

Solve.

1.) At Philip's convenient store the total cost of one medium and one large soda is \$1.74. The large soda costs \$0.16 more than the medium soda. Find the cost of each soda.

$$\begin{aligned} M + L &= 1.74 & 2M + .16 &= 1.74 & M &= \$.79 \\ L &= M + .16 & 2M &= 1.58 & L &= \$.95 \\ & & M &= .79 & & \end{aligned}$$

2.) Hank can row a boat 1 mile upstream (against the current) in 24 minutes. He can row the same distance downstream in 13 minutes. If both the rowing and current speed are constant, find Hank's rowing speed and the speed of the current.

$$\begin{aligned} 24 \text{ min} &= .4 \text{ hr} & .4(R - C) &= 1 \\ 13 \text{ min} &= .22 \text{ hr} & .22(R + C) &= 1 \\ R &\approx 3.5 \text{ mph} & C &\approx 1 \text{ mph} \end{aligned}$$

~~Handwritten scribbles and a circled formula  $D = RT$ .~~

No Calculator

Perform the indicated operation(s).

$$3.) 2 \begin{bmatrix} -6 & -10 & 2 \\ 4 & -7 & -4 \end{bmatrix} - \begin{bmatrix} -1 & 5 & 13 \\ -3 & -6 & 19 \end{bmatrix} = \begin{bmatrix} -12 & -20 & 4 \\ 8 & -14 & -8 \end{bmatrix} - \begin{bmatrix} -1 & 5 & 13 \\ -3 & -6 & 19 \end{bmatrix} = \begin{bmatrix} -11 & -25 & -9 \\ 11 & -8 & -27 \end{bmatrix}$$

$$4.) 2 \begin{bmatrix} 7 & -7 \\ -1 & 3 \end{bmatrix} + 4 \begin{bmatrix} 2 & -4 \\ -5 & -6 \end{bmatrix} = \begin{bmatrix} 22 & -30 \\ -22 & -18 \end{bmatrix}$$

$$\begin{bmatrix} 14 & -14 \\ -2 & 6 \end{bmatrix} + \begin{bmatrix} 8 & -16 \\ -20 & -24 \end{bmatrix}$$

No Calculator

Solve the matrix equation.

$$5.) \begin{bmatrix} 3x & -2 \\ -1 & 8 \end{bmatrix} + \begin{bmatrix} -4 & 0 \\ -7 & -8 \end{bmatrix} = \begin{bmatrix} -16 & -2 \\ y & 0 \end{bmatrix}$$

$$\begin{aligned} 3x - 4 &= -16 & -1 + -7 &= y \\ 3x &= -12 & & \\ x &= -4 & y &= -8 \end{aligned}$$

$$6.) 2 \left( \begin{bmatrix} 3x & -1 \\ 8 & 5 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ -2 & -y \end{bmatrix} \right) = \begin{bmatrix} 26 & 0 \\ 12 & 8 \end{bmatrix}$$

$$\begin{aligned} 2(3x + 4) &= 26 & x &= 3 \\ 6x + 8 &= 26 & & \\ 6x &= 18 & \text{or } 2(5 - y) &= 8 \\ & & 10 - 2y &= 8 \end{aligned}$$

No calculator

Multiply.

$$7.) \begin{bmatrix} 2 & -8 & 1 \\ 0 & 5 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ 8 & -2 & -5 \end{bmatrix}$$

~~Handwritten:  $2 \times 3 \times 2 \times 3$  NOT possible.~~

$$8.) \begin{bmatrix} -\frac{1}{6} & \frac{1}{2} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 12 \\ 0 \\ -12 \end{bmatrix} = -2 + 0 - 4 = -6$$

No calculator

Solve.

$$9.) \begin{bmatrix} 4 & 1 & 3 \\ -2 & x & 1 \end{bmatrix} \begin{bmatrix} 9 & -2 \\ 2 & 1 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} y & 5 \\ -13 & 11 \end{bmatrix}$$

$$\begin{aligned} 36 + 2 - 3 &= y & -22 & \\ 35 &= y & 4 + x + 4 &= 11 \\ & & x &= 3 \end{aligned}$$

Use Cramer's Rule to solve.

10.) The atomic weights of three compounds are shown. Use a linear system and Cramer's Rule to find atomic weights of Carbon (C), Hydrogen (H), and Oxygen (O).

| Compound | Formula                                      | Atomic weight |
|----------|--|---------------|
| Methane  | CH <sub>4</sub>                              | 16            |
| Glycerol | C <sub>3</sub> H <sub>8</sub> O <sub>3</sub> | 92            |
| Water    | H <sub>2</sub> O                             | 18            |

$$\begin{vmatrix} 1 & 4 & 0 \\ 3 & 8 & 3 \\ 0 & 2 & 1 \end{vmatrix} = -10$$

$$D_x \begin{vmatrix} 16 & 4 & 0 \\ 92 & 8 & 3 \\ 18 & 2 & 1 \end{vmatrix} = -120$$

$$D_y \begin{vmatrix} 1 & 16 & 0 \\ 3 & 92 & 3 \\ 0 & 18 & 1 \end{vmatrix} = -10$$

$$D_z \begin{vmatrix} 1 & 4 & 16 \\ 3 & 8 & 92 \\ 0 & 2 & 18 \end{vmatrix} = -160$$

$$(12, 1, 16)$$

**No Calculator. Show all work.**  
Solve using inverse matrices.

11.)  $3x + y = 8$   
 $5x + 2y = 11$

$$\det = 6 - 5 = 1$$

$$\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} 8 \\ 11 \end{bmatrix} = \begin{bmatrix} 16 - 11 \\ -40 + 33 \end{bmatrix}$$

$$\begin{bmatrix} 5 \\ -7 \end{bmatrix} \quad (5, -7)$$

12.)  $x - y - 3z = 9$   
 $5x + 2y + z = -30$  given:  $A^{-1} = \begin{bmatrix} 1 & 3 & 5 \\ -3 & -9 & -16 \\ 1 & 4 & 7 \end{bmatrix}$   
 $-3x - y = 4$

$$\begin{bmatrix} 1 & 3 & 5 \\ -3 & -9 & -16 \\ 1 & 4 & 7 \end{bmatrix} \begin{bmatrix} 9 \\ -30 \\ 4 \end{bmatrix} = \begin{bmatrix} 9 - 90 + 20 \\ -27 + 270 - 64 \\ 9 - 120 + 28 \end{bmatrix} = \begin{bmatrix} -61 \\ 179 \\ -83 \end{bmatrix}$$

$$= (-61, 179, -83)$$

**No calculator.**  
Solve using Gaussian elimination.

$x + 2y - 6z = 23$   
13.)  $x + 3y + z = 4$   
 $2x + 5y - 4z = 24$   
 $x + 4 + 18 = 23$

$$\begin{array}{r} -x + 2y + 6z = -23 \\ x + 3y + z = 4 \\ \hline y + 7z = -19 \end{array}$$

$$\begin{array}{r} -2x - 6y - 2z = -8 \\ + 2x + 5y - 4z = 24 \\ \hline -y - 6z = 16 \end{array}$$

$$\begin{array}{r} y + 7z = -19 \\ + -y - 6z = 16 \\ \hline z = -3 \end{array}$$

$$z = -3$$

$$y = 2$$

$$x = 1$$

$$(1, 2, -3)$$

Solve using reduced row echelon form.

$$x + 2y - z = 3$$

$$14.) -x + y + 3z = -5$$

$$3x + y + 2z = 4$$

Process will vary.

$$(2, 0, -1)$$

Find the partial fraction decomposition.

$$15.) \frac{3x-5}{2x^2-5x-3}$$

$$\begin{aligned} & \overset{-6x^2}{-6x} \overset{+1}{+1} \\ & 2x^2 - 6x + x - 3 \\ & 2x(x-3) + 1(x-3) \\ & (2x+1)(x-3) \end{aligned}$$

$$\frac{A}{2x+1} + \frac{B}{x-3}$$

$$\frac{13}{7(2x+1)} + \frac{4}{7(x-3)}$$

$$Ax - 3A + 2Bx + B$$

$$A + 2B = 3$$

$$-3A + B = -5$$

$$\begin{aligned} & \textcircled{A+2B=3} \\ & \textcircled{-3A+B=-5} \\ & \textcircled{A=13/7} \\ & \textcircled{B=4/7} \end{aligned}$$

$$A = 13/7 \quad B = 4/7$$

$$16.) \frac{x^2+4x+12}{x^3-2x^2+4x-8}$$

$$x^2(x-2) + 4(x-2)$$

$$(x^2+4)(x-2)$$

$$\frac{A}{x-2} + \frac{Bx+C}{x^2+4}$$

$$Ax^2 + 4A + (Bx+C)(x-2)$$

$$Ax^2 + 4A + Bx^2 + Cx - 2Bx - 2C$$

$$A+B=1$$

$$C-2B=4$$

$$4A-2C=12$$

$$A=3$$

$$B=-2$$

$$C=0$$

$$\frac{3}{x-2} + \frac{-2x}{x^2+4}$$

Find the partial fraction decomposition.

$$17.) \frac{x^3 - 4x^2 + 9x - 5}{(x^2 - 2x + 3)^2} = \frac{Ax + B}{x^2 - 2x + 3} + \frac{Cx + D}{(x^2 - 2x + 3)^2}$$

$$(Ax + B)(x^2 - 2x + 3) + Cx + D$$

$$Ax^3 - 2Ax^2 + 3Ax + Bx^2 - 2Bx + 3B + Cx + D$$

$$\begin{aligned} A &= 1 & A &= 1 \\ -2A + B &= -4 & B &= -2 \\ 3A - 2B + C &= 9 & C &= 2 \\ 3B + D &= -5 & D &= 1 \end{aligned}$$

$$\frac{x-2}{x^2-2x+3} + \frac{2x+1}{(x^2-2x+3)^2}$$

Divide then find the partial fraction decomposition.

$$18.) \frac{x^4 + 5x^3 + 16x^2 + 26x + 22}{x^3 + 3x^2 + 7x + 5} = x + 2 + \frac{3x^2 + 7x + 12}{x^3 + 3x^2 + 7x + 5}$$

$$\begin{array}{r} x + 2 \\ 3x^3 + 7x^2 + 5x \overline{) x^4 + 5x^3 + 16x^2 + 26x + 22} \\ \underline{- x^4 + 3x^3 + 7x^2 + 5x} \phantom{+ 22} \\ 2x^3 + 9x^2 + 21x + 22 \\ \underline{- 2x^3 + 6x^2 + 14x + 10} \\ 3x^2 + 7x + 12 \end{array}$$

$$\begin{array}{r} x^2 + 2x + 5 \\ x + 1 \overline{) x^3 + 3x^2 + 7x + 5} \\ \underline{- x^3 + x^2} \phantom{+ 5} \\ 2x^2 + 7x + 5 \\ \underline{- 2x^2 + 2x} \\ 5x + 5 \end{array}$$

$$(x+1)(x^2 + 2x + 5)$$

$$\frac{A}{x+1} + \frac{Bx+C}{x^2+2x+5}$$

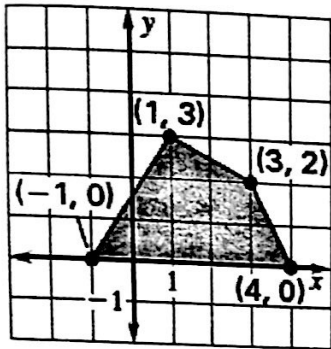
$$= x + 2 + \frac{2}{x+1} + \frac{x+2}{x^2+2x+5}$$

$$\begin{aligned} Ax^2 + 2Ax + 5A + (Bx + C)(x + 1) \\ Ax^2 + 2Ax + 5A + Bx^2 + Cx + Bx + C \end{aligned}$$

$$\begin{aligned} A + B &= 3 & A &= 2 \\ 2A + B + C &= 7 & B &= 1 \\ 5A + C &= 12 & C &= 2 \end{aligned}$$

19.) Find the minimum and maximum values of the objective function for the given feasible region.

Objective Function:  $C = 3x + 4y$



$(1, 3) = 15$   
 $(3, 2) = 17$   
 $(4, 0) = 12$   
 $(-1, 0) = -3$

Max 17 @ (3, 2)  
 Min -3 @ (-1, 0)

20.) Graph the inequalities. Then find the max and min values.

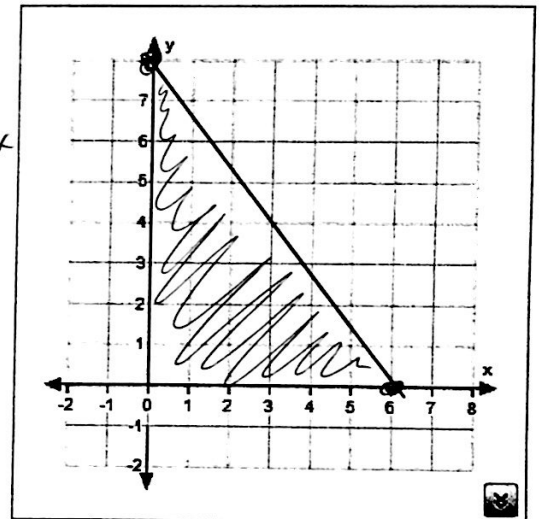
$C = 7x + 4y$

$x \geq 0$

$y \geq 0$

$4x + 3y \leq 24$

$(0, 8) = 32$   
 $(6, 0) = 42$  Max  
 $(0, 0) = 0$  Min



Solve using linear programming.

21.) A seafood restaurant owner orders at least 50 fish. He cannot use more than 30 amberjack or more than 35 flounder. Amberjack costs \$4 each and flounder costs \$3 each. How many of each fish should be used to minimize his cost?

$x + y \geq 50$

$x \leq 30$

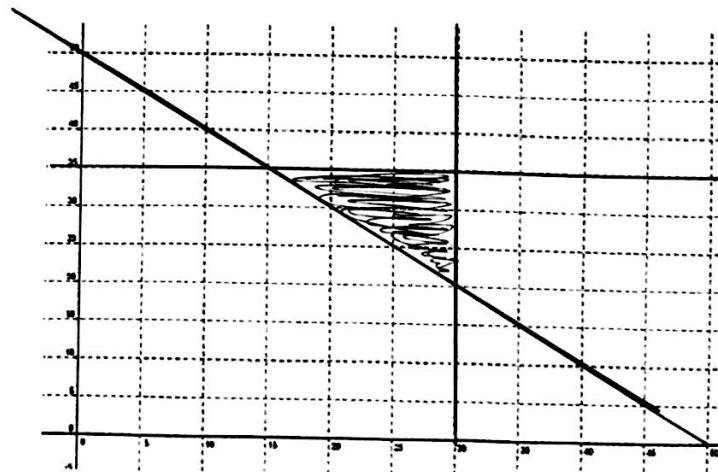
$y \leq 35$

$4x + 3y = C$

$(15, 35)$   
 165

$(30, 35)$   
 225

$(30, 20)$   
 180



**Solve using linear programming.**

22.) A shoe manufacturer makes outdoor and indoor soccer shoes. There is a two-step manufacturing process for both kinds of shoes. Each pair of outdoor shoes requires 2 hours of processing in step one and 1 hour in step two. Indoor shoes require 1 hour of processing in step one, and 3 hours of processing in step two. The company has only 40 hours of labor available for step one and 60 hours available for step two. If outdoor shoes make a profit of \$20 per pair and indoor shoes make a profit of \$15 per pair, how many pairs of each shoe should be made to maximize profit?

|         | Step 1 | Step 2 |
|---------|--------|--------|
| Outdoor | 2      | 1      |
| Indoor  | 1      | 3      |

$$2x + y \leq 40$$

$$x + 3y \leq 60$$

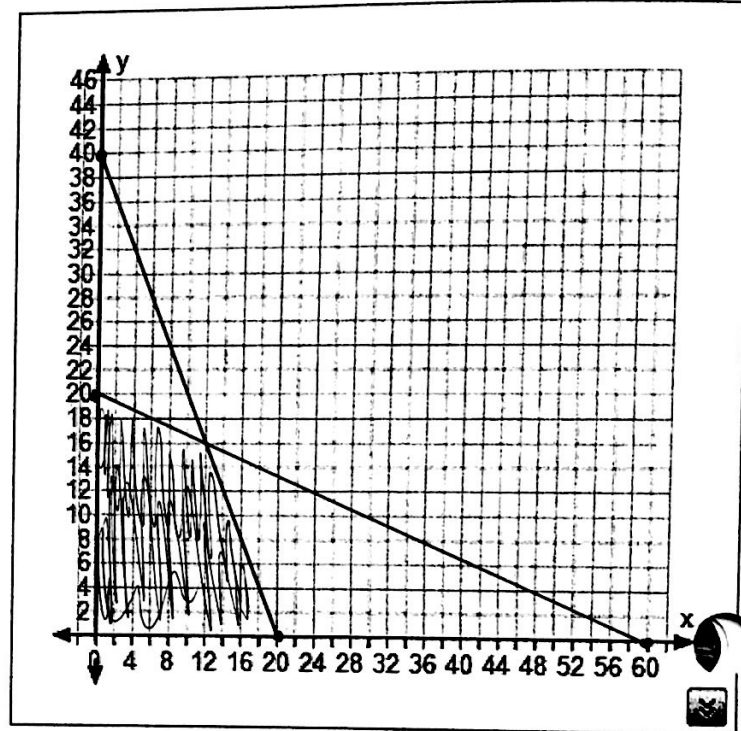
$$x \geq 0$$

$$y \geq 0$$

$$20x + 15y = P$$

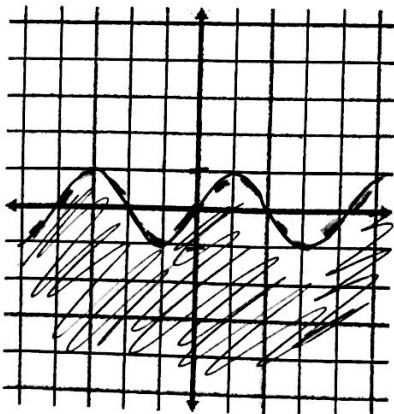
- $(0, 20) = 300$
- $(12, 16) = 480$  ✓
- $(20, 0) = 400$
- $(0, 0) = 0$

12 outdoor  
16 indoor

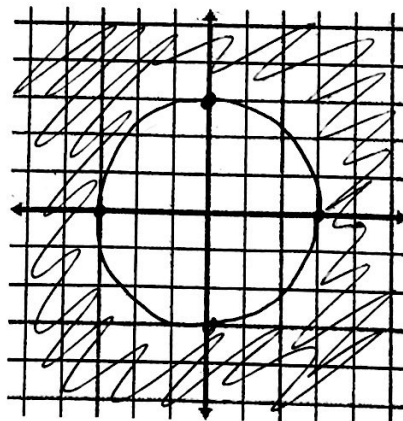


**Sketch each of the following:**

23.)  $y < \sin x$



24.)  $x^2 + y^2 \geq 9$



25.)  $y \geq 2^x$

