

Bounded Intervals

Let a and b be real numbers with $a < b$. The numbers a and b are the endpoints of each interval.

Interval Notation	Interval Type	Inequality Notation	Graph
$[a, b]$	closed	$a \leq x \leq b$	
(a, b)	Open	$a < x < b$	
$[a, b)$	Half open closed-open	$a \leq x < b$	
$(a, b]$	Half open open-closed	$a < x \leq b$	

Unbounded Intervals

Let a and b be real numbers. Each of these intervals has exactly one endpoint, a or b .

Interval Notation	Interval Type	Inequality Notation	Graph
$[a, \infty)$	closed	$x \geq a$	
(a, ∞)	Open	$x > a$	
$(-\infty, b]$	closed	$x \leq b$	
$(-\infty, b)$	open	$x < b$	

Using Interval Notation

Write each interval as an inequality.

1.) $[-3, 9)$ $-3 \leq x < 9$

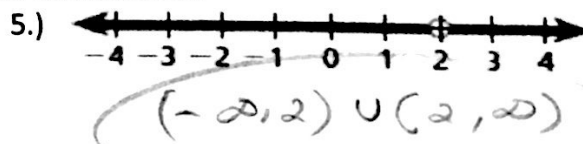
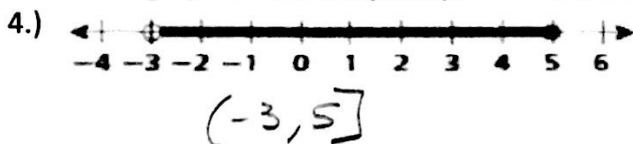
2.) $[-3, \infty)$

$-3 \leq x$

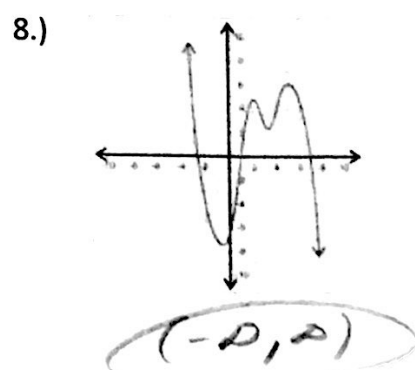
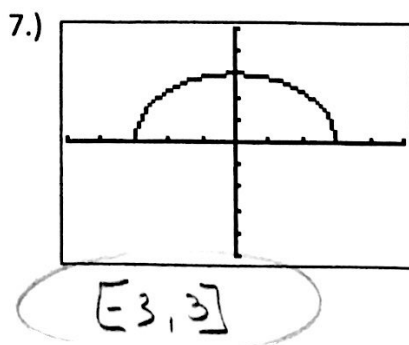
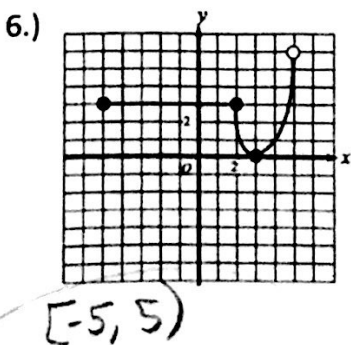
3.) $(-\infty, 4) \cup [5, \infty)$

$x < 4$ ~~and~~ $x \geq 5$
or

Given the graph of the inequality, write the domain in interval notation.

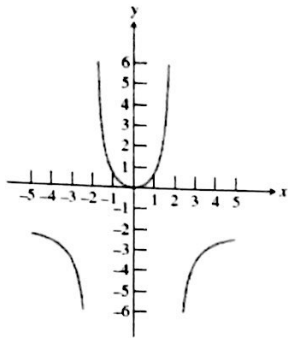


Given each graph, write the domain in interval notation.



Domain: Find the domain of each function.

9.)

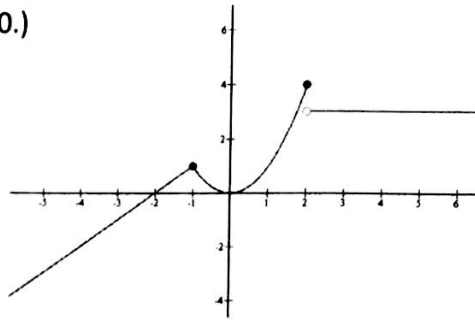


$$(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$$

11.) $f(x) = \frac{\sqrt{x}}{x-5}$ $x \geq 0$
 $x \neq 5$

$$[0, 5) \cup (5, \infty)$$

10.)



$$(-\infty, \infty)$$

12.) $f(x) = \frac{\sqrt{4-x}}{(x+1)(x^2+1)}$ $x \neq -1$

$4-x \geq 0$
 $4 \geq x$

$$(-\infty, -1) \cup (-1, 4]$$

Increasing & Decreasing Intervals: Identify intervals where the function is increasing or decreasing.

13.) $f(x) = (x+2)^2$

I: $(-2, \infty)$

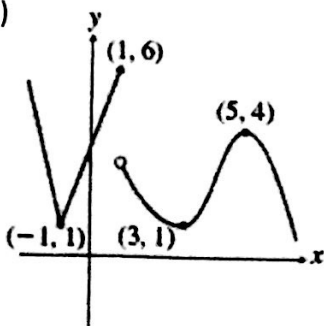
D: $(-\infty, -2)$

14.) $f(x) = \frac{x^2}{x^2-1}$

I: $(-\infty, -1)$, $(-1, 0)$

D: $(0, 1)$, $(1, \infty)$

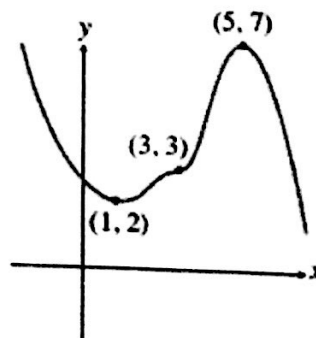
15.)



I: $(-1, 1)$, $(3, 5)$

D: $(-\infty, -1)$, $(1, 3)$, $(5, \infty)$

16.)



I: $(1, 5)$

D: $(-\infty, 1)$, $(5, \infty)$

a.) State the domain of the function in interval notation.

$$[-5, \infty)$$

b.) State the range of the function in interval notation.

$$(-\infty, 3]$$

c.) Find $f(-3)$.

$$2$$

d.) Find the intervals on which the function is decreasing.

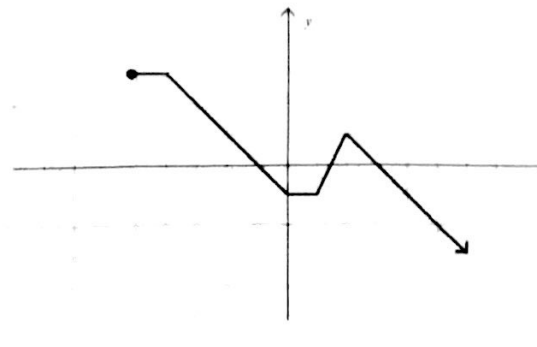
$$(-4, 0) \quad (2, \infty)$$

e.) Find the intervals on which the function is increasing.

$$(1, 2)$$

f.) Find the intervals on which the function is constant.

$$(-5, -4) \quad (0, 1)$$



Find the domain of each function algebraically. Show your work.

18.) $h(x) = \sqrt{3-x}$

$$3-x \geq 0$$

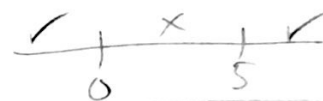
$$3 \geq x$$

$$x \leq 3$$

$$(-\infty, 3]$$



19.) $p(x) = \sqrt{x^2 - 5x}$ $\sqrt{x(x-5)}$



$$(-\infty, 0] \cup [5, \infty)$$

20.) $r(x) = \frac{1}{x^2 - 4x - 12}$ $x = 6, -2$

$$\frac{1}{(x-6)(x+2)}$$

$$(-\infty, -2) \cup (-2, 6) \cup (6, \infty)$$

21.) $k(x) = x^3$

$$(-\infty, \infty)$$

Prove algebraically that the function is even, odd, or neither.

22.) $p(x) = -3x^2 + 4$

$$p(-x) = -3x^2 + 4$$

$$p(-x) = p(x)$$

even

23.) $p(x) = 2x^3 - 3x^2 - 4x + 4$

$$p(-x) = -2x^3 - 3x^2 + 4x + 4$$

$$p(-x) \neq p(x)$$

$$p(-x) \neq -p(x)$$

neither

Prove algebraically that the function is even, odd, or neither.

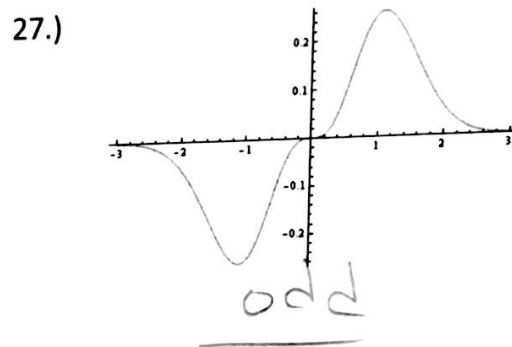
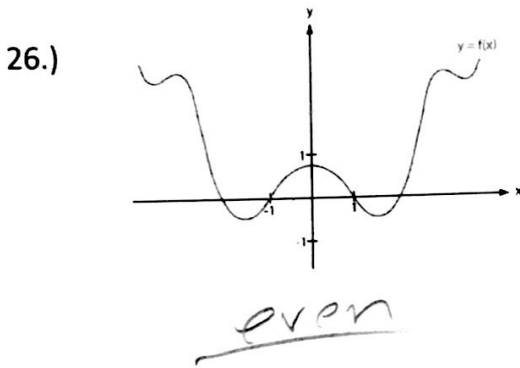
24.) $h(x) = |x| + 4$

$h(-x) = |-x| + 4$
 $h(-x) = h(x)$
even

25.) $t(x) = 2x^3 - 4x$

$t(-x) = -2x^3 + 4x$
 $t(-x) = -t(x)$
odd

Determine whether the graph represents a function that is even, odd, or neither.



Find all asymptotes.

28.) $f(x) = \frac{5x^2}{x^2 - 1}$

$x = 1$
 $x = -1$
 $y = 5$

29.) $f(x) = \frac{3x - 1}{x^2 - 3x + 2}$

$x = 2$
 $x = 1$

$y = 0$

Use the twelve basic functions to answer the questions.

30.) Domain

- Nine of the functions have domain the set of all real numbers. Which three do not?
 Rational function, Logarithm, square root
 *(Reciprocal)
- One of the functions has domain the set of all reals except 0. Which function is it, and why isn't zero in its domain?
 reciprocal* rational function
 when $x = 0$, denominator = 0.
- Which two functions have no negative numbers in their domains? Of these two, which one is defined at zero?
 Square root, logarithm
 square root is defined at 0.

31.) Continuity

- Only two of the twelve functions have points of discontinuity. Are these points in the domain of the function?

reciprocal
rational function ; no \rightarrow continuous in domain

greatest integer function ; yes. \rightarrow discontinuous in domain

32.) Boundedness

\rightarrow explain, not in video.

- Only three of the twelve basic functions are bounded (above and below). Which three?

cosine, sine, logistic

33.) Symmetry

- Three of the twelve basic functions are even. Which are they?

cosine, quadratic, absolute value.
(squaring)

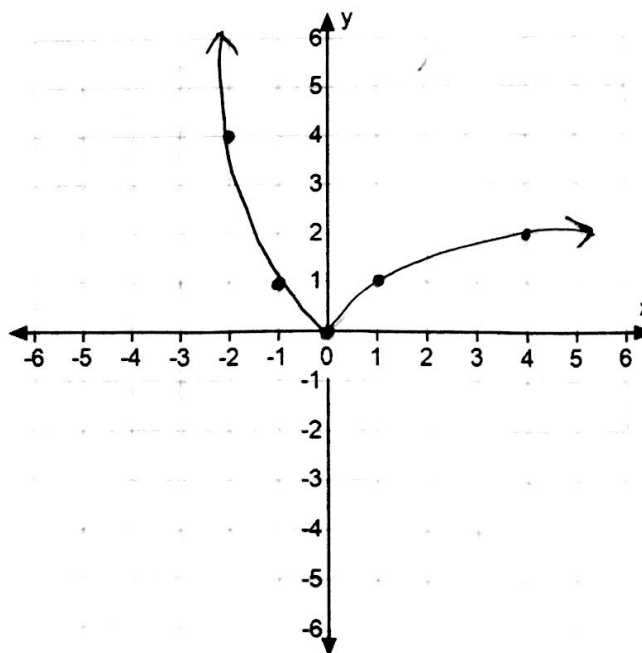
34.) Piecewise Function

- Which of the twelve basic functions has the following piecewise definition over separate intervals of its domain?

$$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases} \quad \text{absolute value.}$$

- Sketch the following function:

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 0 \\ \sqrt{x} & \text{if } x > 0 \end{cases}$$

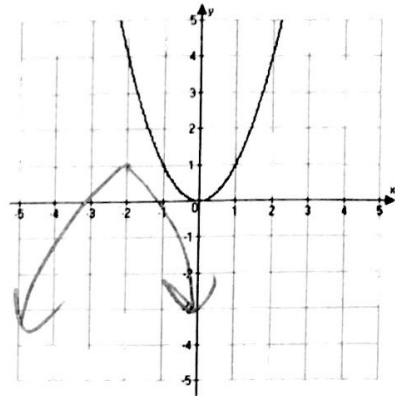
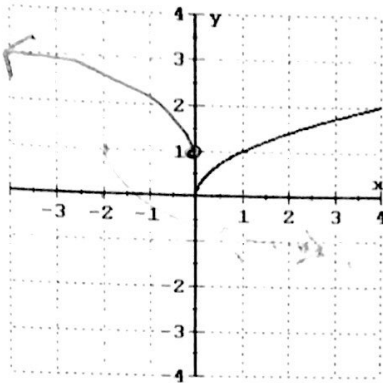


35.) Without using a calculator, sketch the graph of the new functions.

a.) Given: $f(x) = \sqrt{x}$ Graph: $f(x) = \sqrt{-x} + 1$

b.) Given: $f(x) = x^2$ Graph:

$f(x) = -(x+2)^2 + 1$



36.) Write the equation for $g(x) = x^3$, which is changed by the following transformations:

- a.) Horizontal translation to the right 3 units
- b.) Vertical stretch by a factor of 2
- c.) Reflection in the x-axis
- d.) Vertical translation downward 1 unit

$g(x) = -2(x-3)^3 - 1$

37.) Answer the three parts of the question.

- Part 1: Identify the common function f .
- Part 2: Use function notation to write h in terms of f .
- Part 3: Describe the transformation from f to h .

a.) $h(x) = -(x+10)^3 + 5$

$f(x) = x^3$

$h(x) = -f(x+10) + 5$

R_x , Left 10, up 5

b.) $h(x) = |4-x| + 7$

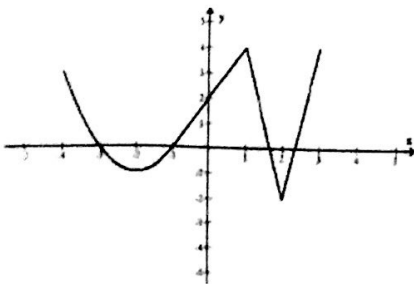
$f(x) = |x|$

$h(x) = f(-x+4) + 7$

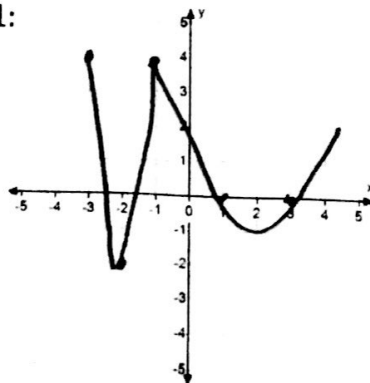
R_y , Right 4, up 7

38.) Given $f(x)$, graph $f(-x) - 2$ in two steps.

Given:



Step 1:



Step 2:

