parabola.

$$y^2 + 2y + 12x - 23 = 0$$

$(x-h)^2 = 4p(y-k),\ p \neq 0$

$(y-k)^2 = 4p(x-h),\ p \neq 0$

$y^2 + 2y = -12x + 23$

$y^2 + 2y + \underline{1} = -12x + 23 + \underline{1}$

$(y+1)^2 = -12x + 24$

$\boxed{(y+1)^2 = -12(x-2)}$

$4p = -12$

$p = -3$

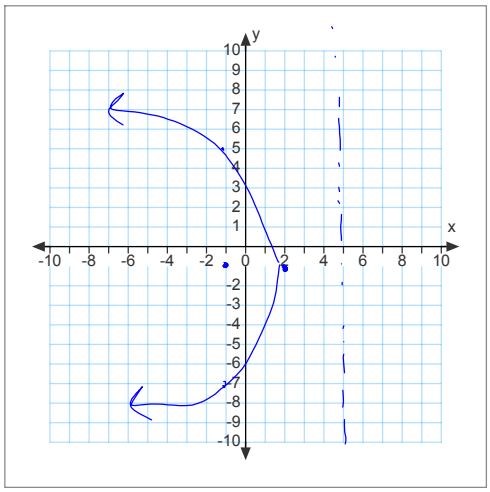
Coordinate of Vertex: $(2, -1)$

Direction it Opens: Left

Axis of Symmetry: $y = -1$

Coordinate of Focus: $(-1, -1)$

Equation of Directrix: $x = 5$



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4.) Find the vertex, focus, and directrix of the parabola given by:

Then graph the parabola.

$$(x-h)^2 = 4p(y-k),\ p \neq 0$$
$$(y-k)^2 = 4p(x-h),\ p \neq 0$$

$$y^2 + 21 = -20x - 6y - 68$$

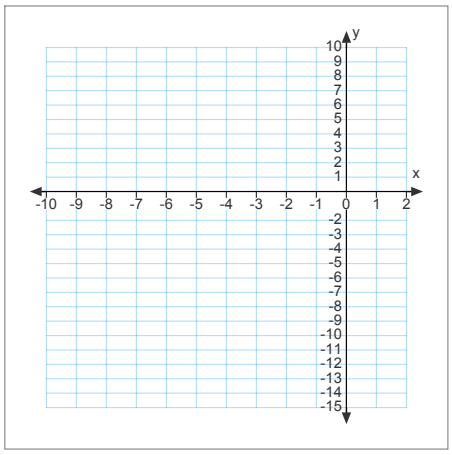
Coordinate of Vertex:

Direction it Opens:

Axis of Symmetry:

Coordinate of Focus:

Equation of Directrix:



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$$y^2 + 21 = -20x - 6y - 68$$

$$(x-h)^2 = 4p(y-k),\ p \neq 0$$
$$(y-k)^2 = 4p(x-h),\ p \neq 0$$

Then graph the parabola.

$y^2 + 6y = -20x - 89$

$y^2 + 6y + \underline{9} = -20x - 89 + \underline{9}$

$(y+3)^2 = -20(x+4)$

$p = -5$

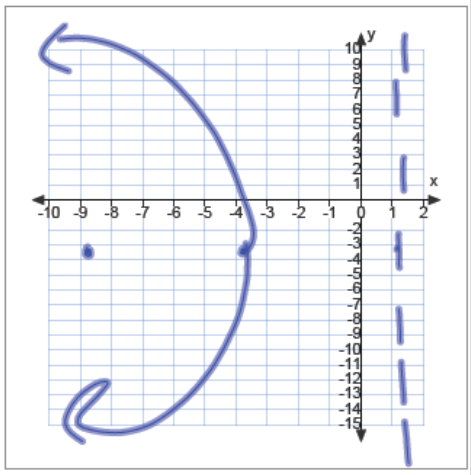
Coordinate of Vertex: $(-4, -3)$

Direction it Opens: Left

Axis of Symmetry: $y = -3$

Coordinate of Focus: $(-9, -3)$

Equation of Directrix: $x = 1$



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5.) Find the vertex, focus, and directrix of the parabola given by:
Then graph the parabola.

$$y = -\frac{1}{2}x^2 - x + \frac{1}{2}.$$

$(x-h)^2 = 4p(y-k),\ p \neq 0$

$(y-k)^2 = 4p(x-h),\ p \neq 0$

$-2\left(y - \frac{1}{2} = -\frac{1}{2}x^2 - x\right)$

$-2y + 1 = x^2 + 2x$

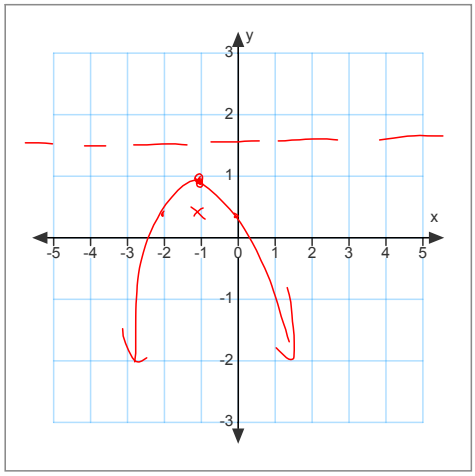
$-2y + 1 + 1 = x^2 + 2x + 1$

$-2y + 2 = (x+1)^2$

$\boxed{-2(y-1) = (x+1)^2}$

$4p = -2$

$p = -\frac{1}{2}$



Coordinate of Vertex: $(-1, 1)$

Direction it Opens: Down

Axis of Symmetry: $x = -1$

Coordinate of Focus: $(-1, 1/2)$

Equation of Directrix: $y = 3/2$

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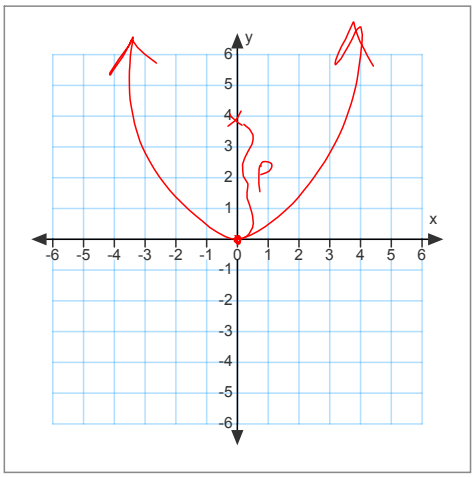
6.) Find the standard form of a parabola with vertex at the origin and focus (0, 4).

$p = 4$

$x^2 = 16y$

$(x-0)^2 = 16(y-0)$



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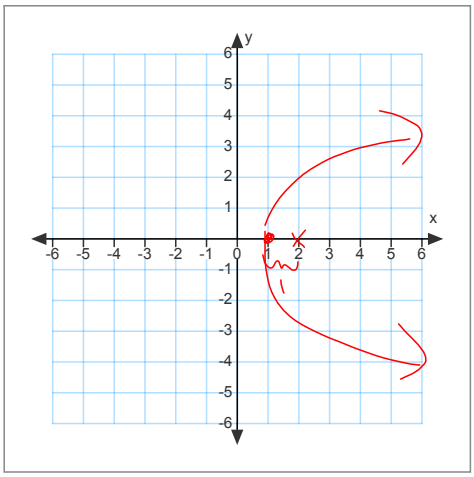
7.) Find the standard form of a parabola with vertex (1, 0) and focus (2, 0).

$p = 1$

y^2

$$\boxed{y^2 = 4(x-1)}$$



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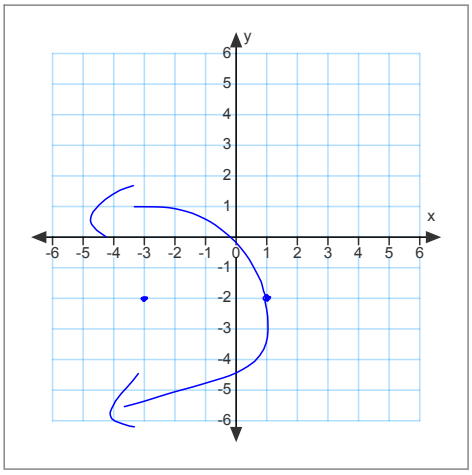
8.) Find the standard form of a parabola with vertex at (1, -2) and focus (-3, -2).

$p = -4$

$$(y+2)^2 = \underbrace{-16}_{4p}(x-1)$$



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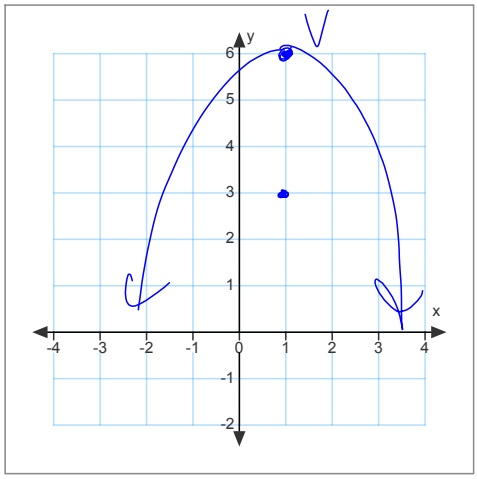
9.) Find the standard form of a parabola with directrix is y =9 and focus (1, 3).

$y=9$

$p=-3$

$(x-1)^2=-12(y-6)$



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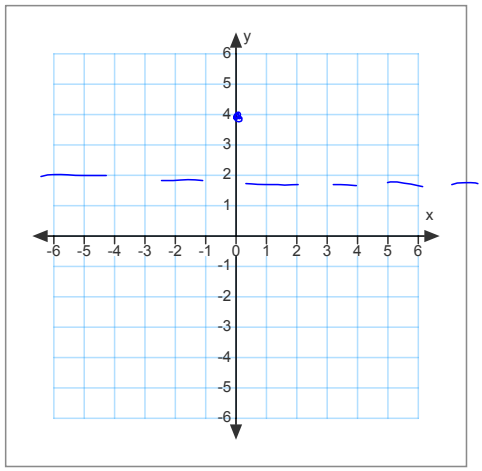
10.) Find the standard form of the equation of the parabola if the Vertex is (0, 4) and the directrix is y = 2.

$p=2$

$V(0,4)$

$x^2=8(y-4)$



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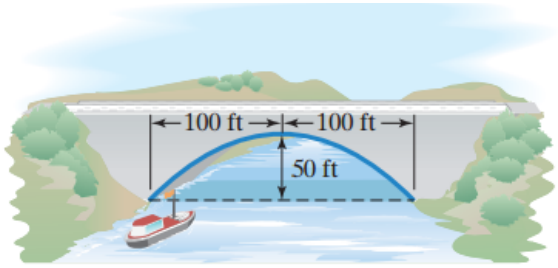
Applications: The parabolic arch shown in the figure is 50 feet above the water at the center and 200 feet wide at the base. Will a boat that is 30 feet tall clear the arch 30 feet from the center?



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