PreCalculus Name KEY
Midterm Exam Review Date 1/7/18 Block____

Chapters 2/7

1. Find the following parts of the function: domain, x-intercept, y-intercept, vertical asymptote(s), horizontal asymptote(s), and/or slant asymptote(s). Domain can just show restrictions (also identify any holes), axis intercepts are points, and asymptotes are equations for x and y. State NONE if the value does not exist.

a.) $f(x)=\dfrac{x-3}{x^2-3x-4} = \dfrac{(x-3)}{(x-4)(x+1)}$

Domain: $x \neq 4, -1$

x-intercept(s): $(3, 0)$

y-intercept(s): $(0, 3/4)$

H.A.: $y = 0$

V.A.: $x = 4$ $x = -1$

S.A.: NONE

Hole(s): NONE

b.) $f(x)=\dfrac{x^2-1}{x^2-2x-3} = \dfrac{\cancel{(x+1)}(x-1)}{(x-3)\cancel{(x+1)}}$

Domain: $x \neq 3, -1$

x-intercept(s): $(1, 0)$

y-intercept(s): $(0, 1/3)$

H.A.: $y = 1$

V.A.: $x = 3$

S.A.: NONE

Hole(s): $(-1, 1/2)$

c.) $f(x)=\dfrac{x^2-x-2}{x-1}$ $\dfrac{(x-2)(x+1)}{x-1}$

Domain: $x \neq 1$

x-intercept(s): $(2, 0)$ $(-1, 0)$

y-intercept(s): $(0, 2)$

H.A.: NONE

V.A.: $x = 1$

S.A.: $y = x$

Hole(s): NONE

$$\begin{array}{r|l} & x \\ \hline x-1 & x^2 - x - 2 \\ - & x^2 - x \end{array}$$

d.) $f(x)=\dfrac{2x+5}{x+1}$

Domain: $x \neq -1$

x-intercept(s): $(-5/2, 0)$

y-intercept(s): $(0, 5)$

H.A.: $y = 2$

V.A.: $x = -1$

S.A.: NONE

Hole(s): NONE

In 2 - 3, find the partial fraction decomposition of each.

2. $\frac{-5x+4}{x^2-x} = \frac{-5x+4}{x(x-1)}$ $\boxed{\frac{-4}{x} - \frac{1}{x-1}}$

$\frac{A}{x} + \frac{B}{x-1}$

$Ax - A + Bx$

$A+B=-5$

$-A=4$

$A=-4$

$B=-1$

3. $\frac{-7x-15}{x^2+6x+9}$

$(x+3)(x+3)$

$(x+3)^2$

$\frac{A}{x+3} + \frac{B}{(x+3)^2}$

$A(x+3)+B$

$Ax+3A+B$

$A=-7$

$3A+B=-15$

$-21+B=-15$

$B=6$

$\frac{-7}{x+3} + \frac{6}{(x+3)^2}$

Chapter 3

In 4 - 6, evaluate each expression WITHOUT A CALCULATOR.

4. $\frac{\log_{12} 12^{36}}{\log_4 4^{18}}$ $= \frac{36}{18} = 2$

5. $\ln e^{5a}$

$5a$

6. $\log_4 320 - \log_4 5$

$\log_4\left(\frac{320}{5}\right)$

$= \log_4 (64) = 3$

7. Use the change of base formula to evaluate: $\log_5 7$

$\frac{\log 7}{\log 5} = 1.2091$

8. Use the properties of logarithms to expand: $\ln \frac{\sqrt{x^3 y^2}}{z}$.

$\frac{3}{2}\ln x + \ln y - \ln z$

9. Use the properties of logarithms to expression the following expression as a single logarithm:

$3\ln(x-2) + 2\ln(x+2)$

$\ln (x-2)^3(x+2)^2$

In 10 - 12, solve each equation algebraically. When necessary, round your result to the nearest thousandth.

10. $3^{2x} - 5 = 9$

$3^{2x} = 14$

$\frac{\log_3 14}{2} = \frac{2x}{2}$

$1.201 = x$

11. $3 + \log_2 3x = 5$

$\log_2 3x = 2$

$4 = 3x$

$\frac{4}{3} = x$

12. $\log(x) + \log(x-21) = 2$

$\log x(x-21) = 2$

$10^2 = x^2 - 21x$

$x^2 - 21x - 100 = 0$

$(x-25)(x+4) = 0$

$x = 25$ $\cancel{x = -4}$

13. The number of bacteria present in culture $N(t)$ at time t hours is given by $N(t) = 3000(2)^t$.

a. What is the initial population?

3000

b. How much bacteria are present after 24 hours? 5.033×10^{10}

c. How long will it take the population to triple in size? 1.58 hours

14. The number of students infected with flu after t days at Washington High School is modeled by the following function: $$P(t) = \frac{1600}{1+99e^{-0.4t}}$$

a. What was the initial number of infected students? 16

b. After 5 days, how many students will be infected? 111.13 $\Rightarrow$ 111 students

c. What is the maximum number of students that will be infected? 1600

15. The number of bacteria in a cup of water is modeled by a logistic curve. The limit to growth of the bacteria is 3500. The initial bacteria count is 100. After 3 hours, the bacteria count rises to 1450. Write the logistic function of the bacteria count.

$c = 3500$ $\quad (3, 1450) \quad 100 = \frac{3500}{1+a} \quad 1450 = \frac{3500}{1+34b^3}$

$y = \frac{c}{1+ab^x} \quad (0, 100) \quad 100 + 100a = 3500$

$100a = 3400$

$a = 34$

$1450(1+34b^3) = 3$

$1+34b^3 = 2.4$

$b = .346$

$y = \frac{3500}{1+34(.346)^x}$

Chapter 4, Part I

16. Convert the angle measure from degrees to radians.

a.) $-270° \cdot \frac{\pi}{180} = -\frac{3\pi}{2}$

b.) $144° \cdot \frac{\pi}{180}$ $\quad \frac{4\pi}{5}$

17. Convert the angle measure from radians to degrees.

a.) $\frac{7\pi}{3} \cdot \frac{180}{\pi} = 420°$

b.) $\frac{-13\pi}{60} \cdot \frac{180}{\pi} = -39°$

18. a.) If the Earth rotates once every 24 hours, find the angular speed in radians/hour.

1 rev/24 hrs $\quad \frac{1\text{ rev}}{24\text{ hrs}} \cdot \frac{2\pi\text{ rad}}{1\text{ rev}} = \frac{2\pi\text{ rad}}{24\text{ hrs}} = .2618\text{ rad/hr}$

b.) If a fan rotates 30 times in a minute, find the angular speed in radians/hour.

$\frac{30\text{ rev}}{\text{min}} \cdot \frac{2\pi\text{ rad}}{1\text{ rev}} \cdot \frac{60\text{ min}}{1\text{ hr}} = 11309.73\text{ rad/hr}$

c.) If a ferris wheel rotates 4 times per minute, find the angular speed in radians/second.

$\frac{4\text{ rev}}{\text{min}} \quad \frac{2\pi\text{ rad}}{1\text{ rev}} \cdot \frac{1\text{ min}}{60\text{ sec}} = .4189\text{ rad/sec}$

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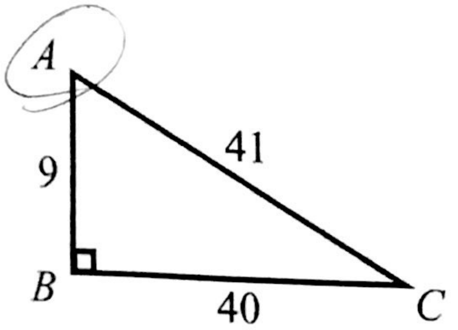
19. Find the six trigonometric ratios of $\angle A$.

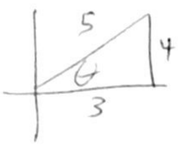


$\sin A = \frac{40}{9}$ $\csc A = \frac{9}{40}$

$\cos A = \frac{9}{41}$ $\sec A = \frac{41}{9}$

$\tan A = \frac{40}{9}$ $\cot A = 9/40$

20. Given $\sin\theta = \frac{4}{5}$ in Quadrant I, find the remaining 5 trig ratios.



$\cos\theta = 3/5$ $\csc\theta = 5/4$ $\cot\theta = 3/4$

$\tan\theta = \frac{4}{3}$ $\sec\theta = 5/3$

21. Given $\csc\theta = \frac{17}{4}$ in Quadrant I, find the remaining 5 trig ratios.

$\cos\theta = \frac{\sqrt{273}}{17}$ $\sec\theta = \frac{17\sqrt{273}}{17}$ $\cot\theta = \frac{\sqrt{273}}{4}$

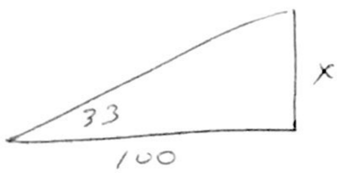
$\tan\theta = \frac{4\sqrt{273}}{273}$ $\sin\theta = 4/17$

22. Use a calculator to evaluate each function.

a.) $\sin 41^\circ$ b.) $\cot 71.5^\circ$ c.) $\cot\frac{\pi}{16}$ d.) $\tan\frac{\pi}{8}$

.6561   .3346   5.0273   .4142

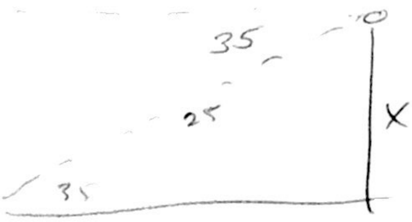
23. John wants to measure the height of a tree. He walks exactly 100 feet from the base of the tree and looks up. The angle from the ground to the top of the tree is 33°. How tall is the tree?



$\tan 33 = \frac{x}{100}$

$x = 64.94\text{ft}$

24. A bird sits on top of a lamppost. The angle of depression from the bird to the feet of an observer standing away from the lamppost is 35°. The distance from the bird to the observer is 25 meters. How tall is the lamppost?



$\sin 35 = \frac{x}{25}$

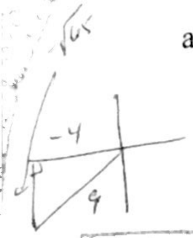
$x = 14.34\text{ft}$

Determine two co-terminal angles (one positive and one negative) for each angle.

a.) $\theta = 52^\circ$ 412   -308

b.) $\theta = \dfrac{7\pi}{8}$ $\dfrac{23\pi}{8}$ $-\dfrac{9\pi}{8}$

26. Find the indicated trigonometric value in the specified quadrant.



a.) $\sec\theta = -\dfrac{9}{4}$; QIII; $\tan\theta$

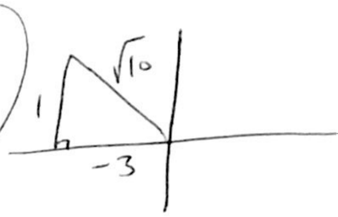
H, A

$\dfrac{\sqrt{65}}{4}$

b.) $\cot\theta = -3$; QII; $\sin\theta$

A/o

$\dfrac{\sqrt{10}}{10}$



Chapter 4, Part II

27. Find the period and amplitude.

a.) $y = 3\sin 2x$

$P = \pi$ $A = 3$

b.) $y = \dfrac{2}{3}\sin \pi x$

$P = 2$ $A = 2/3$

c.) $y = \dfrac{3}{4}\cos\dfrac{\pi}{12}x$

$P = \dfrac{2\pi}{\pi/12} = \dfrac{2\pi}{1}\cdot\dfrac{12}{\pi}$

$P = 24$

$A = 3/4$

28. Identify the transformation from f to g.

a.) $f(x) = \sin x$
$g(x) = -4\sin x$

Reflection x-axis
Vertical stretch x4

b.) $f(x) = \cos x$
$g(x) = -\cos(x - \pi)$

Right π
Rx

c.) $f(x) = 4\sin \pi x$
$g(x) = 4\sin \pi x - 2$

↓ 2

29. Find the max and min.

a.) $y = 3\sin x$

Max: 3
Min: -3

b.) $y = \dfrac{1}{2}\sin(x - \pi)$

Max: $\frac{1}{2}$
Min: $-1/2$

30. Graph the following:

BoTo13

a.) $y = 3\sin 2x - 1$ OToBO

$P = \pi$ $\frac{\pi}{4}$ = Intervals

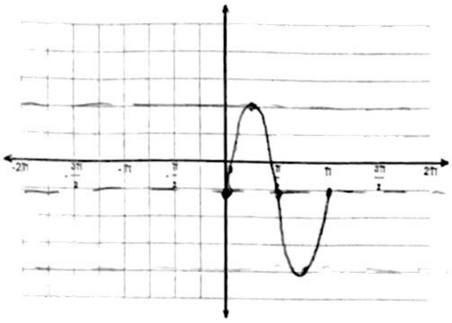
b.) $y = -\cos(2x + \pi)$

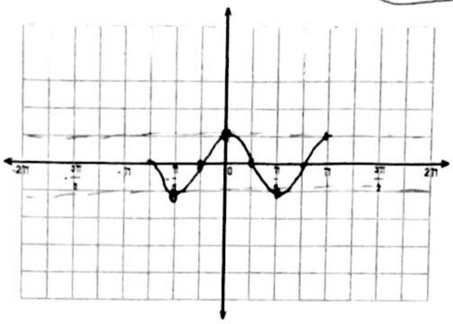
P.S. $P = \pi$ $2x + \pi = 0$

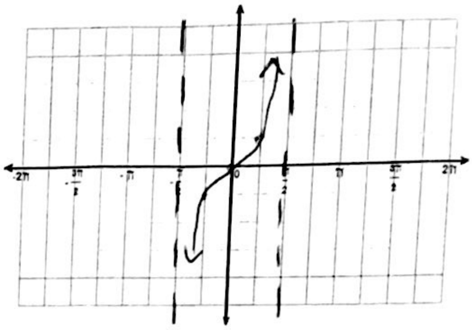
Inter: $\frac{\pi}{4}$ $2x = -\pi$

$x = -\pi/2$

c.) $y = \tan x$

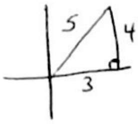




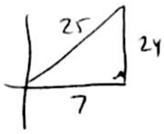


31. Find the exact value of the expression.

a.) $\sin\left(\arctan\frac{4}{3}\right)$ $\frac{4}{5}$

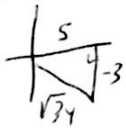


b.) $\cos\left(\arcsin\frac{24}{25}\right)$ $\frac{7}{25}$



c.) $\sec\left(\arctan\left(-\frac{3}{5}\right)\right)$

$\frac{\sqrt{34}}{5}$



32. Find the exact value of y without a calculator.

a.) $y = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

b.) $y = \arctan(1)$ $\frac{\pi}{4}$

Chapter 5

33. Simplify the expression: $\cos\theta - \cos\theta\sin^2\theta$.

$\cos\theta\,(1 - \sin^2\theta)$

$\cos\theta \cdot \cos^2\theta = \cos^3\theta$

34. Simplify the expression: $\dfrac{\cos^2 x + \sin^2 x}{\cot^2 x - \csc^2 x}$.

$\dfrac{1}{\frac{\cos^2\theta}{\sin^2\theta} - \frac{1}{\sin^2\theta}} = \dfrac{1}{\frac{\cos^2\theta - 1}{\sin^2\theta}} = \dfrac{1}{\frac{-\sin^2\theta}{\sin^2\theta}} = -1$

35. Simplify the expression: $\cos x + \sin x \tan x$. → $\cos x + \sin x \cdot \frac{\sin x}{\cos x}$

$\dfrac{\cos^2 x + \sin^2 x}{\cos x} = \dfrac{1}{\cos x} = \sec x$

36. Factor: $\sin^2 x + \sin x - 2$.

$(\sin x + 2)(\sin x - 1)$

37. Simplify the expression: $\dfrac{\sin^2 x - 1}{1 + \sin x}$.

$\dfrac{-\cos^2\theta}{1 + \sin x} \cdot \dfrac{1 - \sin x}{1 - \sin x} = \dfrac{-\cos^2\theta\,(1 - \sin x)}{1 - \sin^2\theta} = \dfrac{-\cancel{\cos^2\theta}(1 - \sin x)}{\cancel{\cos^2\theta}}$

$= -(1 - \sin x) = \sin x - 1$

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