Honors Pre-Calc
Final Exam Review

Name KEY
Date _____ **Period** ____

(6.1 – 6.2 Extra Practice)

1.) An airplane is flying bearing 148°, with an air speed of 875 kph. Because of the wind, its ground speed and direction are 800 kph and bearing 140°, respectively. Find the direction and speed of the wind.

plane: 875 kph at -58° $\langle 875\cos -58, 875\sin -58\rangle$ $\langle 50.55, 129.21\rangle$

ground: 800 kph at -50° $+\langle w\cos\theta, w\sin\theta\rangle$

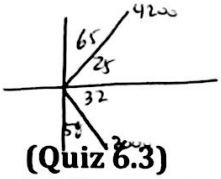
$\langle 800\cos -50, 800\sin -50\rangle$

$w = 138.74$ kph at bearing 21.37

68.63 = 21.37 bearing

(6.1 – 6.2 Quiz)

2.) The magnitude and direction exerted by two tugboats towing a ship are 4200 lbs, N65°E, and 3000 lbs, S58°E, respectively. Find the magnitude and the direction angle of the resultant force. Round to the nearest pound and tenth of a degree.



$\langle 4200\cos 25 + 3000\cos 32, 4200\sin 25 + 3000\sin 32\rangle$

$\theta = 1.67°$

$\langle 6350.64, 185.24\rangle$

6353.3 bearing 88.3°

(Quiz 6.3)

3.) Hannah is playing a game on the boardwalk. If she can toss a Ping-Pong ball into a fish bowl, she wins the fish! Hanna is 20 feet from the center of the bowl and releases the ball 4 feet above the bowl. The fish bowl has a radius of 18 inches. (Yes this is a large fish bowl). Hannah throws the ball with an initial velocity of 25ft/sec at a 55° angle. Will Hannah win the fish?

a.) Write a set of parametric equations for the path of the Ping-Pong.

$x = 25\cos 55T$

$y = -16T^2 + 25\sin 55T + 4$

b.) Will Hannah win the fish? Justify your answer algebraically!

$20 = 25\cos 55T$ $T = 1.39$ sec $y = 1.43$ ft

$21.5 = 25\cos 55T$ $T = 1.499$ sec $y = -1.26$ ft.

Yes between center of bowl and end of bowl - height goes from + to − which means lands in bowl.

(Quiz 6.3)

4.) The center field fence in Yankee Stadium is 7 feet high and 408 feet from home plate. A baseball is hit 3 feet above the ground. It leaves the bat at an angle of 26° with the horizontal at a speed of 90 mph.

a) Write a set of parametric equations for the path of the baseball.

$\frac{90 \text{ miles}}{\text{hr}} \cdot \frac{1\text{hr}}{60\text{min}} \cdot \frac{1\text{m}}{60\text{sec}} = \frac{5280 \text{ ft}}{1 \text{ mile}}$

132 ft/sec

$x = 132\cos 26T$

$y = -16T^2 + 132\sin 26T + 3$

b) At what time does the ball reach a horizontal distance of 408 feet? (Show algebraic work).

$408 = 132\cos 26T$

$T = 3.43895$ seconds

c) What is the vertical height of the ball when the horizontal distance is 408 ft? (Show algebraic work).

$y(3.43895) = -16(\text{ANS})^2 + 132\sin 26(\text{ANS}) + 3$

$= 12.77$ feet

d) Is the hit a homerun?

Yes

(Quiz 6.4-6.5)

5.) Classify the polar graph, identify any symmetry and the location of the maximum r-value.

Polar Curve	Type	Symmetry	Max r-value point
a. $r = -3\cos\theta$	Circle	x-axis	$(3, \pi)$
b. $r = 4 - 3\sin\theta$	Dimple Limacon	y-axis	$(7, 3\pi/2)$
c. $r^2 = 25\sin 2\theta$	Lemniscate	origin	$(5, \frac{\pi}{4} + \pi n)$

6.) Write an equation for a Dimpled Limacon with a max r value at (-8,0) and y-ints at (0, -5) & (0, 5).

$$r = 5 - 3\cos\theta$$

(Chapter 6 Test)

7.) Convert the rectangular equation to a polar equation: $(x-3)^2 + y^2 = 9$

$x^2 - 6x + 9 + y^2 = 9$

$r^2 - 6r\cos\theta = 0$

$r(r - 6\cos\theta) = 0$

$r = 6\cos\theta$

8.) Convert the polar equation to a rectangular equation: $r^2 \sin 2\theta = 6$

$r^2 \cdot 2\sin\theta\cos\theta = 6$

$2r\sin\theta r\cos\theta = 6$

$r\sin\theta\, r\cos\theta = 3$

$xy = 3$

$y = 3/x$

(7.4 Partial Fraction Decomposition)

9.) Find the partial fraction decomposition of the following rational functions.

a.) $\dfrac{x+17}{2x^2+5x-3}$

$\dfrac{A}{2x-1} + \dfrac{B}{x+3}$

$Ax + 3A + 2Bx - B$

$\begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix} X = \begin{bmatrix} 1 \\ 17 \end{bmatrix}$

$\dfrac{5}{2x-1} + \dfrac{-2}{x+3}$

b.) $\dfrac{3x^3+6x-1}{(x^4+4x^2+4)}$

$\dfrac{Ax+B}{x^2+2} + \dfrac{Cx+D}{(x^2+2)^2}$

$(Ax+B)(x^2+2) + Cx + D$

$Ax^3 + Bx^2 + 2Ax + 2B + Cx + D$

$A = 3$

$B = 0$

$2A + 2B + C = 6$

$C = 0$

$D = -1$

$\dfrac{3x}{x^2+2} + \dfrac{-1}{(x^2+2)^2}$

(Chapter 7 Test)

10.) Lenhard Manufacturing has two factories that produce three grades of paper: low grade, medium grade, and high grade. It needs to supply 24 tons of low grade, 6 tons of medium grade, and 30 tons of high grade. Factory A produces 8 tons of low grade, 1 ton of medium grade, and 2 tons of high grade paper daily and costs $2000 a day to operate. Factory B produces 2 tons of low grade, 1 ton of medium grade, 8 tons of high grade paper daily and costs $4000 a day to operate. How many days should each factory operate to fill the orders at minimum cost? *Be sure to state all constraints. Find the cost values at each vertex and identify your final answer.

$C = 2000x + 4000y$

$8x + 2y \geq 24$

$x + y \geq 6$

$2x + 8y \geq 30$

$x \geq 0 \quad y \geq 0$

$(2,4) = 20000$

$(3,3) = 18000$

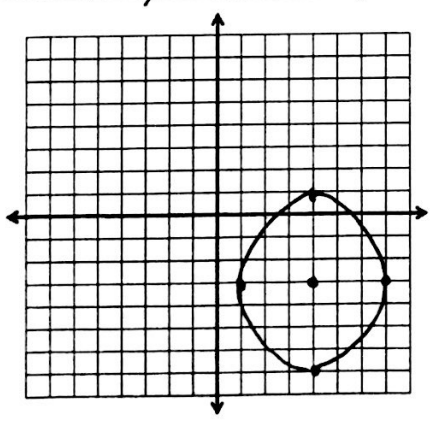
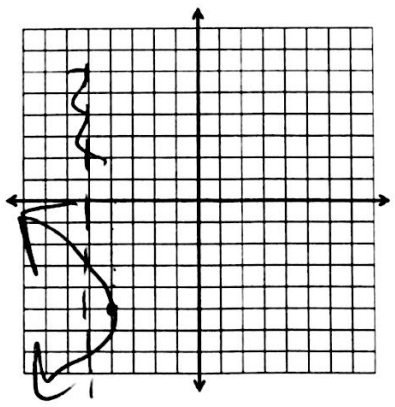
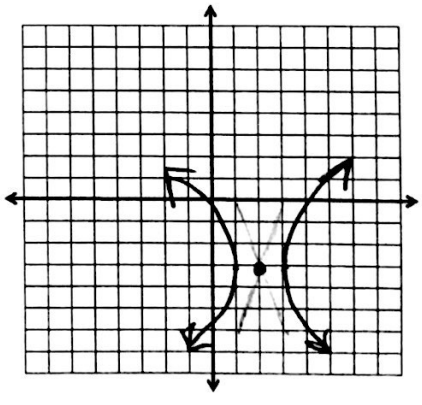
$(15,0) = 30000$

$(0,12) = 48000$

3 days each.

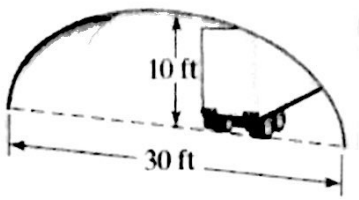
(8.1 – 8.3 Quiz)

Find the standard form of the equation. Classify and graph the conic. Be sure to identify key points.

11.) $16x^2 + 9y^2 - 128x + 54y + 193 = 0$	12.) $y^2 + 10y + 4x + 41 = 0$	13.) $9x^2 - y^2 - 36x - 6y + 18 = 0$
$\frac{(x-4)^2}{9} + \frac{(y+3)^2}{16} = 1$	$(y+5)^2 = -4(x+4)$	$(x-2)^2 - \frac{(y+3)^2}{9} = 1$
Classify: Ellipse	Classify: Parabola	Classify: Hyperbola
Center: $(4,-3)$	Vertex: $(-4,-5)$	Center: $(2,-3)$
Vertices: $(4,1)$ $(4,-7)$	Axis of Symmetry: $y=-5$	Transverse Axis Endpoints: $(3,-3)$ $(1,-3)$
Co-Vertices: $(7,-3)$ $(1,-3)$	Focus: $(-5,-5)$	Conjugate Axis Endpoints: $(2,0)$ $(2,-6)$
Foci: $(4, -3 \pm \sqrt{7})$	Directrix: $x=-3$	Asymptotes: $y = -3x + 3$, $y = 3x - 9$
		

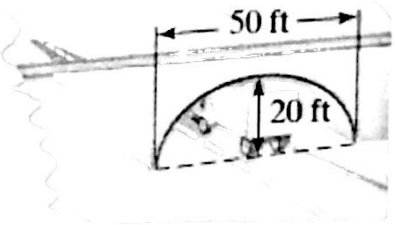
(8.1 - 8.3 Applications WS)

14.)Will a truck that is 14 feet wide carrying a load that reaches 9 feet above the ground clear the semielliptical arch on the one-way road that passes under the bridge shown in the figure?



NO $y = 8.84$

15.) A semielliptic archway has a height of 20 feet and a width of 50 feet, as shown in the figure. Can a truck 14 feet high and 10 feet wide drive under the archway without going into the other lane?



YES $y = 18.3$

(8.1 – 8.3 Applications Notes)

16.) A comet following a hyperbolic path about the Sun has a perihelion distance of 90 Gm. When the line from the comet to the Sun is perpendicular to the focal axis of the orbit, the comet is 281.25 Gm from the Sun. Calculate a, b, c, and e. What are the coordinates of the center of the Sun if we coordinatize space so that the hyperbola is given by $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$?

$a^2 + b^2 = c^2$

$a^2 + 281.25a = (a+90)^2$

$a^2 + 281.25a = a^2 + 180a + 8100$

$101.25a = 8100$

$a = 80$

$(170, 0)$ sun is focus

$c = 170$

$c - a = 90$

$\frac{b^2}{a} = 281.25$

$b^2 = 281.25a$

$b = 150$

$e = \frac{170}{90} = 2.125$

(8.4 Notes)

17.) Rotate the axes to eliminate the xy-term in the equation. Then write the equation in standard form. Sketch the graph showing both axes.

$2x^2 - 3xy - 2y^2 + 10 = 0$

$A = 2$

$B = -3$

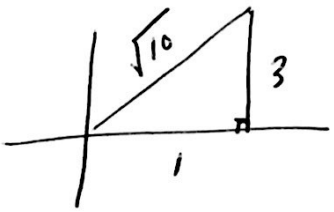
$C = -2$

$\cot 2\theta = \frac{A - C}{B} = \frac{-4}{3}$

$\cot 2\theta = \frac{\cot^2\theta - 1}{2\cot\theta} = \frac{-4}{3}$

$(3\cot\theta - 1)(\cot\theta + 3)$

$\cot\theta = \frac{1}{3} \quad \cot\theta = -3$



$x = \frac{1}{\sqrt{10}}(x' - 3y')$

$y = \frac{1}{\sqrt{10}}(3x' + y')$

$\frac{(x')^2}{4} - \frac{(y')^2}{4} = 1$

(8.4 Notes)

18.) Use the discriminant to classify the graph.

a.) $12x^2 - 6xy + 7y^2 - 45 = 0$ b.) $x^2 - 6xy - 5y^2 + 4x - 22 = 0$ c.) $36x^2 - 60xy + 25y^2 + 9y = 0$

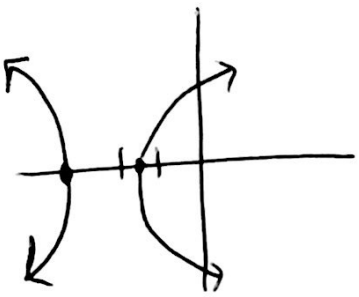
-300 56 0

Ellipse Hyperbola parabola

Circle

(8.5 HW Quiz)

19.) Find a polar equation for the hyperbola with a focus at the pole and the given polar coordinates as the endpoints of its transverse axis: $(-3, 0)$ *and* $(1.5, \pi)$.



$C(-2.25, 0)$

$c = 2.25$

$a = .75$

$e = \frac{2.25}{.75} = 3$

$e = 3$

$-3 = \frac{3k}{1-3}$

$6 = 3k$

$2 = k$

$r = \frac{6}{1 - 3\cos\theta}$

20.) Sketch the conic, and find the values of e, a, b, and c: $r = \frac{11}{6 - 5\sin\theta}$.

$r = \frac{11/6}{1 - 5/6\sin\theta}$

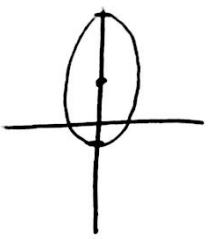
$e = 5/6$

$a = 6$

$c = 5$

$(11, \pi/2)$ $C(0,5)$

$(1, \frac{3\pi}{2})$



$a^2 = b^2 + c^2$

$36 = b^2 + 25$

$b = \sqrt{11}$

(Chapter 9 Quest)

Find the following:

a.) Identify whether the sequence is arithmetic, geometric, or neither.
b.) If arithmetic or geometric, identify the common difference or common ratio.
c.) If arithmetic or geometric, write a recursive rule.
d.) Write an explicit rule.
e.) Find a_{11}.

21.) 12, 15, 18, 21, 24, ... Arithmetic $cd = 3$

$a_n = a_{n-1} + 3$

$a_n = 9 + 3n$

$a_{11} = 42$

22.) $-3, \frac{-3}{4}, \frac{-3}{9}, \frac{-3}{16}, \ldots$ Neither

$a_n = -3/n^2$

$a_{11} = -3/121$

23.) 5, 11, 21, 35, 53, ... Neither / Quadratic

$a_n = 2n^2 + 3$

$a_{11} = 245$

24.) 3, -12, 48, -192, 768, ... Geometric $CR = -4$

$a_n = a_{n-1} \times (-4)$

$a_n = 3(-4)^{n-1}$ $a_n = 3145728$

25.) Evaluate the expression by hand (using the formula – show all work): $\binom{15}{11}$

$$\frac{15!}{11!4!} = 1365$$

26.) Find the coefficient of x^3y^2 term in the expansion of $(3x-2y)^5$.

$$\binom{5}{2}(3x)^3(-2y)^2 = 10 \cdot 27x^3(4y^2)$$

$$= 1080x^3y^2$$

27.) Fully expand and simplify the binomial: $(x+2y)^6$.

$$x^6 + 6x^5(2y) + 15x^4(2y)^2 + 20x^3(2y)^3 + 15x^2(2y)^4 + 6x(2y)^5 + (2y)^6$$

$$x^6 + 12x^5y + 60x^4y^2 + 160x^3y^3 + 240x^2y^4 + 192xy^5 + 64y^6$$

28.) Find the sum of the first 75 terms for the series: $4+11+18+25+\ldots$

$$S = \frac{75(4+522)}{2}$$

$$a_n = 7n-3$$

$$a_{75} = 522$$

$$S_{75} = 19725$$

29.) For the given series, $117+110+103+\ldots$, find which term gives the sum of 975.

$$975 = \frac{n(117+124-7n)}{2}$$

$$1950 = 241n - 7n^2$$

$$7n^2 - 241n + 1950 = 0$$

$$\frac{241 \pm \sqrt{3481}}{14}$$

$$n = 13$$

$$a_n = 124 - 7n$$

30.) Find "n" if you know that $S_n = 147620$ in the series $5+15+45+135+\ldots$

$$147620 = \frac{5(1-3^n)}{1-3}$$

$$-295240 = 5(1-3^n)$$

$$-59048 = 1-3^n$$

$$-59049 = -3^n$$

$$\log_3 59049 = n$$

$$n = 10$$

31.) Find: $\sum_{n=1}^{8}(5n^2+12n)$

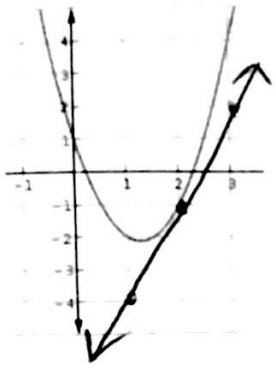
$$17 + 44 + 81 + 128 + 185 + 252 + 329 + 416$$

$$= 1452$$

(10.3 Derivatives WS)

32.) Find (a) the slope of the graph at the given point, (b) the equation of the tangent line to the graph at the point, (c) graph tangent line with the graph of the function.

$f(x) = 2x^2 - 5x + 1; x = 2$



$$\lim_{h \to 0} \frac{2(x+h)^2 + 5(x+h) + 1 - (2x^2 - 5x + 1)}{h}$$

$$\lim_{h \to 0} \quad 4x + 2h - 5$$

$(2, -1)$

$f'(x) = 4x - 5$

$f'(2) = 3$

(10.1 – 10.3 Review)

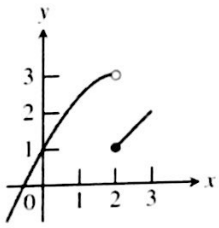
33.) Find each limit.

a.) $\lim_{x \to 2^-} f(x) =$ 3

b.) $\lim_{x \to 2^+} f(x) =$ 1

c.) $\lim_{x \to 2} f(x) =$ DNE

d.) $f(2) =$ 1



(10.2 Notes)

34.) Find the limit using the definition:

a.) $f(x) = x^2 - 2x$

b.) $f(x) = \sqrt{2x - 1}$

c.) $f(x) = \dfrac{1}{5 - x}$

(a) $f'(x) = 2x - 2$

(b) $f'(x) = \dfrac{1}{\sqrt{2x-1}}$

(c) $f'(x) = \dfrac{1}{(5-x)^2}$