9.2 Ellipses

Definition of an Ellipse

An **ellipse** is the set of all points, P, in a plane the sum of whose distances from two fixed points, F_1 and F_2, is constant (see **Figure 9.3**). These two fixed points are called the **foci** (plural of **focus**). The midpoint of the segment connecting the foci is the **center** of the ellipse.

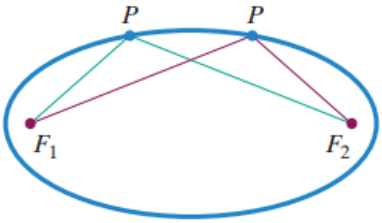

FIGURE 9.3

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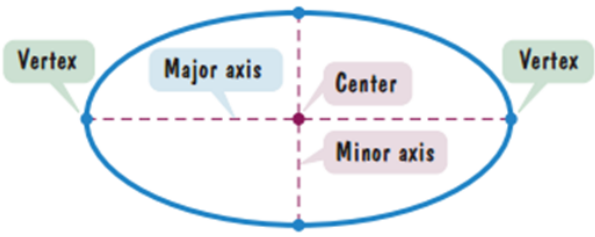
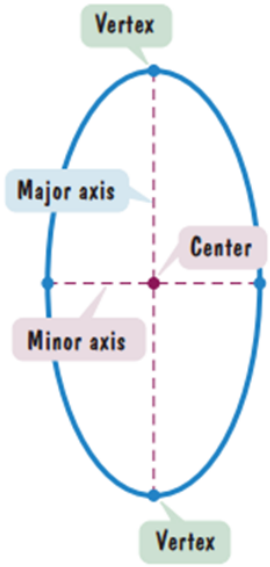
$$a > b$$



Major axis is horizontal

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$



Major axis is vertical

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

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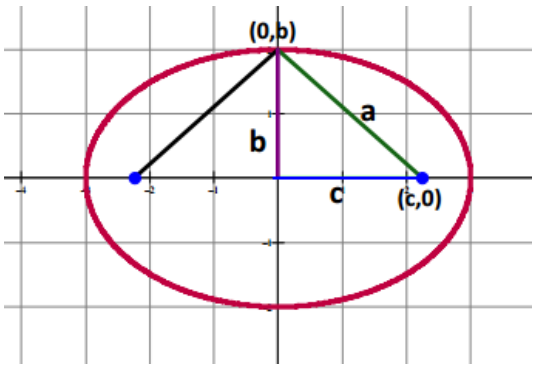
Relationship between a, b, and c.



$a^2 = b^2 + c^2$

Major Axis = 2a

Minor Axis = 2b

Foci: $c^2 = a^2 - b^2$

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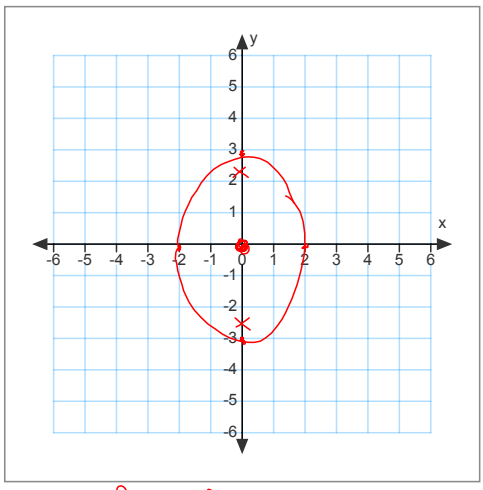
Find the indicated values and graph:

1.) $\frac{9x^2}{36} + \frac{4y^2}{36} = \frac{36}{36}$

$\boxed{\frac{x^2}{4} + \frac{y^2}{9} = 1}$

$a = 3$

$b = 2$



center: $(0,0)$

vertices: $(0,3)$ $(0,-3)$

co-vertices: $(-2,0)$ $(2,0)$

foci: $(0, \pm\sqrt{5})$

$c^2 = a^2 - b^2$

$c^2 = 9 - 4$

$c^2 = 5$ $c = \sqrt{5}$

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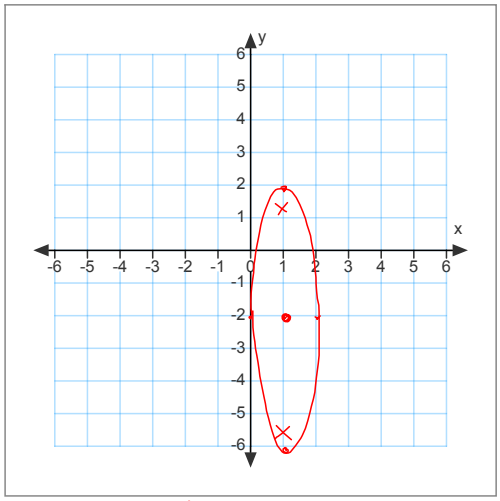
Find the indicated values and graph:

2.) $\frac{(x-1)^2}{1}+\frac{(y+2)^2}{16}=1$

$a=4$

$b=1$



center: $(1,-2)$

vertices: $(1,2)(1,-6)$

co-vertices: $(0,-2)\ (2,-2)$

foci: $(1,-2\pm\sqrt{15})$

$c^2=a^2-b^2$

$c^2=16-1$

$c^2=15$

$c=\sqrt{15}$

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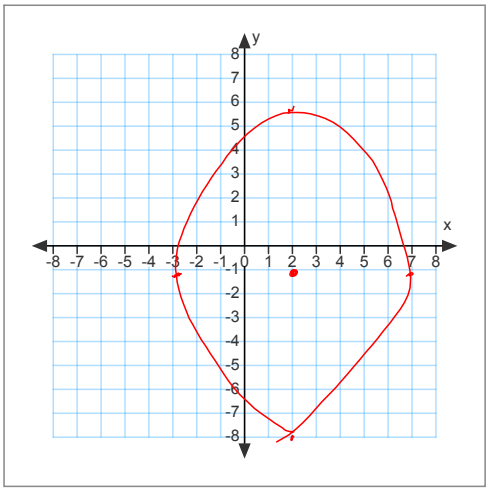
Find the indicated values and graph:

3.) $49(x-2)^2+25(y+1)^2=1225$

$\frac{(x-2)^2}{25}+\frac{(y+1)^2}{49}=1$



center: $(2,-1)$

vertices: $(2,6)\ (2,-8)$

co-vertices: $(-3,-1)\ (7,-1)$

foci: $(2,-1\pm2\sqrt{6})$

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Find the indicated values and graph:

4.) $9x^2+25y^2-36x+50y-164=0$

$9x^2-36x+25y^2+50y=164$

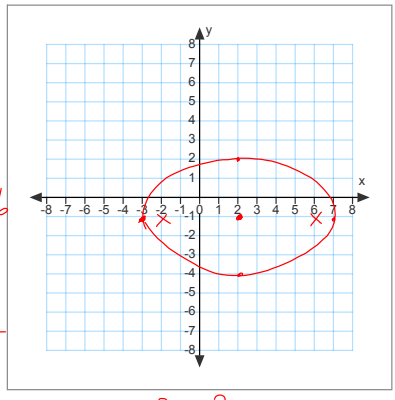
$9(x^2-4x+\underline{4})+25(y^2+2y+\underline{1})=164+36+25$

$\rightarrow 9(x-2)^2+25(y+1)^2=225$

$\boxed{\frac{(x-2)^2}{25}+\frac{(y+1)^2}{9}=1}$

$a=5$

$b=3$



$c^2=a^2-b^2$

$c^2=25-9$

$c^2=16$

$c=4$

center: $(2,-1)$

vertices: $(-3,-1)$ $(7,-1)$

co-vertices: $(2,2)$ $(2,-4)$

foci: $(2\pm4,-1)$

$\rightarrow (6,-1)$ $(-2,-1)$

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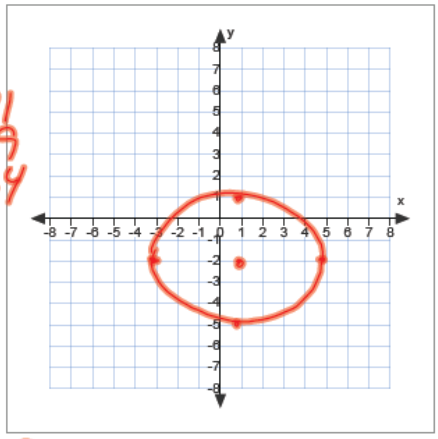
Find the indicated values and graph:

5.) $9x^2+16y^2-18x+64y-71=0$

$9(x^2-2x+1)+16(y^2+4y+4)=71+9+64$

$9(x-1)^2+16(y+2)^2=144$

$\frac{(x-1)^2}{16}+\frac{(y+2)^2}{9}=1$



$c^2=16-9$

$c=\sqrt{7}$

center: $(1,-2)$

vertices: $(5,-2)$ $(-3,-2)$

co-vertices: $(1,1)$ $(1,-5)$

foci: $(1\pm\sqrt{7},-2)$

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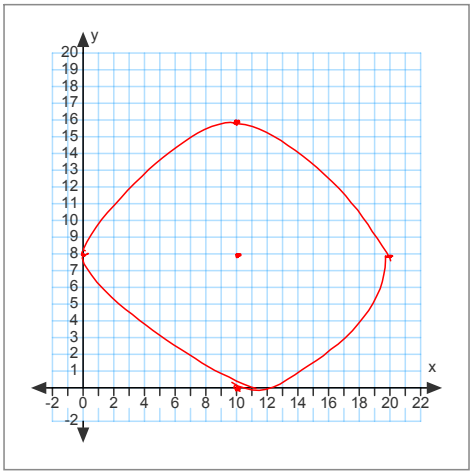
Write the standard equation for an ellipse with the given information:

6.) Vertices: (20, 8) and (0, 8) — Major: a

Co-Vertices: (10, 16) and (10, 0) — Minor: b



$C(10, 8)$ $a = 10$

$b = 8$

$$\boxed{\frac{(x-10)^2}{100} + \frac{(y-8)^2}{64} = 1}$$

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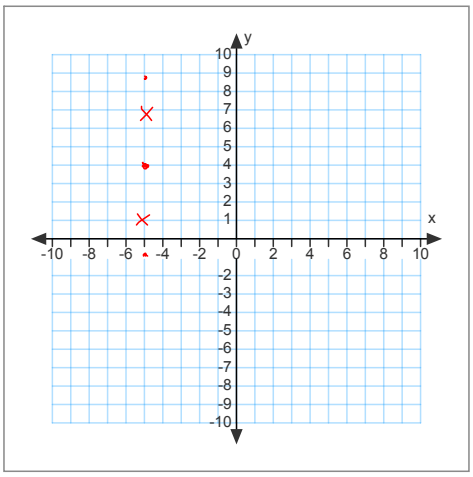
Write the standard equation for an ellipse with the given information:

7.) Vertices: (-5, 9) and (-5, -1)

Foci: (-5, 7) and (-5, 1)



$a = 5$

$c = 3$

$C(-5, 4)$

$a = 5$ $b = 4$

$c^2 = a^2 - b^2$

$9 = 25 - b^2$

$b = 4$

$$\boxed{\frac{(x+5)^2}{16} + \frac{(y-4)^2}{25} = 1}$$

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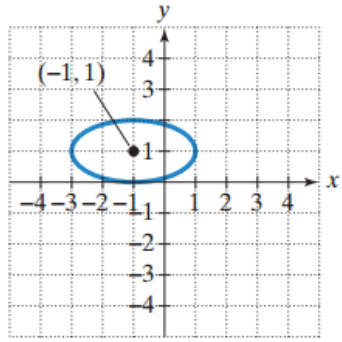
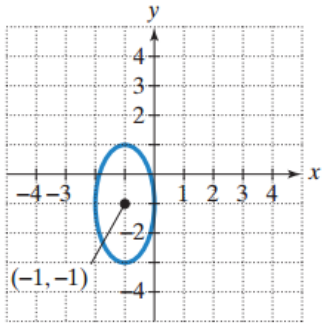
8.) Write the standard equation for an ellipse with the given information:

a.)



$$\frac{(x+1)^2}{4} + \frac{(y-1)^2}{1} = 1$$

b.)



$$\frac{(x+1)^2}{1} + \frac{(y+1)^2}{4} = 1$$

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9.) A semi-elliptical archway over a one-way road has a height of 10 feet and a width of 40 feet (see Figure 9.11). Your truck has a width of 10 feet and a height of 9 feet. Will your truck clear the opening of the archway?

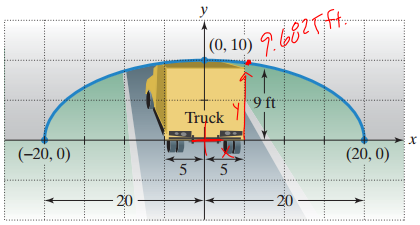
$$\frac{x^2}{400} + \frac{y^2}{100} = 1$$

$$\frac{5^2}{400} + \frac{y^2}{100} = 1$$

$$\frac{y^2}{100} = \frac{15}{16}$$

$$16y^2 = 1500$$

$$\boxed{y = 9.6825\text{ft}}$$



The truck will fit.

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