9.2 The Binomial Theorem

Multiply:

1.) $(a+b)^0$ $\quad 1$

2.) $(a+b)^1$ $\quad a+b$

3.) $(a+b)^2$ $\quad a^2+2ab+b^2$

4.) $(a+b)^3$ $\quad a^3+3a^2b+3ab^2+b^3$

5.) $(a+b)^4$ $\quad a^4+4a^3b+6a^2b^2+4a^3b^3+b^4$

Each expression has n + 1 terms

sum of the powers of each term = n

Pascal's Triangle: Used to Calculate Binomial Coefficients

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Find the first 7 rows of Pascal's Triangle:

1							
1	1						
1	2	1					
1	3	3	1				
1	4	6	4	1			
1	5	10	10	5	1		
1	6	15	20	15	6	1	
1	7	21	35	35	21	7	1

We name the nth row and the rth element by using the notation:

$$nCr = \binom{n}{r} = \frac{n!}{r!(n-r)!};\ n! = n(n-1)(n-2)\ldots 1$$

"n choose r"

# of ways to select r things from n

$4\cdot 3\cdot 2\cdot 1$

We starting counting rows and elements at 0.

Locate the first 6 in the triangle: $\binom{4}{2}$ $\quad \frac{4!}{2!2!} = \frac{4\cdot 3\cdot \cancel{2!}}{2!\cdot \cancel{2!}} = 6$ $\quad 2\cdot 1$ $\quad 2\cdot 1$

Find the following using the formula and check your answer with Pascal's Triangle.

a.) $\binom{5}{3}$ $\quad \frac{5!}{3!2!} = \frac{5\cdot 4\cdot \cancel{3!}}{\cancel{3!}\,2!}$ $\quad \frac{20}{2} = 10$

b.) $\binom{7}{5}$ $\quad \frac{7!}{2!5!}$ $\quad \frac{7\cdot 6\cdot \cancel{5!}}{2!\,\cancel{5!}} = 21$

c.) $\binom{12}{9} = 220$

Use Pascal's Triangle to expand the binomial:

a.) $(2x-y)^6$ $\quad 1\ 6\ 15\ 20\ 15\ 6\ 1$ $\quad a=2x$ $\quad b=-y$

$1(2x)^6 + 6(2x)^5(-y) + 15(2x)^4(-y)^2 + 20(2x)^3(-y)^3$

$+ 15(2x)^2(-y)^4 + 6(2x)(-y)^5 + (-y)^6$

$= 64x^6 - 192x^5y + 240x^4y^2 - 160x^3y^3 + 60x^2y^4 - 12xy^5 + y^6$

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The Binomial Theorem:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

$$= a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \binom{n}{3} a^{n-3} b^3 + \cdots + \binom{n}{n-1} a b^{n-1} + \binom{n}{n} b^n$$

Expand each binomial using the binomial theorem.

a.) $(a+b)^3$

$\binom{3}{0}a^3 + \binom{3}{1}a^2b + \binom{3}{2}ab^2 + \binom{3}{3}b^3$

$\frac{3!}{0!\,3!}$

$a^3 + 3a^2b + 3ab^2 + b^3$

b.) $(2x-3y)^3$

$(2x)^3 + 3(2x)^2(-3y) + 3(2x)(-3y)^2 + (-3y)^3$

$8x^3 - 36x^2y + 54xy^2 - 27y^3$

c.) $(5x+3y)^4$

1 4 6 4 1

$(5x)^4 + 4(5x)^3(3y) + 6(5x)^2(3y)^2 + 4(5x)(3y)^3 + (3y)^4$

$625x^4 + 1500x^3y + 1350x^2y^2 + 540xy^3 + 81y^4$

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Answer the following:

a.) Find the term containing: x^3 in: $(x+y)^{11}$

$\binom{11}{8}x^3y^8$ $165x^3y^8$

b.) Find the coefficient of the term: a^6b^5 in the expansion: $(2a-5b)^{11}$

$\binom{11}{5}(2a)^6(-5b)^5$

$462(64)(-3125) = -92{,}400{,}000$

c.) Find the fourth term of $(2x+y)^9$.

$\binom{9}{3}(2x)^6y^3$

$84 \cdot 64x^6y^3 = 5376x^6y^3$

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