PreCalculus
Final Exam Review

Name KEY
Date ______ **Block** ______

Chapter 5 (Sections 5.3 – 5.5)

1. Find all solutions for the variable in the interval $[0, 2\pi)$.

a.) $2\sin^2 x + 3\cos x - 3 = 0$

$2(1-\cos^2 x) + 3\cos x - 3 = 0$

$2 - 2\cos^2 x + 3\cos x - 3 = 0$

$2\cos^2 x - 3\cos x + 1 = 0$

$(2\cos x - 1)(\cos x - 1) = 0$

$\cos x = 1/2$

$x = \pi/3, 5\pi/3$

$\cos x = 1$

$x = 0$

c.) $3\tan^3 x - \tan x = 0$

$\tan x (3\tan^2 x - 1) = 0$

$\tan x = 0$

$x = 0, \pi$

$\tan^2 x = 1/3$

$\tan x = \pm \sqrt{3}/3$

$x = \pi/6, 5\pi/6, 7\pi/6, 11\pi/6$

b.) $2\cos^2 x = \cos x$

$2\cos^2 x - \cos x = 0$

$\cos x (2\cos x - 1) = 0$

$\cos x = 0$

$x = \pi/2, 3\pi/2$

$\cos x = 1/2$

$x = \pi/3, 5\pi/3$

d.) $\sin 2x - \cos x = 0$

$2\sin x\cos x - \cos x = 0$

$\cos x (2\sin x - 1) = 0$

$\cos x = 0$

$x = \pi/2, 3\pi/2$

$\sin x = 1/2$

$x = \pi/6, 5\pi/6$

2. Use half-angle formulas to find the exact value.

a.) $\cos\left(\frac{\pi}{8}\right)$ $\pi/4$ $= \sqrt{\frac{1+\cos\pi/4}{2}} = \sqrt{\frac{1+\sqrt{2}/2}{2}} = \sqrt{\frac{\frac{2+\sqrt{2}}{2}}{2}} = \sqrt{\frac{2+\sqrt{2}}{2}\cdot\frac{1}{2}} = \sqrt{\frac{2+\sqrt{2}}{4}}$

$= \frac{\sqrt{2+\sqrt{2}}}{2}$

b.) $\tan\left(\frac{3\pi}{8}\right)$ $3\pi/4$ $= \frac{1-\cos\frac{3\pi}{4}}{\sin 3\pi/4} = \frac{1-(-\sqrt{2}/2)}{\sqrt{2}/2} = \frac{2+\sqrt{2}}{\cancel{2}}\cdot\frac{2}{\sqrt{2}} = \frac{2+\sqrt{2}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{\cancel{2}\sqrt{2}+\cancel{2}}{\cancel{2}}$

$= \sqrt{2}+1$

c.) $\sin\left(\frac{\pi}{12}\right)$ $\pi/6$ $= \sqrt{\frac{1-\cos\pi/6}{2}} = \sqrt{\frac{1-\sqrt{3}/2}{2}} = \sqrt{\frac{\frac{2-\sqrt{3}}{2}}{2}} = \sqrt{\frac{2-\sqrt{3}}{2}\cdot\frac{1}{2}} = \sqrt{\frac{2-\sqrt{3}}{4}}$

$= \frac{\sqrt{2-\sqrt{3}}}{2}$

3. Write the expression as the sine, cosine, or tangent of an angle.

a.) $\cos 60^\circ \cos 10^\circ - \sin 60^\circ \sin 10^\circ$ $= \cos(70)^\circ$

b.) $\frac{\tan 152^\circ - \tan 47^\circ}{1 + \tan 152^\circ \tan 47^\circ}$ $= \tan(105)^\circ$

c.) $\sin\frac{4\pi}{9}\cos\frac{\pi}{8} + \cos\frac{4\pi}{9}\sin\frac{\pi}{8}$ $\sin\left(\frac{4\pi}{9}+\frac{\pi}{8}\right) = \sin\left(\frac{32\pi}{72}+\frac{9\pi}{72}\right)$

$= \sin\left(\frac{41\pi}{72}\right)$

4. Verify each identity.

a.) $\cos\left(\theta+\frac{\pi}{2}\right)-\cos\left(\theta-\frac{\pi}{2}\right)=-2\sin\theta$

$= \cancel{\cos\theta\cos\frac{\pi}{2}} - \sin\theta\sin\pi/2 - (\cancel{\cos\theta\cos\pi/2} + \sin\theta\sin\pi/2)$

$= -\sin\theta - \sin\theta = -2\sin\theta$

b.) $\sec 2x = \dfrac{\sec^2 x}{2-\sec^2 x}$ $\dfrac{\frac{1}{\cos^2 x}}{2-\frac{1}{\cos^2 x}} = \dfrac{1}{\cancel{\cos^2 x}} \cdot \dfrac{\cancel{\cos^2 x}}{2\cos^2 x - 1} = \dfrac{1}{2\cos^2 x - 1} = \dfrac{1}{\cos 2x} = \sec 2x$

c.) $\dfrac{\cos x+\cos 3x}{\sin 3x-\sin x}=\cot x$

$\dfrac{2\cos\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}{2\cos\left(\frac{3x+x}{2}\right)\sin\left(\frac{3x-x}{2}\right)} = \dfrac{2\cancel{\cos(2x)}\cos(-x)}{2\cancel{\cos(2x)}\sin(x)} = \dfrac{\cos x}{\sin x} = \cot x$

d.) $(\sin x+\cos x)^2 = 1+\sin 2x$

$\sin^2 x + 2\sin x\cos x + \cos^2 x$

$1 + 2\sin x\cos x = 1 + \sin 2x$

Chapter 6

In 5 - 7, solve the triangle for all angles and sides. If two solutions exist, find both.

5. $c = 13,\ b = 8,\ B = 31°$ Sines

$\dfrac{a}{\sin A} = \dfrac{8}{\sin 31} = \dfrac{13}{\sin C}$

$C = 56.8$ $\quad C = 123.2$

$A = 92.2$ $\quad A = 25.8$

$a = 15.5$ $\quad a = 6.8$

6. $A = 55°,\ b = 12,\ c = 7$ Cosines

$\cancel{\dfrac{a}{\sin 55} = \dfrac{12}{\sin 13} = \dfrac{7}{\sin C}}$

$a^2 = 12^2 + 7^2 - 2(12)(7)\cos 55$

$a = 9.83$

$\dfrac{9.83}{\sin 55} = \dfrac{12}{\sin B}$

$B = 89.66$

$C = 35.34$

7. $A = 33°,\ B = 70°,\ b = 7$ Sines

$\dfrac{a}{\sin 33} = \dfrac{b}{\sin 70} = \dfrac{c}{\sin C}$

$C = 77$

$a = 4.1$

$c = 7.3$

In 8 - 9, find the area of the triangle to the nearest tenth.

8. $A = 52°,\ b = 14$ m, $c = 21$ m $\quad A = \frac{1}{2} bc\sin A = \frac{1}{2}(14)(21)\sin 52 = 115.8\,m^2$

9. $a = 7$ cm, $b = 8$ cm, $c = 9$ cm

$s = \dfrac{7+8+9}{2} = 12$ $\quad A = \sqrt{12(12-7)(12-8)(12-9)}$

$A = 26.8\,cm^2$

Chapter 9

10 - 12, write the equation in standard form and then classify the graph as a parabola, circle, ellipse, or hyperbola.

10. $x^2 + y^2 - 6x + 4y + 9 = 0$

$x^2 - 6x + 9 + y^2 + 4y + 4 = -9 + 9 + 4$

$(x-3)^2 + (y+2)^2 = 4$

Circle

11. $x^2 - 6x + 16y + 21 = -4y^2$

$x^2 - 6x + 9 + 4(y^2 + 4y + 4) = -21 + 9 + 16$

$\frac{(x-3)^2}{4} + \frac{4(y+2)^2}{4} = \frac{4}{4}$

$\frac{(x-3)^2}{4} + \frac{(y+2)^2}{1} = 1$

Ellipse

12. $y^2 - 6y - 4x + 21 = 0$

$y^2 - 6y + 9 = 4x - 21 + 9$

-12

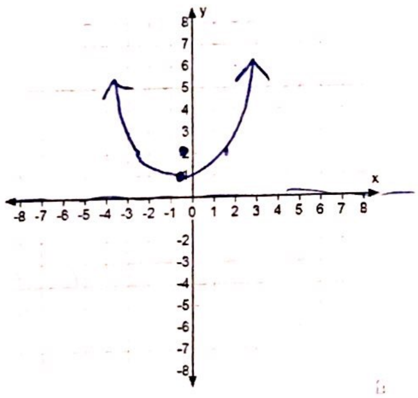
$(y-3)^2 = 4x$

$(y-3)^2 = 4(x - 3)$

parabola

13. Find the vertex, axis of symmetry, focus, and directrix of the parabola and sketch its graph.

$$\left(x + \frac{1}{2}\right)^2 = 4(y-1)$$



$4p = 4$

$p = 1$

up

$V(-1/2, 1)$

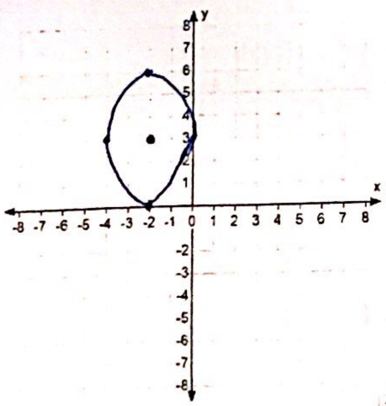
A.O.S: $x = -1/2$

Focus: $(-1/2, 2)$

directrix: $y = 0$

14. Identify the conic as a circle or ellipse. Then find the center and radius (if it's a circle); find the center, vertices, co-vertices, and foci (if it's an ellipse). Sketch its graph.

$$9x^2 + 4y^2 + 36x - 24y + 36 = 0$$



$9(x^2 + 4x + 4) + 4(y^2 - 6y + 9) = \cancel{-36 + 36}$ $+36$

$\frac{9(x+2)^2}{36} + \frac{4(y-3)^2}{36} = \frac{36}{36}$

Ellipse

$\frac{(x+2)^2}{4} + \frac{(y-3)^2}{9} = 1$

$C(-2, 3)$

Vertices: $(-2, 6)$, $(-2, 0)$

Co-Vertices: $(0, 3)$, $(-4, 3)$

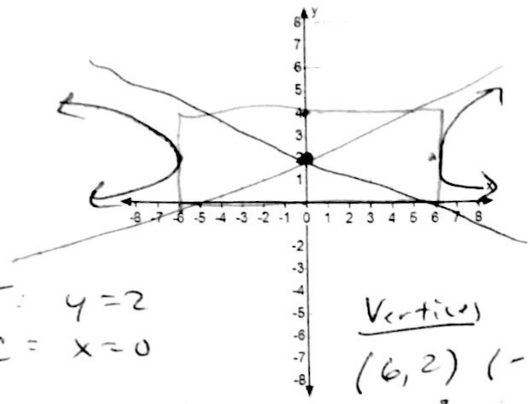
$c^2 = 9 - 4 = 5$

$c = \sqrt{5}$

Foci $= (-2, 3 \pm \sqrt{5})$

15. Find the center, vertices, foci, lines containing the axes, and the equations of the asymptotes of the hyperbola, and then sketch its graph.

$$x^2 - 9y^2 + 36y - 72 = 0$$



$C(0, 2)$

$x^2 - 9(y^2 - 4y + 4) = 72 - 36$

Asym

$y = \pm \frac{1}{3}(x) + 2$

$$\frac{x^2}{36} - \frac{9(y-2)^2}{36} = \frac{36}{36}$$

T: $y = 2$

C: $x = 0$

Vertices

$(6, 2)$ $(-6, 2)$

$$\boxed{\frac{x^2}{36} - \frac{(y-2)^2}{4} = 1}$$

$(\pm 2\sqrt{10}, 2) \leftarrow$ Foci: $c^2 = 36 + 4 = 40$

$c = \sqrt{40} = 2\sqrt{10}$

16. Write the equation of a circle that has a center at (-1, 3) and passes through the point (-5, 6).

$(x+1)^2 + (y-3)^2 = 25$

$4^2 + 3^2 = r^2$

$r = 5$

Sequences and Series

$-4 - 10n + 10$

In 17 - 18, write the explicit formula for each sequence.

17. ~~-3, -6, -12, -24, -48, …~~

$a_n = (-3)(2)^{n-1}$

18. ~~-4,~~ -14, -24, -34, -44, …

$a_n = -4 - 10(n-1) = 6 - 10n$

19. Find "n" if you know that $S_n = 59{,}046$ in the series 6 + 18 + 54 + 162…

$59046 = 6\left(\frac{1-3^n}{1-3}\right)$

$n = 9$

20. Evaluate $\sum_{n=0}^{5}(20 - n^2)$

$20 + 19 + 16 + 11 + 4 + (-5) = 65$

21. Evaluate $\binom{12}{3}$

$\frac{12!}{3!\,9!} = 220$

In 22 - 23, find each term described.

22. 2nd term in expansion of $(x + 3)^3$

$\binom{3}{1} x^2 \cdot 3$

$9x^2$

23. 4th term in expansion of $(3u - 1)^4$

$\binom{4}{3}(3u)(-1)^3$

$4(3u)(-1)$

$-12u$

In 24 - 25 expand completely.

24. $(2y - x)^4$

$16y^4 - 32y^3x + 24y^2x^2 - 8yx^3 + x^4$

25. $(2y + 3x)^3$

$8y^3 + 36y^2x + 54yx^2 + 27x^3$