

Calculator on select problems only!

For problems 1 - 5, use the following vectors: $u = 2i + 4j$ $v = 3i - 9j$ $w = 6i - 4j$ (Calculator)

1. State the unit vector in the direction of v .

$$\left\langle \frac{3}{\sqrt{90}}, \frac{-9}{\sqrt{90}} \right\rangle = \left\langle \frac{3}{3\sqrt{10}}, \frac{-9}{3\sqrt{10}} \right\rangle = \left\langle \frac{1}{\sqrt{10}}, \frac{-3}{\sqrt{10}} \right\rangle^*$$

$$= \left\langle \frac{\sqrt{10}}{10}, \frac{-3\sqrt{10}}{10} \right\rangle^?$$

2. Find the direction angle θ for the vector w in degrees.

$$\tan^{-1}(-4/6) = \theta = 326.3^\circ$$

3. Find the angle, in degree measure, between the vectors u and v .

$$\cos \theta = \frac{-30}{\sqrt{20} \cdot \sqrt{90}} \quad \theta = 135^\circ$$

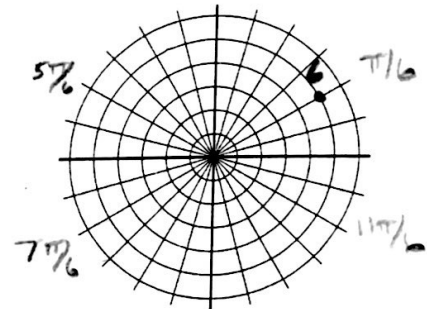
4. Find the projection of v onto u .

$$\text{proj}_u v = \frac{u \cdot v}{|u|^2} \langle u \rangle = \frac{-30}{20} \langle 2, 4 \rangle = \langle -3, -6 \rangle$$

$$\left(\frac{u \cdot v}{|u|^2} \right) \langle u \rangle = \dots$$

6. Graph, then convert to rectangular coordinates:

$$\left(-5, -\frac{5\pi}{6}\right) \quad \left(\frac{5\sqrt{3}}{2}, \frac{5}{2}\right)$$



7. Convert to polar coordinates, then graph:

$$(-6, 6\sqrt{3}); r \leq 0 \text{ and } 0 \leq \theta \leq 2\pi$$

$$r = 12 \quad \tan^{-1}(6\sqrt{3}/-6) \quad (-12, 5\pi/3) \rightarrow \text{doesn't fit}$$

$$\theta = 2\pi/3$$

Eliminate the parameter and state the domain/range restriction if one exists:

8. $x = 5 - 3t, y = 2 + t, -4 \leq t \leq 4$

$$x = 5 + 12 = 17 \quad -7 \leq x \leq 17$$

$$x = 5 - 12 = -7$$

$$\frac{x-5}{-3} = t$$

$$y = 2 - \frac{1}{3}x + \frac{5}{3}$$

$$y = -\frac{1}{3}x + \frac{11}{3}$$

9. $x = 4 + 2\cos\theta, y = -1 + \sin\theta, 0 \leq t \leq \pi$

$$\frac{x-4}{2} = \cos\theta \quad y+1 = \sin\theta$$

$$\left(\frac{x-4}{4}\right)^2 + \left(\frac{y+1}{1}\right)^2 = 1$$

ellipse $C(4, -1)$

$$a = 2$$

$$h = 1$$

$$D: 2 \leq x \leq 6$$

$$R: -2 \leq y \leq 0$$

Change the following polar equations to rectangular equations:

10. $r^2 \sin 2\theta = 16$

$2 r \sin \theta \cos \theta = 16$
 $yx = 8$
 $y = 8/x$

11. $r = 6 \sec \theta$

$r = \frac{6}{\cos \theta}$
 $r \cos \theta = 6$
 $x = 6$

12. $r = 8 \cos \theta + 2 \sin \theta$

$r^2 = 8r \cos \theta + 2r \sin \theta$
 $x^2 + y^2 = 8x + 2y$
 $x^2 - 8x + 16 + y^2 - 2y + 1 = 17$
 $(x-4)^2 + (y-1)^2 = 17$

Change the following rectangular equations to polar equations:

13. $x^2 + (y+3)^2 = 9$

$x^2 + y^2 + 6y + 9 = 9$
 $r^2 + 6r \sin \theta = 0$
 $r(r + 6 \sin \theta) = 0$
 $r = -6 \sin \theta$

14. $x = 3$

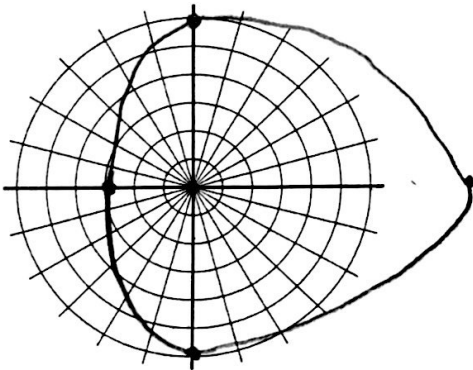
$r \cos \theta = 3$
 $r = 3 \sec \theta$

15. $x + 5y = 8$

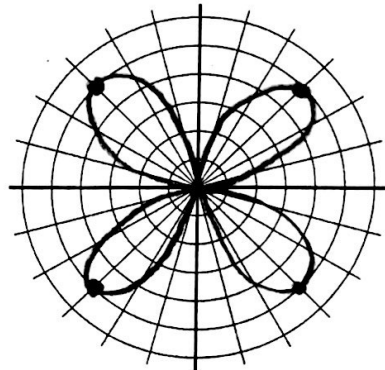
$r \cos \theta + 5r \sin \theta = 8$
 $r = \frac{8}{\cos \theta + 5 \sin \theta}$

Classify and Graph the following polar equations:

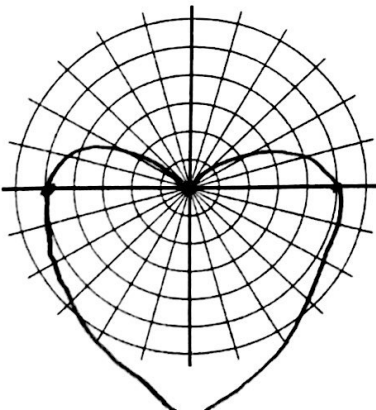
16. $r = 6 + 3 \cos \theta$ convex limaçon



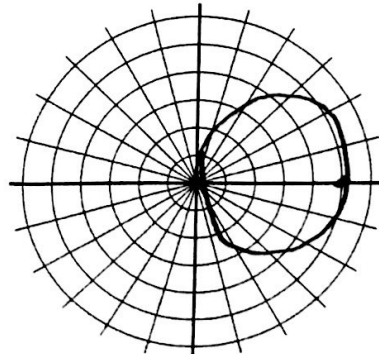
17. $r = -5 \sin 2\theta$ Rose $\frac{1}{2}$ petals



18. $r = 5 - 5 \sin \theta$ Cardioid

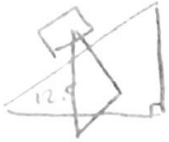


19. $r = 5 \cos \theta$ Circle



Application (Calculator):

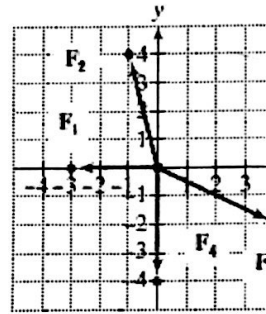
20. Find the magnitude of the force required to keep a 280 lb barrel from sliding down a ramp inclined at 12.5° . Then, find the magnitude of the force of the barrel against the ramp.



$$280 \sin 12.5 = 60.6$$

$$280 \cos 12.5 = 273.4$$

21. Find the magnitude and direction of the resultant force F_R of the four forces shown. What additional force is required for the given forces to be in equilibrium?



$$F_1 = \langle -3, 0 \rangle$$

$$F_R = \langle 0, -2 \rangle \quad \theta = 3\pi/2$$

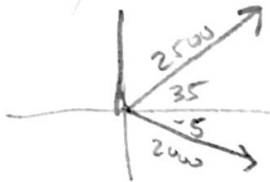
$$F_2 = \langle -1, 4 \rangle$$

$$\text{Add } F \rightarrow \langle 0, 2 \rangle$$

$$F_3 = \langle 4, -2 \rangle$$

$$F_4 = \langle 0, -4 \rangle$$

22. Two tugboats are pulling on a large ship that has gone aground. One tug pulls with a force of 2500 lbs bearing 55° . The second tug pulls with a force of 2000 bearing 95° . Find the magnitude and compass direction (bearing) of the resultant force.



$$\langle 2500 \cos 35, 2500 \sin 35 \rangle$$

$$+ \langle 2000 \cos(-5), 2000 \sin(-5) \rangle$$

$$\langle 4040.3, 1259.6 \rangle$$

$$4232.1$$

$$\text{bearing } 72.7$$

23. A force is given by the vector $F=3i + 2j$. The force moves an object along a straight line from $(4, 9)$ to the point $(10, 20)$. Find the work done if the distance is measured in feet and the force is measured in pounds.

$$W = Fd \cos \theta$$

$$F = \sqrt{13}$$

$$\theta_1 = 61.4$$

$$W = 40Ft - 16s$$

$$d = \sqrt{157}$$

$$\theta_2 = 33.7$$

$$\theta = 27.7$$

24. Greg Maddux throws a slider across the plate about 3 feet above the ground. Derek Jeter hits the ball with an initial velocity of 140 feet per second at an angle of 21° above the horizontal. The ball travels straight at the 420 foot mark on the center field wall which is 15 feet high.

Show algebraic work, not just an answer from your graphing calculator.

- a) Write the parametric equations that represent the position of the ball.

$$x(t) = 140 \cos 21t$$

$$y(t) = -16t^2 + 140 \sin 21t + 3$$

- b) Find the height of the ball after it has traveled 420 feet horizontally.

$$\frac{420}{140 \cos 21} = \frac{140 \cos 21 t}{140 \cos 21}$$

$$t = 3.21 \text{ seconds}$$

- c) Will the ball clear the fence for a home run or will the center fielder be able to catch it?

$$y(3.21) = -1 \text{ feet}$$

It's still in the park. It can be caught.

- d) If there were no outfield fences or seats, how far would the ball travel before it hits the ground?



N/A