① $\cos(4x) = 8\cos^4 x - 8\cos^2 x + 1$

$\cos 2(2x) = 2\cos^2(2x) - 1$

$= \cos(2x+2x) = \cos 2x \cos 2x - \sin 2x \sin 2x$

$= (2\cos^2 x - 1)(2\cos^2 x - 1) - 2\sin x \cos x \cdot 2\sin x \cos x$

$= 4\cos^4 x - 4\cos^2 x + 1 - 4\sin^2 x \cos^2 x$

$= 4\cos^4 x - 4\cos^2 x + 1 - 4(1-\cos^2 x)\cos^2 x$

$= 4\cos^4 x - 4\cos^2 x + 1 - (4 - 4\cos^2 x)\cos^2 x$

$= 4\cos^4 x - 4\cos^2 x + 1 - 4\cos^2 x + 4\cos^4 x$

$= 8\cos^4 x - 8\cos^2 x + 1$

② $\cot x - \tan x = \cos 2x \sec x \csc x$

$\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} = \frac{\cos^2 x - \sin^2 x}{\sin x \cos x} = \frac{\cos 2x}{\sin x \cos x} = \cos 2x \sec x \csc x$

③ $\cos^2 x = \frac{\csc x \cos x}{\tan x + \cot x}$

$\frac{\frac{1}{\sin x} \cdot \cos x}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}} = \frac{\frac{\cos x}{\sin x}}{\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}} = \frac{\cos x}{\cancel{\sin x}} \cdot \frac{\cancel{\sin x} \cos x}{\sin^2 x + \cos^2 x}$

$= \cos^2 x$

④ $\tan \frac{x}{2} = \frac{\sin x}{1+\cos x}$

$\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)} = \frac{\sqrt{\frac{1-\cos x}{2}}}{\sqrt{\frac{1+\cos x}{2}}} = \frac{1-\cos x}{\cancel{2}} \cdot \frac{\cancel{2}}{1+\cos x} = \frac{1-\cos x}{1+\cos x}$

$= \frac{1-\cos x}{1+\cos x} \cdot \frac{1+\cos x}{1+\cos x} = \frac{1-\cos^2 x}{1+2\cos x+\cos^2 x} = \sqrt{\frac{\sin^2 x}{(1+\cos x)^2}} = \frac{\sin x}{1+\cos x}$

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(5) $\cot^2 x = 1/3$

$\cot x = \pm\sqrt{3}/3$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

(6) $\frac{1+\sin x}{\cos x} + \frac{\cos x}{1+\sin x} = 4$

$$\frac{1+2\sin x+\sin^2 x+\cos^2 x}{(\cos x)(1+\sin x)} = 4$$

$$\frac{2+2\sin x}{\cos x\,(1+\sin x)} = 4$$

$$\frac{2\cancel{(1+\sin x)}}{\cos x\cancel{(1+\sin x)}} = 4$$

$$\frac{2}{\cos x} = 4$$

$4\cos x = 2$

$\cos x = 1/2$

$x = \pi/3, 5\pi/3$

(7) $\cos 2x + 5\cos x = 2$

$2\cos^2 x - 1 + 5\cos x - 2 = 0$

$2\cos^2 x + 5\cos x - 3 = 0$

$(2\cos x - 1)(\cos x + 3) = 0$

$\cos x = 1/2$ $\qquad$ $\cos x = -3$

$x = \pi/3, 5\pi/3$

⑧ $\sin(x+\pi) + \cos(x+\pi) = 0$

$\sin x \cos\pi + \cos x \cancel{\sin\pi} + \cos x \cos\pi - \cancel{\sin x}\sin\pi = 0$

$-\sin x - \cos x = 0$

$-\sin x = \cos x$

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

⑨ $u = 225°$

$\sin\frac{225}{2} = \sqrt{\frac{1-\cos 225}{2}} = \sqrt{\frac{1+\sqrt{2}/2}{2}} = \sqrt{\frac{2+\sqrt{2}}{2}\cdot\frac{1}{2}}$

$= \frac{\sqrt{2+\sqrt{2}}}{2}$

$\cos\frac{225}{2} = -\sqrt{\frac{1+\cos 225}{2}} = -\sqrt{\frac{1-\sqrt{2}/2}{2}} = -\sqrt{\frac{2-\sqrt{2}}{2}\cdot\frac{1}{2}}$

$= -\frac{\sqrt{2-\sqrt{2}}}{2}$

tangent $\frac{225}{2} = \frac{1-\cos 225}{\sin 225} = \frac{1+\sqrt{2}/2}{-\sqrt{2}/2} = \frac{2+\sqrt{2}}{\cancel{2}}\cdot\frac{\cancel{2}}{-\sqrt{2}}$

$= \frac{2+\sqrt{2}}{-\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}+2}{-2}$

$= -\sqrt{2}-1$

$\frac{\pi}{3} - \frac{\pi}{4}$

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$$\sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \sin\frac{\pi}{3}\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\cos\pi/3$$
$$= \sqrt{3}/2 \cdot \sqrt{2}/2 - \sqrt{2}/2 \cdot \frac{1}{2}$$
$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$\cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\pi/3 \sin\pi/4$$
$$= \frac{1}{2} \cdot \sqrt{2}/2 + \sqrt{3}/2 \cdot \sqrt{2}/2$$
$$= \frac{\sqrt{2} + \sqrt{6}}{4}$$

$$\tan(\pi/3 - \pi/4) = \frac{\tan\pi/3 - \tan\pi/4}{1 + \tan\pi/3 \tan\pi/4}$$

$$= \frac{\sqrt{3} - 1}{1 + \sqrt{3}} \cdot \cancel{\frac{1+\sqrt{3}}{1+\sqrt{3}}}$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} \cdot \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{3 - 2\sqrt{3} + 1}{3 - 1} = \frac{4 - 2\sqrt{3}}{2}$$

$$= 2 - \sqrt{3}$$