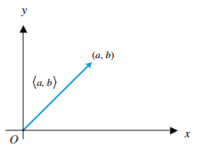
6.1 Vectors on a Plane

Before we get started, a vector is a line having **direction** as well as **magnitude**. Vectors go on forever, as any line would, but for calculation purposes we will look at vectors as directed **line segments** only.



As a result, the section of a vector we are dealing with has an **initial point **** and a **terminal point **. This is where the vector starts (usually, but not limited to, at the origin) and ends, respectively.

terminal point (head)

Vectors are written in **component form**, not to be confused with the vector’s endpoint, .

initial point (tail)

The Basics: Finding Equivalent Vectors (and vectors in general!)

Equivalent vectors are vectors with the same direction and magnitude. To start off your understanding of vectors, if you are given an initial point and a terminal point and told to find the resultant vector, use the **Heads Minus Tails** rule. This will become useful later as a part of finding vectors to project.

*HMT rule:*

the vector!

terminal point (head)

initial point (tail)

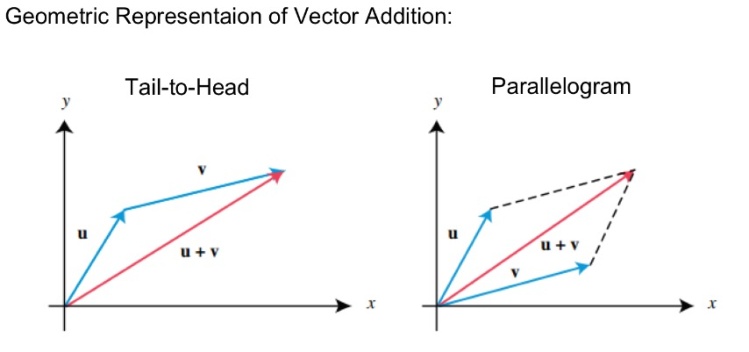
The Basics: Finding Magnitude

This is another topic that will be built upon and heavily used in vector computations. Magnitude tells the size, or in this situation, length of its object.

The formula for magnitude is , where is the initial point and is the terminal point of the vector.

Another way to write this is, where vector . This method is more commonly applied.

The Basics: Vector Operations

Where vector, vector, and *k*  is a real number (scalar)...

The sum (or **resultant**) of the two vectors is:

And the **product of the scalar k and the vector u** is:

Unit Vectors

A vector **u** with a magnitude is a **unit vector**. A vector **v** with a magnitude not equal to 1 can be converted into a unit vector **u** with the following formula:

\*note: vector cannot be\*

If **v** is not the zero vector , then the vector **u** is a **unit vector in the direction of v**.

**Example 1** *(found on page 3 of 6.1 Vectors in a Plane guided class notes)***:**

Find a unit vector in the direction of

The **standard unit vectors** are: and

Any vector can be written as an expression in terms of standard unit vectors:

Horizontal component:

Vertical Component:

Component form:

Linear Combination:

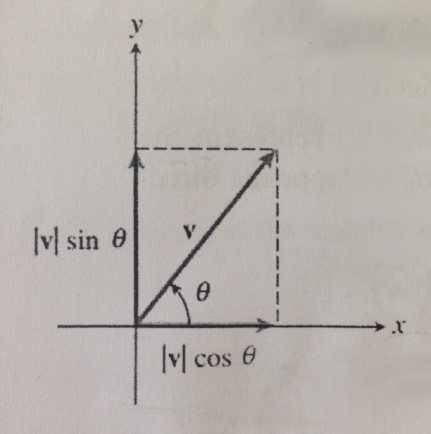
**Example 2** *(textbook homework pg 464, #27)***:**

Find the unit vector in the direction of vector when . Write your answer in (a) component form and (b) as a linear combination.

First, find the magnitude of

a) Therefore, the unit vector in component form is:

b) And the unit vector as a linear combination is:

Direction Angles and Resolving Vectors

A vector’s **direction angle** is the angle from the positive x-axis to the vector itself.

If has a direction angle , the components of can be computed using the formula:

\*For the formula above, it follows that the unit vector in the direction of is:

6.2 Vector Projection

Dot Products

The dot product of vectors and , where vector and vector is:

Properties of dot products *(let* ***u****,* ***v****, and* ***w*** *be vectors, and let c be a scalar)*:

1.

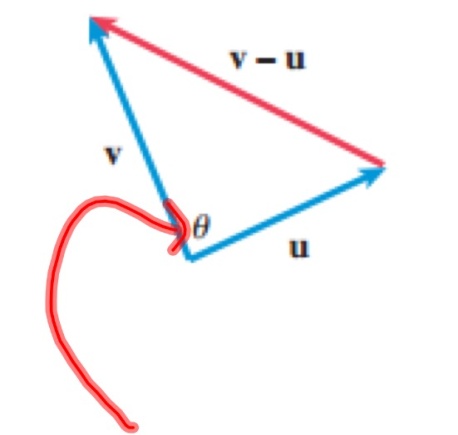
2.

3.

4.

5.

The Angle between Two Vectors \*IMPORTANT\*

If is the angle between two nonzero vectors and , then:

And therefore

Vectors and are **orthogonal** if and only if (Hint: the angle between the two vectors will be 90˚)

Vectors and are **parallel** if the angle between them is 180˚

Projecting One Vector onto Another

If and are nonzero vectors, the projection of onto is:

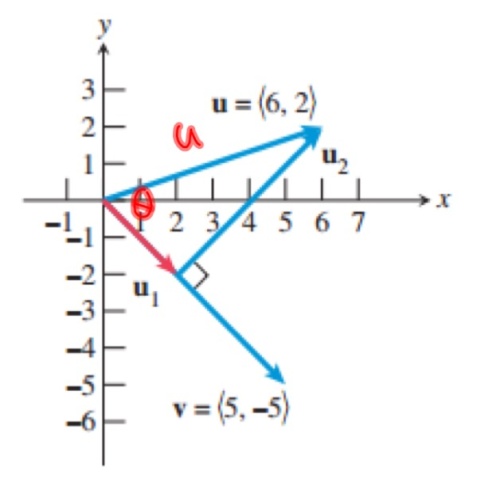
**Helpful reminders**

(when projecting u onto v)

You can also find the projection of onto by

using the formula:

**Example 1** *(number 9 from 6.2 Dot Product of Vectors Notes)***:**

Find the vector projection of u onto v when and

Looking back at the Helpful Reminders on the previous page, we see , so we can substitute out

We can now cancel out from the first term . We will also now plug in the appropriate values in the second term .

We will now substitute the appropriate values in the first term . The dot product of is 20. If you do not understand how we got this value, review the content under the title Dot Product in Section 6.2.

Now, we simplify numerically.

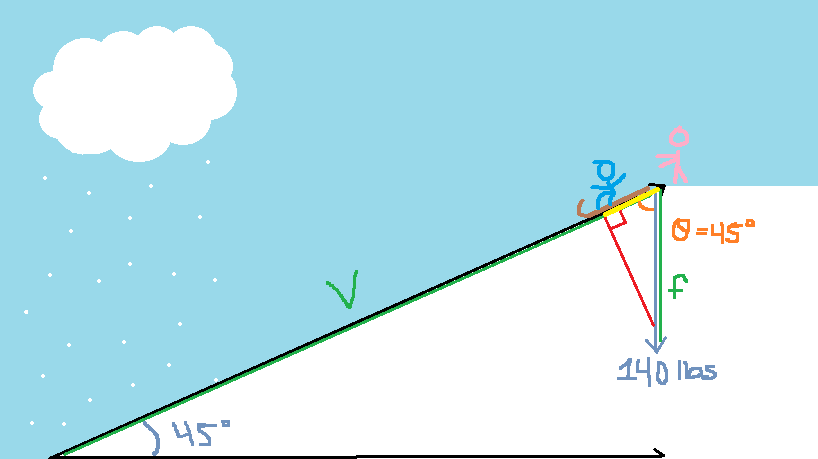
Vector Applications

The best way to explain how to apply vectors in real-world situations is by going through an example to illustrate how the mathematical concept intertwines with real problems.

Let's go through some examples together:

**Example 2** *("example 6" on page 5 of 6.2 dot Product of Vectors notes)***:**

Juan is sitting on a sled on the side of a hill inclined at 45˚. The combined weight of Juan and the sled is 140 lbs. What force is required for Rafaela to keep the sled from sliding down the hill?



It helps to draw a picture first!

To find the weight in pounds Rafaela will have to pull to keep Juan from sledding, we must project vector *f* onto vector *v* (vectors labeled in green).

Therefore, we are finding the magnitude of the segment in yellow!!!

What we know so far:

The magnitude of vector is 140

Vector is (because its direction is going directly downwards, vertically, with a force of 140 lbs)

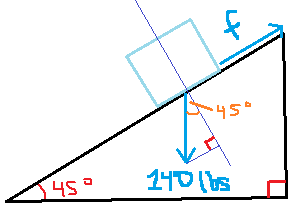
The angle labeled in orange is because the angle of the hill originally given to us (in blue) is . Therefore, the opposite interior angles theorem allows us to derive from this given that the orange angle is in fact also .

Since vector 's direction angle is , vector in component form is written as (this is derived from the Unit Circle!) This also means vector ’s magnitude is 1.

Now we can just plug values to the vector projection formula:

Now, find the magnitude of .

Rafaela would have to pull 99 pounds if she wishes to keep Juan from sledding.

That process was pretty complicated, BUT there is an alternate method you can use to solve these problems: **Mr. Kanuga's Method.** He suggests drawing a diagram like the one below, and using the formula Force .

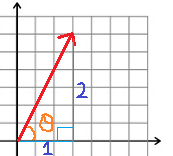
Force =

Vectors can also be applied in finding **work**. Work is measured in foot-lbs and given as

with being the angle between the force vector and the displacement vector (displacement vector: the shortest distance from the initial to the final position of a point).

**Example 3** *("example 7" on page 5 of 6.2 Dot Product of Vectors Notes)***:**

Find the work done by a force of 10 pounds acting in the direction in moving an object 3 feet.

We know F = 10 lbs and D = 3 ft, so we are looking for .

We can find by taking inverse tangent of :

Now plug all the variables into the work formula:

6.3 Parametric Equations and Motion

In a ***parametric equation***, x and y are functions of a third variable, also called the parameter, represented by t

We are used to seeing equations in ***Rectangular/Cartesian Form***, , but a ***parametric equation*** is comprised of two parts: and

The graph of the parametric equation is restricted it the *parametric interval*, which is

Thus, the***initial point***on a parametric graph is (g(a), f(a)) and the ***terminal point***is (g(b), f(b))

**Example 1** *(found on first page of 6.3 Parametric Equations and Motion guided class notes)***:**

|  |  |  |
| --- | --- | --- |
| t | x | y |
| -3 | -2 | 3 |
| -2 | -1 | 0 |
| -1 | 0 | -1 |
| 0 | 1 | 0 |
| 1 | 2 | 3 |
| 2 | 3 | 8 |



Terminal point

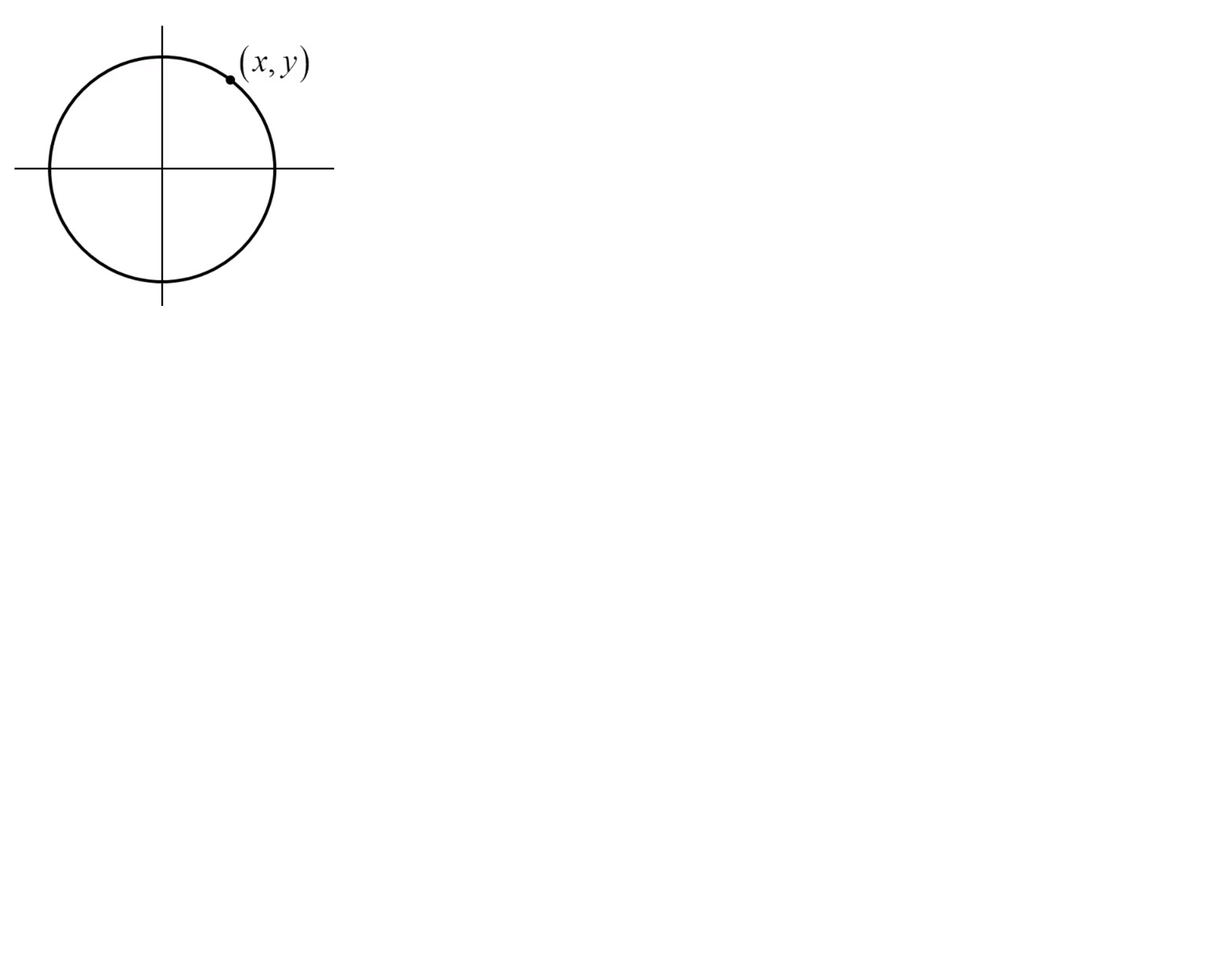
(3, 8)

10

10

Initial point

(-2, 3)

**The Unit Circle**

**Parametric Form:**

**Cartesian/ Rectangular Form:**

In order to convert an equation from parametric form to rectangular form, you must *eliminate the parameter*

**Example 2** *(number 1 in the 6.3 Parametric Equations and Motion guided class notes)***:**

**Eliminate the parameter when:**

Domain:

**Example 3** *(number 2 on Quiz 6.3)***:**

**Eliminate the parameter and state the domain or range restriction if one exists. Describe the graph of the function.**

As we can see based off the Cartesian formula for this equation, the shape of this graph will resemble a circle. However, the restrictions given to t translate to domain and range restrictions that we must take into account when finding our new graph. Finding the domain restrictions were easy; all you have to do is plug in the lowest and highest possible t values into the equation in order to find the lowest and highest possible values of x. However, in this situation, finding the range requires a bit more intuition. The y will hit its lowest possible value twice, once when the lowest possible t values is plugged in, and again when the highest possible t value is plugged in. Therefore, in order to find the highest point this graph will reach in terms of y, we have to plug in the t value that is the average of the highest and lowest t value.

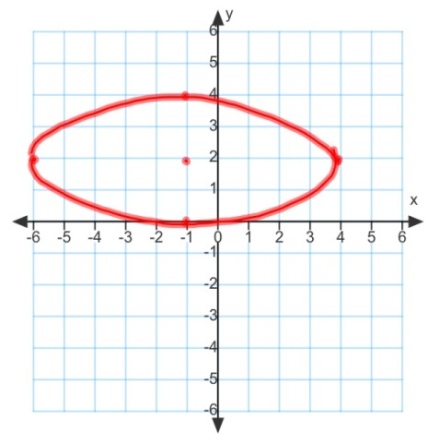
D:

The graph is semi circle (laying above the x axis) with a center at (0,0) and a radius of 3

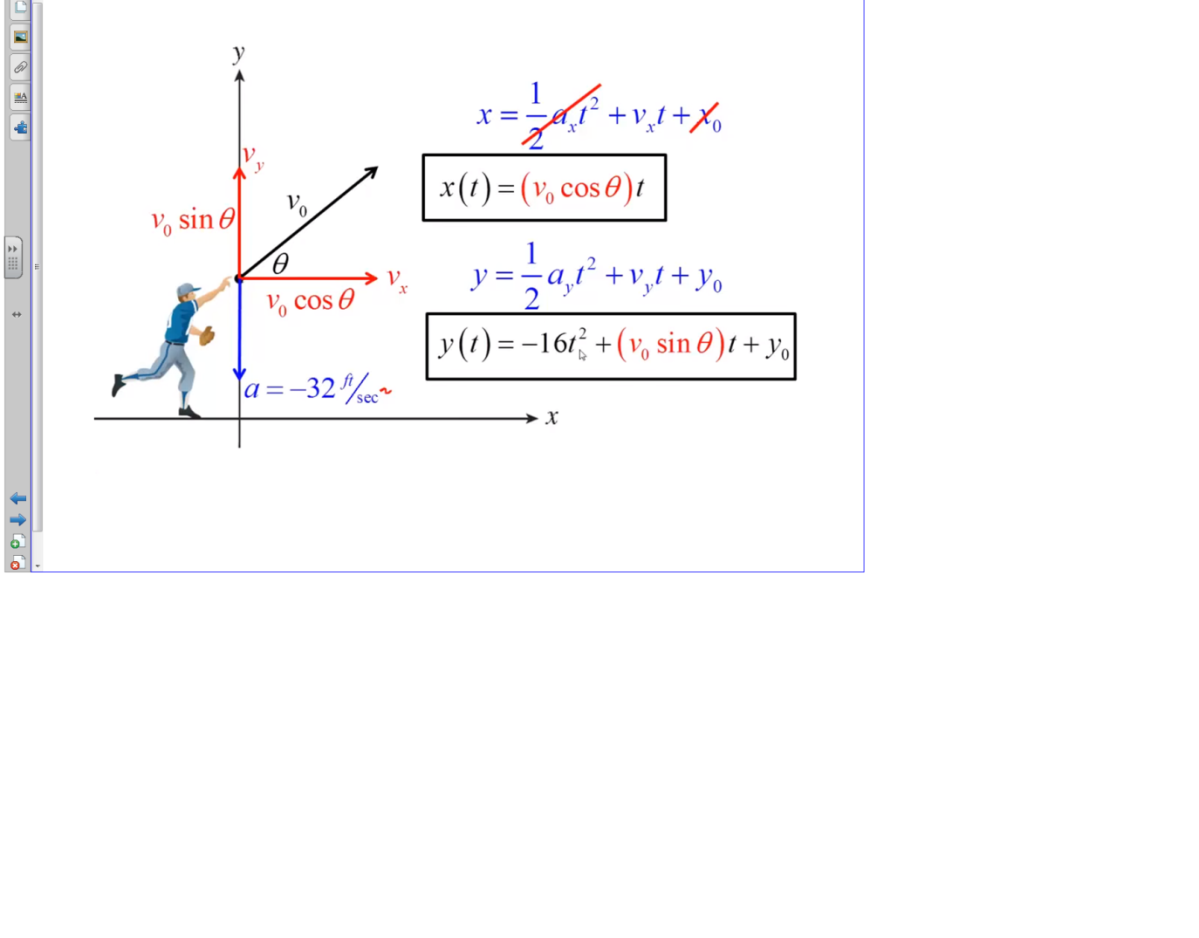
R:

**Example 4** *(number 5 in the 6.3 Parametric Equations and Motion guided class notes)***:**

**Eliminate the parameter and graph.**



In application problems using parametric, you will often see problems involving parabolic and/or sinusoidal motion. *Parabolic motion* can be observed in problems where a projectile is being thrown (i.e. a baseball, an arrow, yard darts, etc.). *Sinusoidal motion* is commonly observed in problems dealing with circular motion (i.e. Ferris wheels, a dog on a leash/tether (running around in a circle), merry-go-rounds).

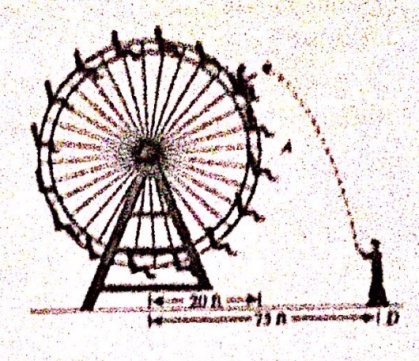
****

**Using parametric equations to find the distance a projectile travels:**

x(t) gives you the horizontal distance in relation to time and y(t) gives you the height in relation to time

***Parabolic Motion***

**Example 5** *(number 1 from the 6.3 Parametric Application Notes)*

A 20 ft Ferris wheel turns counter clockwise one revolution every 12 seconds. Eric stands at point D, 75 ft from the base of the wheel. At the instant Jane is at point A, Eric throws the ball at the Ferris wheel, releasing it from the same height as the bottom of the wheel. If the ball's initial speed is 60 ft/sec and it is released at an angle of 60˚ with ,the horizontal, does Jane have a chance to catch the ball? Follow the steps below to obtain the answer.

**a)** What are the parametric equations and , for Jane's path, in terms of time, ? Think of Jane as a point moving around the outer edge of a circle (the Ferris wheel).

***Sinusoidal Motion***

**b)** What are the parametric equations and , for the path of the ball, in terms of time, ? Think of the ball as a projectile being released at an angle.

**c)** Graph the two paths simultaneously and determine if Jane and the ball arrive at the point of intersection of the two paths at the same time.

First, make sure your calculator is in *radian mode*. Then, graph equations and (plugging them into the calculator as and ) and calculate the intersection point. You will end up with the approximate point . This means that when , . Therefore, Jane and the ball will arrive at the point of intersection after about 2.2 seconds.

**d)** Find a formula for the distance between Jane and the ball at any time .

**e)** Use the graph of the parametric equations , , to estimate the minimum distance between Jane and the ball and when it occurs. Do you think Jane has a chance to catch the ball? Why or why not?

\*\*\*Graph the parametric equations , in the calculator. Based on your answer for section c of this question, you know the minimum occurs at about . Therefore, use the Trace function on your calculator to find the point that occurs when . The value is the distance between the ball and Jane.

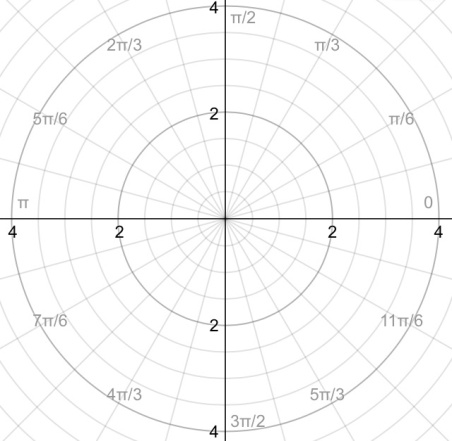
Jane and the ball will be closest to each other at about 2.2 seconds. At this time, they are about 1.642 ft away from each other, so Jane can in fact catch the ball.

6.4 Polar Coordinates

While **rectangular coordinates** are written in the format , **polar coordinates** are written in the format , where r is the radius (distance from pole) and is the angle of rotation from positive x axis

**Example 1** *(on page 1 of 6.4 Polar Coordinates Notes)***:**

Plot the polar point



Polar coordinates are not unique, all other points found by using:

or \*Note: π, NOT πn\*

Coordinate Conversion

Polar to Rectangular:

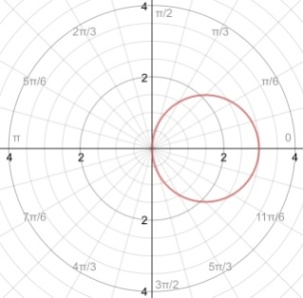
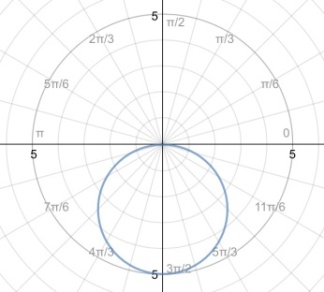
Rectangular to Polar:

\*You can also use these conversion factors to convert polar equations to rectangular equations and vice versa\*

6.5 Graphs of Polar Equations

Identifying Polar Graphs

**Circle** *(6a and 7b from 6.5 Graphs of Polar Equations Application)***:**

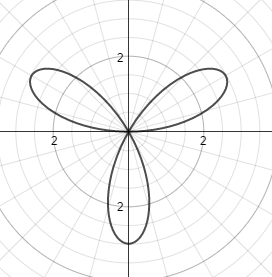
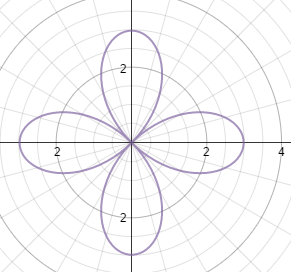
**Equation for a Rose Curve:** AND

When is odd,

When is even,

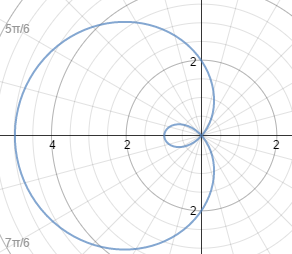
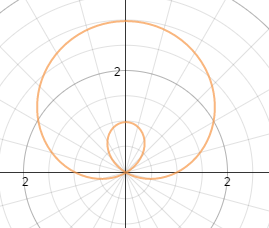
When is used, the first petal starts at

When is used, the first petal starts on the polar axis (think x-axis)

**Rose Curve** *(10a and 10b from 6.5 Graphs of Polar Equations Application)***:**   
 

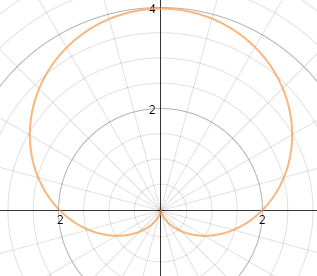
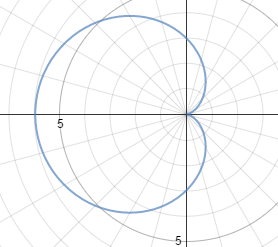
**Equation for a limacon**: AND

**Limacon with a Loop** *(page 5 of 6.5 Graphs of Polar Equations Application)***:**

\*occurs when   


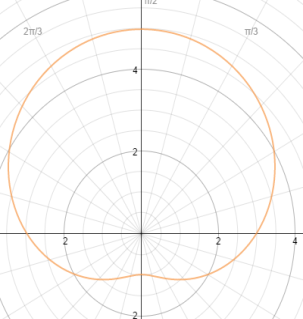
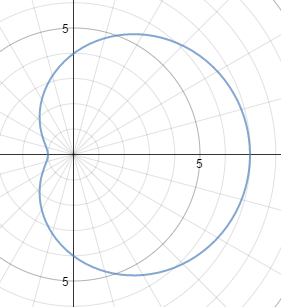
**Cardioid Limacon** *(page* *5 of 6.5 Graphs of Polar Equations Application)***:**

\*occurs when

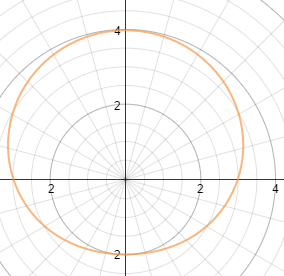
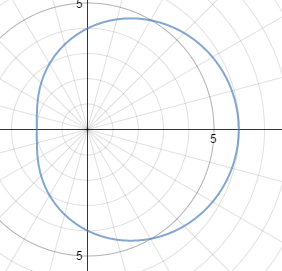
 

**Limacon with a dimple** *(page 6 of 6.5 Graphs of Polar Equations Application)***:**

\*occurs when

**Convex Limacon** *(page 6 of 6.5 Graphs of Polar Equations Application)***:**

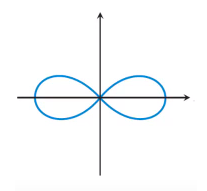
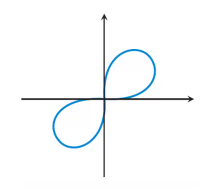
\*occurs when   
 

In limacons using , the x-intercepts are and the y-intercepts are the sum/difference values of and (depending on the type of graph).

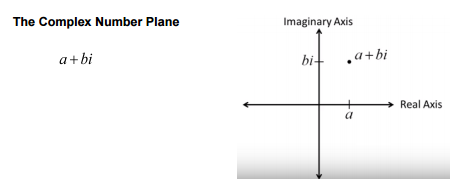
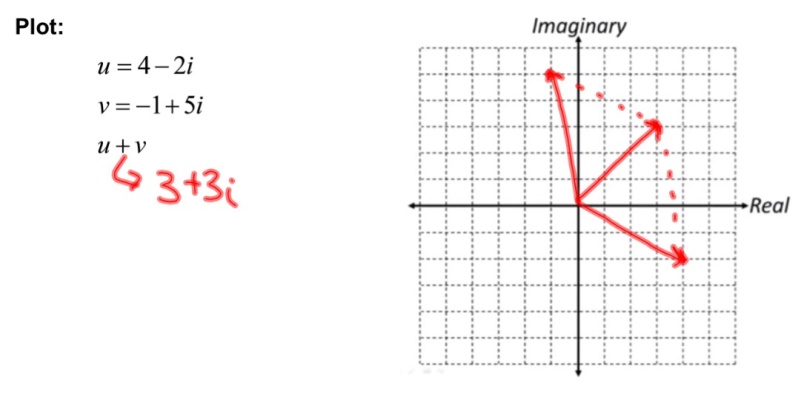
In limacons using , the y-intercepts are and the x-intercepts are the sum/difference values of and (depending on the type of graph).

**Equation for a lemniscate:** AND

**Lemniscate** *(page 6 of 6.5 Graphs of Polar Equations Application)***:**

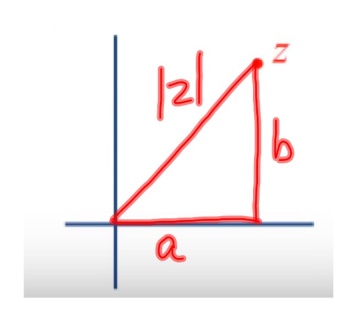


6.6 DeMoivre’s Theorem

The Complex Number Plane

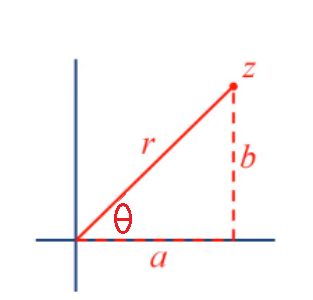
\*remember, complex numbers are written in the form

The Absolute Value of a Complex Number



Trigonometric Form of Complex Numbers

Standard form:  Trigonometric Form:



;

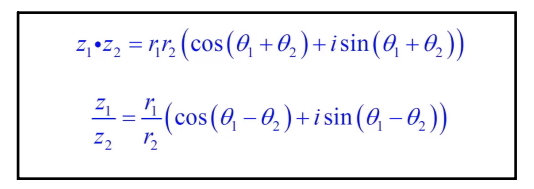
**Example 1** *(#1 on 6.6 HW Quiz)*

Write the trigonometric form of the complex number where the argument satisfies: .

**Example 2** *(#2 on 6.6 HW Quiz)*

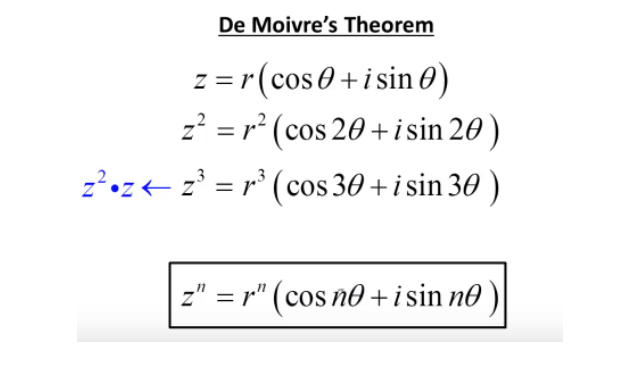
Write the complex number in standard form.

Product and Quotient of Complex Numbers



**Example 3** *("a" on page 3 of 6.6 Polar Coordinates Notes)***:**

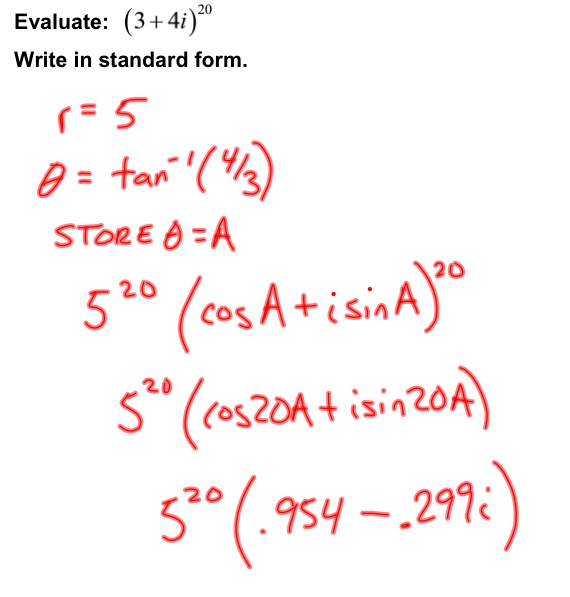
Find when and



DeMoivre’s Theorem allows us to efficiently find the values of complex numbers to certain powers or the value of roots of complex numbers.

\*Note: when finding the roots of complex numbers, the number of answers is equal to the degree of the root; i.e. When being asked for the fourth root of a complex number, there are 4 possible answers\*

**Example 4** *(on page 4 of 6.6 Polar Coordinates Notes)***:**

****

**Example 5** *(on page 5 of 6.6 Polar Coordinates Notes)***:**

